Multi Scale Systematic Risk
(An Application on Tehran Stock Exchange)

Roghayeh Torki Samaei
Department of Economics, science and research, Islamic Azad University, Tehran, Iran

ABSTRACT

The capital asset pricing model (CAPM) states that the expected return on an asset depends upon its level of systematic risk. The asset’s systematic risk is measured relative to that of the market portfolio. This paper attempts to estimate the CAPM at different time scales. In this study, we use wavelets analysis, as the key empirical method for examining the relationship between the return of the stock and its systematic risk at different time scales. The proposed procedure is acted on a sample, composed of 15 selected stocks, listed on Tehran Stock Exchange (TSE) actively traded over 2004-2009. It has proved that the relationship between the return of a stock and its beta is more robust at medium and short scales. This evidence shows that the Tehran Stock Exchange market is more efficient at the 1st to 4th scales (2 – 32 days). This finding, therefore, shows that the predictions of the CAPM are more relevant at short and medium-term horizon and in a multi-scale framework as compared to the other horizons.

KEYWORDS: Wavelets, CAPM, systematic risk, TSE, scaling.

1. INTRODUCTION

Because of the particular importance of the factor of risk in financial investments, this factor and the relationship between risk and expected return has always been under scrutiny by the experts of financial markets. Since risk constitutes one of the major factors that influence the expected return of financial assets, the stock managers and other investors need to evaluate the degree of sensitivity of their financial assets to risk. For the optimization of their financial assets in proportion to the risk level, these individuals are always trying to search for and identify, and also measure and control, those factors that affect the return of investment. The relationship between the risk and return on a financial security has been in use is the capital asset pricing model (CAPM) which states that the expected return on an asset depends upon its level of systematic risk. Asset risk is segmented into two components: systematic (diversifiable) and unsystematic (nondiversifiable) risk. The popular measure of systematic risk is the CAPM beta. The systematic risk explains how a stock acts in relation to the market. The asset’s systematic risk is measured relative to that of the market portfolio. The model was developed by Sharpe (1964), Lintner (1965a, b) and Mossin (1966), which follows logically from the Markowitz (1952) mean-variance portfolio theory.

Most of the latest empirical researches on CAPM reasonably confirm the tenet of the model. Fama and French, (2004) found linear relationship between expected return and beta. They stated that the beta premium is positive, and asset which does not have any correlation with the market has expected returns equal to the risk-free interest rate. Black, Jensen and Sholes (1972) found that Higher returns should be expected from stocks that have higher beta. Kothari, Shanken and Sloan (1995) found that there are a strong relationship between beta and returns. They used annual returns instead of monthly ones for computing betas over the period 1927-1990. Fama and French (1992) reported that there is no relationship between beta and returns over the period 1963-1990 in the US stock market. Hawawin, Mayer, Schwartz and Witcomb (1986) and Handa, Kothari and Wasley (1989; 1993) implied that according to the time scale, systematic risk coefficient would change. In the research by Fernandez (2006), through the use of wavelet analysis, the relationship between the return of stocks and the systematic risk in at different time scales has been investigated for 24 stocks at the Santiago Stock Exchange. The results of their investigation favor medium-term investment models for the Santiago Stock Exchange. For the calculation of the systematic risk in the U.S. stock market, Gencay, Whitcher and Selcuk (2003) used the wavelet analysis to explore the relationship between the return of stocks and systematic risk at different time scales. According to the results of their article, the association between the return of a stock and its beta gets stronger as the time scale increases. Therefore, the model has more favorable predictions for medium-term and long-term investment horizons. In the research conducted by Rhaeim, Ben Ammou and Ben Mabrouk (2007) on the French stock market, they used the wavelet analysis to investigate the relationship between the return of stocks and systematic risk in the capital asset pricing model, at

*Corresponding Author: Roghayeh Torki Samaei, Department of Economics, science and research, Islamic Azad University, Tehran, Iran Tel: +98191 3250728 E-mail: roghayeh.torki.samaei@yahoo.com
different time scales. Their results indicate that at the short-term and long-term scales, the association between the stock return and the level of systematic risk is stronger; therefore, the French stock market is more efficient for short-term and long-term investments. In another research by Aktan, Mabrouk and Rhaiem (2009) on Istanbul stock exchange, they found that the relationship between the return of a stock and its beta is more robust at medium scale. Their result indicate that the Turkish stock market is more efficient in the 3rd scale (8 – 16 days). In this study, by using the data of 15 actively-traded stocks at the Tehran Stock Exchange, the wavelet analysis (which is employed in the modeling of financial data) has been applied as a new method in the estimation of CAPM at different time scales. On this basis, besides estimating the beta associated with those 15 active stocks, the existing relationship between the stock’s beta and the average risk premium of the stock at different time scales is evaluated in order to calculate the risk premium of the market at different time scales. This task is carried out in order to determine the scales at which CAPM predictions are more appropriate in the Tehran Stock Exchange. This article has been organized as follows: in section two, brief theoretical principles of CAPM, wavelet analysis, and wavelet variance and covariance have been presented; in section three, the data and variables used in the research have been introduced; in section four, the obtained results have been analyzed and discussed; and finally section five, concludes the final results of the paper.

2- Theoretical background and method
2.1- The capital asset pricing model

Most conventional asset pricing theories are consumption-based models, which gained under the assumption of rational utility maximization of all agents under uncertainty. Consumption-based models falls into the general equilibrium markets paradigm under the low of one price and no arbitrage theorems. The fundamental idea under asset pricing is that asset price equals the sum of all discounted future cash flows and investors aim to have a stable consumption flows (marginal utility growth) in the future. Historically, theoretical framework of asset pricing is developed under the theories of portfolio selection (mean-variance efficiency frontier), Beta representation (CAPM), and stochastic discount factor, which are essentially the same idea with different forms. Stochastic discount factor is a more generalized form to express the theory of consumption-based asset pricing and can be easily applied to different securities as a special case with explicit model. CAMP (Sharp, Linter) (Sharpe 1964) (Lintner 1965) as a factor pricing model (consumption-based) has been intensively empirical tested by both academic and practitioners. In empirical modeling, additional modeling assumptions are imposed on the properties of asset prices behavior. Stationarity and normality are two basic assumptions on empirical data. Rapid growth of computer technology and easy access financial data allowed researchers to examine whether various theoretical asset pricing models are support by statistical studies. In the mean time stylized facts revealed in empirical data influence the development of new theoretical models. Black, Jensen and Scholes (1972) (Black, Jensen et al. 1972) performed the earliest rigorous tests of the CAPM. They found that: "the cross sectional plots of the mean excess returns on the portfolios against the estimated betas indicate that the relation between mean excess return and beta was linear". This is potentially consistent with CAPM". The intercept and the slope of the cross-sectional relation varied with different sub periods and were not consistent with the traditional form of the capital asset pricing model." Fama and MacBeth (1973) (Fama and MacBeth 1973) and Blume and Friend (1973) (Blume and Friend