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Economic Evaluation of End User Contracts from Electricity Retailer Companies Viewpoint

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ABSTRACT

As the structure of power networks changes, a new group of companies one of which is retailers have stepped into the field of the electricity market. One of the major problems of the retailers is to contract with the suppliers and the end users in a way that the risks are minimized. Risk is the uncertainty in the price paid by the retailer to supply power. This paper analyzes the retailer contracts with both supplier and end user to maximize the retailer profit; holding the risk at an acceptable level. The proposed stochastic optimization model helps retailer to choose the optimal strategy in terms of price and value of buying and selling electricity in order to gain the maximum profit. One of the most costly problems usually faced by retailers and distribution companies is the losses issue, and the losses allocated to the participants are considered in model through the proportional sharing method. The proposed model is implemented in the network of 24 bus IEEE (RTS) and the results are then evaluated.

KEYWORDS: Electricity market, retailing, pricing, losses allocation

1. INTRODUCTION

Retailers on one side contract with electricity suppliers to provide the electricity with a fixed price and on the other side; they contract with the end users to meet their power needs. In electrical energy supply, retailer strategy should help determine how much power should be provided through bilateral contracts and how much through the spot market, with regard to bilateral contracts and spot market interactions. In [1], the retailers' strategies selected unreasonably for future load determination (considering over estimation and under estimation) are analyzed hourly for one year interval using the Mont-Carlo simulation and the best strategy, maximizes the retailers profit, was determined. In [2], the probable optimization method used to maximize the profit of the retailers considering the real market data PJM was presented, assuming that the operation of the retailer is specified before the program is executed. The final model was formulated in form of a mixed integer linear program with binary variables, which comes from considering the allowed limits for load variations of retailer's end users. The end user can provide his electricity requirements through three methods: purchasing from the spot market, through the bilateral contracts, and producing by his own facilities. In [3], a combination of all techniques was presented in order to supply the end user power in medium term (of a few months) in a way that the electricity supply cost of the end user is minimized. It was assumed that the end user load is known and the load is modeled without considering the uncertainty. In addition, it was supposed that the retailer suffers no risk in this case.

Reference [4] as well as [3] discussed about supplying electrical energy for a large end user with difference that in [4] the risk is also modeled. In other words, the consumer aims to minimize the costs of supplying electrical energy, so that the risk of costs come from electricity price fluctuation are limited at the spot market. Supplying electric energy for a large end user considering the risk is also discussed in [5], with a difference that the stochastic programming was applied to solve the problem.

Supplying the electric energy for a local distribution company was discussed in [6]. The electricity is supplied by spot markets or by the bilateral contracts. In [6], the aim was to minimize supplying electric energy costs considering risk. What is carried out in this paper is similar to [3], but the statistical approaches are applied there to conduct the analysis. In addition, the internet-based multi-round auction method is proposed for bilateral contracts since they are non-standard. It is important to note that in [3] the load uncertainty is not modeled and the load is intended to be a specific load. In [7], the topic of risk management in long-term contracts of retailer purchasing is under consideration. Here, the amount of electrical energy that must be provided through any source considering the load in uncertain form and estimating the electricity price in long-term contracts and spot markets (daily) is determined. In addition, the statistical techniques were applied in [7] for the optimization. The topic of electricity purchase by buyer from the Nordic market was discussed in [8]. It wa aimed to minimize the electricity purchase costs assuming that the energy should be supplied by the day-ahead market or the regulation

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market. This analysis was based on the assumption that market distribution function is clarified. The market distribution function expresses the probability of electricity purchase for the buyer in different prices. The estimation of this curve was investigated in [9].

In [10] a model of Time-of-Use (TOU) pricing for a retail electricity market on Pareto Improvement method was proposed based on the short-term marginal cost, considering the effect of consumer response by price elasticity matrix of electricity demand. In [11] a risk-constrained stochastic programming framework was proposed to decide which forward contracts the retailer should sign and at which price it must sell electricity so that its expected profit is maximized at a given risk level. In [12] a multistage stochastic optimization approach was developed which accounts for the uncertainties of both electricity prices and loads, and which permits the specification of conditional-value-at-risk requirements to optimize hedging across intermediate stages in the planning horizons. The result of [12] has shown that a risk neutral retailer is more susceptible to price-related than load-related uncertainties in terms of the expected cost of satisfying the load, and that a risk averse retailer is especially sensitive to the drivers of the forward risk premium.

The most complete topics raised in retailing field were discussed in [13]. Here, the optimum price and amount of retailing contracts were determined. Nevertheless, the losses cost was not implied. In continuous, the losses allocation methods and the technique applied in this paper are initially mentioned. The problem model is obtained in section 3. In sections 4, 5, and 6, the proposed network and the model implementation results are presented and the conclusion comes finally in section 7.

2. Losses Allocation

In this section, different losses allocation techniques are compared. Several techniques are applied for losses allocation. The most usual of which are the following four methods:

a. Pro Rata Method
b. Incremental Allocation Method
c. The Loss Formula or the Circuit Based Method
d. Proportional Sharing method

Currently, some electricity markets such as Spain and Brazil apply the Pro Rata method to allocate the losses between the market participants [14], whereas in markets such as Australia and New Zealand the Incremental Allocation method is applied. This paper utilizes the Proportional Sharing allocation method, which would be detailed in continuous.

2.1. The Proportional Sharing Allocation Method

The Proportional Sharing method initially presented by J. Bialek created a fundamental change in losses allocation process. Bialek presented a topological tracing method considers each bus as an ideal node so that the power passes outside of the bus can equal to the sum of the powers passed into the bus. This allows the load demands to track the generators' output (upstream looking) or track the generator output to load (downstream looking) where the first method is applied in this paper.

2.2. The Upstream Looking Algorithm

Assume that it is possible to break up total transmission losses into its particles i.e. the losses associated with each specific load. The sum of real demand of a specific load in addition to the associated allocated transmission losses is called the gross demand. It is obvious that the total gross demand of the system equals to total real generation. Now, if P_i^g is considered as the unknown gross power streams to i^{th} bus and P_{ij}^g is considered as the unknown power passes through *i-j* line, they both would flow in the network if the network is considered to have no losses (the gross demand is considered to equal to real power generation). This would lead to losses less power flow with equal gross currents at the beginning and at the end of each line. Generally, the gross power balance equation of i^{th} bus can be defined as follows if the bus is seen from input bus viewpoint:

$$P_i^g = \sum_{j \in \alpha_i^u} \left| P_{ij}^g \right| + P_{Gi} \qquad for \;\; i = 1,2,...,n \qquad (1)$$

where α_i^u is the sum of the buses fed by i^{th} bus directly, or in other words the power in the related line should flow towards i^{th} bus and P_{Gi} is the power generated in i^{th} bus. The flown power $\left|P_{ij}^g\right|$ can be replaced by

$$\left(\frac{\left|P_{ij}^g\right|}{P_j^g} \right) P_j^g \;\; \text{since} \left|P_{ij}^g\right| = \left|P_{ji}^g\right|.$$

In normal conditions, the transmission losses are small and it is possible to assume that: $\left|P_{ji}^g\right| \Big/ P_j^g \cong \left|P_{ji}\right| \Big/ P_j$

where P_{ji} is the real power passes from j^{th} bus in *j-i* line and P_j is total real power flown in j^{th} bus. These equations are based on the assumption that the gross power flow in each bus is similar to the real flown power. Relation (1) can be changed as follow due to the mentioned assumptions:

$$A_u P_{gross} = P_G \text{ or } \quad P_i^g - \sum_{j \in \alpha_i^u} \frac{\left|P_{ji}\right|}{P_j} P_j^g = P_{Gi} \qquad (2)$$

where P_{gross} is the unknown vector of the gross power flows towards the i^{th} bus, P_G is the generation vector of the buses, and A_u is the upstream-looking distribution matrix, i,j^{th} element of which is as follows:

$$[A_u]_{ij} = \begin{cases} 1 & for \quad i = j \\ -\left|P_{ij}\right| / P_j & for \quad j \in \alpha_i^u \\ 0 & otherwise \end{cases} \qquad (3)$$

It is necessary to mention that A_u is sparse and non-symmetric. If A_u^{-1} exists, $P_{gross}=A_u^{-1}$ and its i^{th} element equals to the follows:

$$P_i^g = \sum_{k=1}^{n} \left[A_u^{-1}\right]_{ik} P_{Gk} \qquad i = 1,2,...,n \qquad (4)$$

This equation well shows how all system generators provide the gross power of the i^{th} bus. On the other hand, from the perspective of the power flows outside the bus, the similar P_i^g, equals to the sum of all gross powers flown outside the i^{th} bus. Therefore, the gross power passes through ***i-j*** line can be calculated as follows applying the proportional sharing method:

$$P_{ij}^g = \frac{P_{ij}^g}{P_i^g} P_i^g = \frac{P_{ij}^g}{P_i^g} \sum_{k=1}^{n} \left[A_u^{-1}\right]_{ik} P_{Gk} = \sum_{k=1}^{n} D_{ij,k}^g P_{Gk} \quad for \quad j \in \alpha_i^d \qquad (5)$$

where α_i^d is the set of the buses directly fed by i^{th} bus and $D_{ij,k}^g = P_{ij}^g \left[A_u^{-1}\right]_{ik} / P_i^g$. This equation defines $D_{ij,k}^g$ as the topological generation distribution factor, where a share of the generation belongs to the k^{th} generator provided on *i-j* line. In other words, it shows the share of a specific generator in all lines' current flow. The gross demand in i^{th} bus can be presented as follows:

$$P_{Li}^g = \frac{P_{Li}^g}{P_i^g} P_i^g = \frac{P_{Li}}{P_i} P_i^g = \frac{P_{Li}}{P_i} \sum_{k=1}^{n} \left[A_u^{-1}\right]_{ij} P_{Gk} \qquad (6)$$

where P_{Li} shows the net demand of the i^{th} bus. This equation is very important since it shows the amount of the load demand in an assumed bus when a losses less network is fed by a real generation. Therefore, the difference between gross demand and the real one is the losses associated with the power flow from all generators towards a specific load and can be expressed as follows:

$$\Delta P_{Li} = P_{Li}^g - P_{Li} \qquad (7)$$

In other words, the upstream-looking algorithm not only allows the operator to determine the participation rate of each generator in a specific load's demand satisfaction but it is possible to allocate total transmission losses to the specific loads of the network. This is an important result, through which one can calculate the share of each specific load in real lost power amount of the network.

3. Modeling the Problem

3.1. Assumptions of the Problem

Followings are assumed for the proposed model:

a. The major aim of the retailer is to maximize its profit considering the risk associated with uncertainties.
b. Electricity is purchased through the bilateral contracts from the spot market.
c. Retailer is able to sell its surplus electricity to day-ahead market or purchase its electricity shortage from it.
d. The factors influence the probability of accepting retailer's offer and generally, the acceptance function of retailer and its subscribers are considered to be known.
e. The upper and the lower limits of the prices offered to the subscribers are considered to be known in order to more close the model to the reality. In addition, the retailer is prevented to purchase load in high amount in order to adapt the model with the reality and to prevent retailers to suffer high level of losses due to the spot decrease of the prices and the purchased load is considered to be in a high level.
f. It is assumed that there exist many suppliers and the retailer chooses the supplier according to the economical considerations.
g. The risk management problem is taken in to account in optimum retailing strategy determination.

The indices, variables, and the information applied in this paper are introduced in continuous and the problem is then modeled by introducing the functions applied in paper.

3.2. Indices

t: time period (for example day, hour, and etc)
T: a set of considered periods
b: blocks of time periods
$B(b)$: set of time periods in block b
δ_{tb}: the marker function, which equals to 1 if $t \in B(b)$ and equals to zero if not
Ω: Possible realization sets of the random variables
ω: the realization of Ω

It should be noted that the sets of periods can consist of hours, or groups of hours (such as day, week, etc), which depends on its special application. In addition to these periods, there are blocks of periods, which are applied on end users and suppliers' prices. For example, if the periods are hours, the blocks can be divided into peak and non-peak hours, which are used to diagnose the prices. Therefore, there exists a mapping for each block belongs to the block consists of periods associated with that block. For example, consider a time horizon of six hours. Assume T={1,2,…,6} with two blocks, where B(1)={1,2,6} is the first hours block and B(2)={3,4,5} is the second block. Therefore, the marker function can be considered as a 6*2 matrix, the transpose of which is shown in Table 1.

Table 1: different values of marker function

t=	1	2	3	4	5	6
b=1	1	1	0	0	0	1
b=2	0	0	1	1	1	0

3.3. Variables

L^f_b: future load estimation of retailer to purchase from the supplier
p_b^{eu}: the price of the contract exists between the retailer and the end user ($/MWh)
u^1_t, u^2_t, u^3_t: the variables of the end user side sum of which is the difference between the middle estimation value and the real load and on *t*, parts *1*, *3* are punished (Fig.1). The function is discontinuous and has a jump from zero to a positive point.
v^1_t, v^2_t, v^3_t: the binary variables associated with u^1_t, u^2_t, u^3_t.

3.4. Information

$L_t^a(w)$: end user's real load
$\pi(\omega)$: probability mass function of the stochastic events
p_b^{su}: the price of the contract between retailer and the supplier
$p_t^{mcp}(\omega)$: the spot price of the market
B_t: benchmark load of the end user load
σ_{tt}^2: covariance exists between stochastic variables p_t^{mcp} and p_t^{mcp}. If $t = \tilde{t}$, σ_{tt}^2 is the variance of stochastic variable of p_t^{mcp}.
s^1_t, s^3_t: positive slopes of punishments of sections 1, 3 in figure related to the end user's bandwidth
$\underline{p}_b^{eu}, \overline{p}_b^{eu}$: the positive upper and lower limits of the contract with the end user
M: the time period value (very large)
N: the positive constant value

3.5. Functions

3.5.1. Final End User Bandwidth Functions

In a contract between the retailer and the final end user, there exists a bandwidth limit, which is shown in Fig.1. Here, bandwidth means the permissible deviation of end user load around a certain level. If the final end user consumes too more than the given benchmark load, a charge goes to the final end user. On equal terms and as the contract price with the end user exceeds to a higher level, the contract conditions should be beneficial to the end user. Therefore, $\underline{\alpha}(p_b^{eu}), \overline{\alpha}(p_b^{eu})$ functions are proposed in terms of MWh, which show the upper and lower limits of the end user contract price. As the end user contract price is higher, these functions operate in a way that the probability of end user punishment is decreased. It is important to note that the *x*-axis shows the difference between the predicted and the real end user load i.e. $B_t - L_t^a(\omega)$.

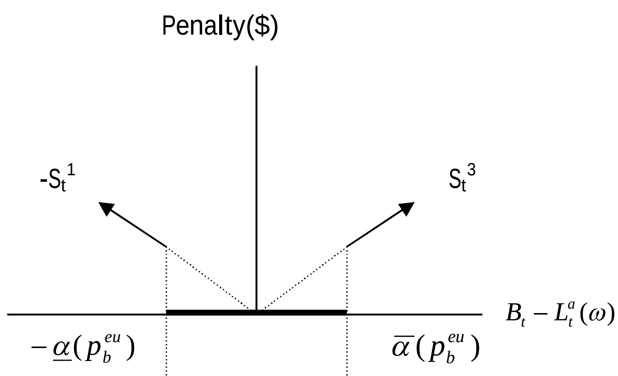

Figure 1: The final end user bandwidth functions

3.5.2. Acceptance Function of supplier and End User

From a retailer viewpoint, offer to end user with high price decreases the acceptance chance, which is because of the fact that the end user chooses the retailer with the lowest price in equal condition. Therefore, it is initially essential for retailer to have a proper processing and strategy for end user behavior and contract with the supplier then.

The acceptance function of A_b^{eu} (which is function of $R_+ \rightarrow [0,1]$) is considered to achieve the above aim. This function can show the market share of that company. This function shows the possibility if the end user accepts the retailer offered price p_b^{eu} . It is important to note that the $A_b^{eu}(p_b^{eu})$ function should be subtractive. In addition, parameter $A_b^{su}(p_b^{su})$ is the rate of price acceptance by supplier. In this investigation, this price is considered constant to simplify the calculations.

3.5.3. Calculating the Probability Distributions

Some points should be under consideration to estimate the load and the price probability distributions:

First, the prices and the loads are generally correlated. Therefore, on a specific hour, that price or that load considerably depends on the former hours. Separate probable different distributions are determined for load and the price depending on their value in former hours (low, middle, and high). Blended with the fact that depends on the season and the peak/non-peak, 24 different distributions for load and 24 distributions for the price are considered.

The other important point for probable distributions determination of the price and the load is the fact that these values can relate to each other and therefore the correlated distributions are required. According to the previous investigations, the load and the price do not depend on each other. This is due to several reasons. First, the load is one of the characteristics of the retailer, while the price is based on the market features. High amount of load for a retailer does not mean high market price, because it depends on the behavior of the rest of the market. This is because many of the retailers have not enough power to influence the price. Of course, it is possible to consider more relation between the price and the load if there are very large retailers. The second is the fact that the spot price depends on some other parameters apart from retailer load such as wholesalers' generation. Therefore, the influence of a retailer is negligible. In any case, the best option is to apply separate distributions.

Based on the real information, the logarithmic normal distribution is applied for the spot price and the triangular distribution is applied for the load. The logarithmic normal distribution is selected based on the fact that the prices of *h*, *h-1*, P_h, and P_{h-1} hours are considered as $P_h = P_{h-1}(1+\varepsilon_h)$, where $\{\varepsilon_h\}$ is an independent set of random variables and is significantly small and is resulted in asymptotic logarithmic normal distribution form. Other distributions are considered due to their own fitting and consequently the triangular distribution is chosen. The triangular distribution is appropriate when the information is minimum i.e. just the minimum, maximum and the mode values are in hand.

3.6. Limitations

The retailer faces with two groups of limitations explained in continuous.

a. The end user bandwidth limitations

As mentioned before, the overall deviation between the real and the estimated load of the end user is broken in to three variables in order to realize ω.

$$B_t - L_t^a(\omega) = u_t^1(\omega) + u_t^2(\omega) + u_t^3(\omega) \qquad (8)$$

where $u_t^1(\omega) \le 0$, $u_t^3(\omega) \ge 0$ and $u_t^2(\omega)$ can be positive, negative or zero, but at least one of the *u* variables would be nonzero. In this relation, $v_t^1(\omega)$, $v_t^2(\omega)$, $v_t^3(\omega)$ binary variables are utilized so that if $B_t - L_t^a(\omega)$ falls in any region of *u*, the associated *v* variable gains 1 and gains zero if not. Therefore three modes occur.

a.1. $B_t - L_t^a(\omega) < -\underline{\alpha}(p_b^{eu})$ falls outside of the bandwidth

a.2. $B_t - L_t^a(\omega) > \overline{\alpha}(p_b^{eu})$ falls outside the bandwidth

a.3. $\underline{\alpha}(p_b^{eu}) \le B_t - L_t^a(\omega) \le \overline{\alpha}(p_b^{eu})$ the estimated load is in range of the real load, and therefore the followings are valid:

- If $B_t - L_t^a(\omega_L) \le \underline{\alpha}(p_b^{eu}) - \varepsilon$, then $u_t^1(\omega_L) = B_t - L_t^a(\omega_L)$ and $u_t^2(\omega_L) = u_t^3(\omega_L) = 0$.
- If $B_t - L_t^a(\omega_L) \ge \overline{\alpha}(p_b^{eu}) + \varepsilon$ then $u_t^3(\omega_L) = B_t - L_t^a(\omega_L)$ and $u_t^1(\omega_L) = u_t^2(\omega_L) = 0$.
- If $\underline{\alpha}(p_b^{eu}) \le B_t - L_t^a(\omega) \le \overline{\alpha}(p_b^{eu})$ then $u_t^2(\omega_L) = B_t - L_t^a(\omega_L)$ and $u_t^1(\omega_L) = u_t^2(\omega_L) = 0$.

Choosing $\varepsilon > 0$ significantly small, results in small intervals of $[-\underline{\alpha}(p_b^{eu}) - \varepsilon, -\underline{\alpha}(p_b^{eu})]$ and $[\overline{\alpha}(p_b^{eu}), \overline{\alpha}(p_b^{eu}) + \varepsilon]$, while 1, 3 modes can be expressed as $B_t - L_t^a(\omega_L) \le \underline{\alpha}(p_b^{eu}) - \varepsilon$ and $B_t - L_t^a(\omega_L) \ge \overline{\alpha}(p_b^{eu}) + \varepsilon$ due to the practical reasons as well as illustrated in Fig.2.

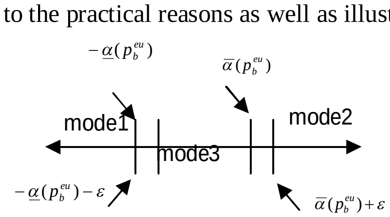

Figure 2: 1,2,3 modes drawing

The rest of the end user bandwidth limitations are expressed as follows:

$$-Mv_t^1(\omega) \le u_t^1(\omega) \le 0, \quad \forall t,\omega \tag{9}$$

$$u_t^1(\omega) + \underline{\alpha}(p_b^{eu}) \le M(1 - v_t^1(\omega)) - \varepsilon, \quad \forall t,\omega \tag{10}$$

$$-\underline{\alpha}(p_b^{eu}) \le u_t^2(\omega) \le \overline{\alpha}(p_b^{eu}), \quad \forall t,\omega \tag{11}$$

$$-Mv_t^2(\omega) \le u_t^2(\omega) \le -Mv_t^2(\omega), \quad \forall t,\omega \tag{12}$$

$$0 \le u_t^3(\omega) \le Mv_t^3(\omega), \quad \forall t,\omega \tag{13}$$

$$-u_t^3(\omega) + \overline{\alpha}(p_b^{eu}) \le M(1 - v_t^3(\omega)) - \varepsilon, \quad \forall t,\omega \tag{14}$$

$$v_t^1(\omega)\,,\, v_t^2(\omega)\,,v_t^3(\omega) \in \{0,1\}, \quad \forall t,\omega \tag{15}$$

$$v_t^1(\omega) + v_t^2(\omega) + v_t^3(\omega) = 1, \quad \forall t,\omega \tag{16}$$

b. Prices Limitations

A positive upper and a positive lower limit should be applied on L_b^f, p_b^{eu} variables in order to achieve a more realistic model. Therefore, the followings are valid:

$$\underline{p}_b^{eu} \le p_b^{eu} \le \overline{p}_b^{eu}, \quad \forall b \tag{17}$$

$$0 \le L_b^f \le \overline{L}_b^f, \quad \forall b \tag{18}$$

3.7. Target Function

It is obvious that this problem can be modeled just if the supplier and the end user contract together with the retailer in which the real load of the end user equals to $L_t^a(\omega)A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})$. Therefore, the expected income of the retailer is as $p_b^{eu}L_t^a(\omega)A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})$ in t time interval for ω realization. The retailer expected cost for purchasing from the supplier for ω realization equals to $p_b^{su}L_b^f A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})$.

In addition, because $P_{Li} \approx L_t^a(\omega)$, the losses allocated the final end-user equals to the follows:

(19)

where the A_u matrix configuration pattern is explained before. Since the load rate of the final end user is not known until the network loading (real time), the losses cost is also calculated on spot price base. Therefore, the losses cost equals to the follows:

$$\frac{L_t^a(\omega)}{P_i}\sum_{k=1}^{n}\left[A_u^{-1}\right]_{ij}P_{Gk} - L_t^a(\omega) = L_t^a(\omega)\left\{\frac{1}{P_i}\sum_{k=1}^{n}\left[A_u^{-1}\right]_{ij}P_{Gk} - 1\right\}$$

$$p_t^{mcp}(\omega)L_t^a(\omega)\left\{\frac{1}{P_i}\sum_{k=1}^{n}\left[A_u^{-1}\right]_{ij}P_{Gk} - 1\right\} \tag{20}$$

Therefore, the final target function is obtained as follows considering the losses:

$$\begin{aligned}
&Max_{p_b^{eu},L_b^f,\ \forall b\in B,t\in T}\\
&\sum_t\sum_b\sum_\omega \delta_{tb}p_b^{eu}L_t^a(\omega)A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})\pi(\omega)\\
&-\sum_t\sum_b \delta_{tb}p_b^{su}L_b^f A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})\\
&+\sum_t\sum_b\sum_\omega \delta_{tb}(-s_t^1u_t^1(\omega)+s_t^3u_t^3(\omega))\\
&\times A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})\pi(\omega)\\
&+\sum_t\sum_b\sum_\omega \delta_{tb}\,p_t^{mcp}(\omega)\left[L_b^f - L_t^a(\omega)\right]\\
&\times A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})\pi(\omega)\\
&-N\sum_t\sum_b\sum_{\tilde t}\sum_{\tilde b}\sum_\omega \Big\{\Big[\delta_{tb}\left[L_b^f - L_t^a(\omega)\right]\\
&\times A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})\Big]\\
&\times\sigma_{t\tilde t}^2\Big[\delta_{\tilde t\tilde b}\left[L_{\tilde b}^f - L_{\tilde t}^a(\omega)\right]\\
&\times A_{\tilde b}^{eu}(p_{\tilde b}^{eu})A_{\tilde b}^{su}(p_{\tilde b}^{su})\Big]\pi(\omega)\Big\} - \sum_t\sum_b\sum_\omega \delta_{tb}\,p_t^{mcp}(\omega)L_t^a(\omega)\left\{\frac{1}{P_i}\sum_{k=1}^{n}\left[A_u^{-1}\right]_{ij}P_{Gk} - 1\right\}\\
&\times A_b^{eu}(p_b^{eu})A_b^{su}(p_b^{su})\pi(\omega)
\end{aligned} \tag{21}$$

The forth term is the charge expected from the market and is based on the spot price and can be profit or the cost, which depends on market price and on the fact if the retailer estimation is higher or lower than the end user side real load. The fifth term relates to the risk. This risk prevents the retailer from buying and selling high amounts of energy in the market, which would decrease the risk value. Generally, this may decrease the profit value beside the risk reduction. Parameter N is determined according to the strategy and the power of the retailer about the risk. Overall and due to the generalities of the market, the profit of the retailer is resulted from the sum of all above-mentioned issues.

4. The Sample Network

The 24 bus (RTS) IEEE is chosen as the test network. This network possesses 24 buses, 38 lines and 14 power plants. In this network, the load peak is 2850 MW and the installed capacity is 3405 MW [12].

Now, assuming that the retailer load (final end user) stands on bus 4 and the supplier is considered bus 13, the results are presented in section 5.

5. The Results of the Proposed Model and their Comparison with the Losses less Model Results

In this section, the results of the proposed model –considering the losses- are compared with that of this model under losses less condition. It is essential to note that the first row of the Tables 2-6 are the purchase price from the supplier in non-peak condition and the first column of these tables are the purchase price from the supplier in peak condition, all in terms of $/MWh. In Table 2, the results of optimum selling to the end user values in peak condition resulted from the proposed model and the losses less model are compared. The first row of the table shows the price of purchase from the supplier variations in non-peak condition and the first column shows the price of purchase from the supplier variations in peak condition. For example, when the retailer purchases the load for 33$/MWh in peak condition and 20$/MWh in non-peak condition from the supplier, the optimum price of selling to the end user considering no losses equals to 44.1$/MWh and equals to 44.569$/MWh considering losses. Here, the peak price considering losses is increased up to 1.1% (0.47$/MWh). The maximum increase occurs in $p^{su}_{peak} = 31(\$/MWh)$ and the least increase occurs in $p^{su}_{peak} = 35\,(\$/MWh)$. Likewise, in Table 3, the results of optimum selling to the end user price value in non-peak condition, which are resulted from the proposed model and the previous model, are presented. Here, the non-peak price considering the losses has meanly 0.93% (0.23$/MWh) increased. Maximum increase occurs in $p^{su}_{off-peak} = 18(\$/MWh)$.

Table 2: Comparison of end user price in peak condition considering the losses and neglecting the losses ($/MWh)

	18		19		20		21		22	
	lossless	with loss	Lossless	with loss	lossless	with loss	lossless	with loss	lossless	with loss
31	43.1	43.575	43.07	43.548	43.05	43.521	43.02	43.494	42.99	43.468
32	43.63	44.1	43.6	44.073	43.58	44.047	43.55	44.02	43.52	43.994
33	44.15	44.622	44.13	44.596	44.1	44.569	44.07	44.542	44.05	44.516
34	44.67	45.141	44.65	45.114	44.62	45.088	44.59	45.061	44.57	45.035
35	45.19	45.656	45.16	45.63	45.14	45.603	45.11	45.577	45.08	45.551

Table 3: Comparison of end user price in non-peak condition considering losses and neglecting the losses ($/MWh)

	18		19		20		21		22	
	Lossless	with loss	lossless	with loss	lossless	with loss	lossless	with loss	lossless	with loss
31	24.054	24.286	24.619	24.851	25.183	25.415	25.746	25.978	26.308	26.54
32	24.054	24.287	24.619	24.851	25.183	25.415	25.746	25.978	26.308	26.54
33	24.055	24.287	24.62	24.852	25.183	25.416	25.747	25.978	26.309	26.541
34	24.055	24.288	24.62	24.852	25.184	25.416	25.747	25.979	26.309	26.541
35	24.056	24.288	24.62	24.853	25.184	25.416	25.747	25.979	26.31	26.541

Table 4: Comparison of the load purchased in peak condition considering losses and neglecting the losses ($/MWh)

	18		19		20		21		22	
	Lossless	with loss	lossless	with loss	lossless	with loss	lossless	with loss	lossless	with loss
31	80.44	80.697	80.56	80.814	80.66	80.93	80.78	81.044	80.88	81.158
32	79.89	80.123	80.01	80.249	80.13	80.373	80.25	80.496	80.36	80.617
33	79.29	79.491	79.42	79.627	79.54	79.761	79.67	79.894	79.79	80.025
34	78.62	78.78	78.76	78.929	78.9	79.076	79.03	79.221	79.16	79.364
35	77.86	77.965	78.01	78.13	78.16	78.293	78.31	78.454	78.46	78.612

Table 5: comparison of the load purchased in non-peak condition considering losses and neglecting the losses ($/MWh)

	18		19		20		21		22	
	Lossless	with loss	lossless	with loss	lossless	with loss	lossless	with loss	lossless	with loss
31	61.72	62.082	51.567	51.503	40.064	39.404	26.538	24.999	9.932	7.025
32	61.881	62.253	51.736	51.684	40.244	39.596	26.731	25.207	10.144	7.255
33	62.049	62.43	51.912	51.87	40.431	39.796	26.933	25.424	10.364	7.494
34	62.223	62.613	52.095	52.064	40.624	40.002	27.141	25.647	10.593	7.741
35	62.402	62.803	52.283	52.264	40.825	40.216	27.357	25.879	10.83	7.997

Table 6: Comparison of the retailer profit considering losses and neglecting losses ($)

	18		**19**		**20**		**21**		**22**	
	Lossless	With loss	lossless	With loss	lossless	With loss	lossless	With loss	lossless	with loss
31	20985.7	18929.2	17809.1	17427.9	17246.3	16320.5	17246.3	15633.7	17246.3	15394
32	19793.2	17799.5	16629.1	16292.4	16031.1	15179.2	16031.1	14486.6	16031.1	14241.1
33	18653.4	16726.3	15507.5	15213.2	14867.7	14094	14867.7	13395.4	14867.7	13144
34	17572.6	15715.9	14450.9	14196.5	13762.4	13071.1	13762.4	12366.4	13762.4	12108.7
35	16557	14774.5	13465.9	13248.7	12721.6	12116.9	12721.6	11405.7	12721.6	11141.7

6. Influence of the Network Topology on the Losses

The major problem of the pro rata and incremental allocation methods is the fact that the losses are distributed just based on the generated or the consumed power level and the losses allocated to loads is independent from their positions in the network and from the power transmission path from supplier to the end user. In this section, the influence of the load position and the power transmission path from supplier to end user on the losses allocated to it is investigated in two case studies. It is assumed once that the resistances of the 33, 32, 26, 25 and 38 lines (lines connected to bus 21) are doubled (Case1). The other time, it is assumed that the resistance of the 5 lines connected to buses 4 and 13 are doubled (Case 2). The results of these two cases are presented in Tables 7-9. As it is obvious, the results of Case 1 do not significantly differ from the losses less condition. Here, the profit of the retailer is meanly 30$ decreased and the price in peak and non-peak conditions is meanly 0.005$/MWh increased, which are considerably small. Here, the purchased load variations are also very small.

However, the results of the Case 2 show a more significant difference in compare with Case1 in losses less condition, this illustrates the influence of the network topology and power transmission path from the supplier to the end user on the allocated losses. Here, the profit of the retailer is meanly 1300$ decreased and the price in peak is more than 0.3$/MWh and in non-peak more than 0.2$/MWh increased. As mentioned before, the variables and different limitations of target function such as acceptance function, end user punishment function (bandwidth), risk, spot price, estimated load, etc, prevent the price to increase or load to increase/decrease and consequently, the profit of the retailer is much decreased. In this Case, similar to the previous one, the variations of the purchased load are not large.

Table 7: Comparison of the end user price in peak condition for Case1 & Case2 ($/MWh)

	Case 1	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**
	18	18	19	19	20	20	21	21	22	22
31	43.58	43.88	43.56	43.85	43.53	43.83	43.5	43.8	43.48	43.77
32	44.11	44.41	44.08	44.38	44.05	44.35	44.03	44.33	44	44.3
33	44.63	44.93	44.6	44.9	44.58	44.87	44.55	44.85	44.52	44.82
34	45.15	45.44	45.12	45.42	45.1	45.39	45.07	45.36	45.04	45.34
35	45.66	45.96	45.64	45.93	45.61	45.91	45.58	45.88	45.56	45.85

Table 8: Comparison of the purchased load in peak condition for Case1 & Case2 ($/MWh)

	Case 1	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**
	18	18	19	19	20	20	21	21	22	22
31	80.7	80.88	80.82	81	80.93	81.12	81.05	81.24	81.16	81.35
32	80.13	80.29	80.25	80.42	80.38	80.55	80.5	80.68	80.62	80.8
33	79.49	79.64	79.63	79.78	79.77	79.92	79.9	80.06	80.03	80.2
34	78.78	78.9	78.93	79.05	79.08	79.21	79.22	79.36	79.37	79.51
35	77.97	78.04	78.13	78.22	78.3	78.39	78.46	78.56	78.61	78.73

Table 9: Comparison of the retailer profit for Case1 & Case2 ($/MWh)

	Case 1	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**	**Case 1**	**Case 2**
	18	18	19	19	20	20	21	21	22	22
31	18895.31	17500.59	17394.74	16035.51	16288.21	14970.56	15602.35	14332.37	15363.73	14147.49
32	17766.65	16412.45	16260.27	14941.5	15147.95	13870.71	14456.33	13226.7	14211.96	13036
33	16694.55	15383.27	15182.15	13906.24	14063.83	12829.39	13366.23	12179.34	13115.9	11982.63
34	15685.31	14419.33	14166.68	12936.01	13042.14	11852.88	12338.35	11196.58	12081.84	10993.63
35	14745.15	13526.81	13220.07	12036.98	12089.11	10947.38	11378.91	10284.61	11116.02	10075.22

Conclusions

Losses cost, which is not well investigated in previous researches is one of the problems retailers as well as the distribution companies usually face with. In this paper, the problem associated with the losses is completely considering the network structure is modeled and analyzed. The results of the proposed model –considering losses- are compared with that of this model neglecting losses. Since the functions, data, and many variables such as acceptance function, end user punishment function (bandwidth), risk, spot price, estimated load, etc play role in target function these issues can face off the retailer with price increase limitation and load decrease/increase. For example, the retailer profit decreases considering losses from 1.6% in 19^{th} and 35^{th} buses until 9.8% in 18^{th} and 31^{st} buses in compare with the losses less condition. In this paper, the influence of the load position in the network, the network topology, and the power transmission path from the supplier towards the end user on the allocated losses are investigated through two case and it is shown that the resistance increase of the lines hold more final end user load distribution factor, considerably affect the network losses. In this paper, just the risk associated with the load uncertainty is under consideration and it is possible to add other risk factors such as financial risks, market behavior risk, and the legislator risk. In addition, it is possible to consider the final end user in the proposed model as different classes of end users holding different load model.

REFERENCES

[1] S. A. Gabriel, M. F. Genc, and S. Balakrishnan, "A simulation approach to balancing annual risk and reward in retail electrical power markets", IEEE Trans. Power Syst., vol. 17, no. 4, pp. 1050–1057, Nov. 2002.

[2] S. A. Gabreil, "A mixed integer stochastic optimization model for settlement in retail electric power markets", Network and Spatial Economics, vol. 4, pp. 323-345, 2004.

[3] A.J. Conejo, J.J. Fernandez-Gonzalez, and N. Alguacil, "Energy procurement for large consumers in electricity markets", IEE Proc.-Gener. Transm. Distrib., vol. 152, no.3, 2005.

[4] A.J. Conejo, and M. Carrion, "Risk-constrained electricity procurement for a large consumer", IEE Proc.-Gener. Transm. Distrib., vol. 153, no.4, 2006.

[5] M. Carrion, A.B. Philpot, A.J. Conejo, and J. M. Arroyo, "A stochastic programming approach to electric energy procurement for large consumers", Submitteed to IEEE Trans. Power Syst.

[6] C.K. Woo, R. I. Karimov, and I. Horowitz, "Managing electricity procurement cost and risk by a local distribution company", Energy Policy (32), pp. 635-645, 2004.

[7] M. Zhou, G. Li, Y. Nie, and Y. Ni, "Risk management on long-term electricity purchase allocation in China power markets", IEEE PEDS 2005 Conference, pp. 909-914, 2005.

[8] A. B. Philpot, and E. Pettersen, "Optimizing demand-side bids in day-ahead electricity markets", IEEE Trans. Power Syst., vol.21, no.2, pp. 488-498, 2006.

[9] E. J. Anderson, and A. B. Philpot,"Estimation of electricity market distribution functions", Oper. Res., vol. 50, no. 3, pp. 21-32, 2003.

[10] L. Yingchen, C. Lu, C. Xingying, "An Efficient Time-of-Use Pricing Model for a Retail Electricity Market Based on Pareto Improvement", Power and Energy Engineering Conference (APPEEC), 2011 .

[11] M. Carrión, Antonio J. Conejo, and J. M. Arroyo, "Forward Contracting and Selling Price Determination for a Retailer", IEEE Trans. Power Syst., vol. 22, no. 4, Nov 2007.

[12] J. Kettune, A. Salo, and D. W. Bunn, "Optimization of Electricity Retailer's Contract

Portfolio Subject to Risk Preferences", IEEE Trans. Power Syst.,, Vol. 25, NO. 1, Feb 2010.

[13] S. A. Gabriel, , Antonio J. Conejo, , Miguel A. Plazas, and S. Balakrishnan "optimal price and quantity determination for retail electric power contracts", IEEE Trans. Power Syst., vol. 21, no. 1, Feb 2006.

[14] Janusz Bialek, "topological generation and load distribution factors for supplement charge allocation in transmission open access" Power Systems, IEEE Transactions on, vol.12, No. 3, August 1997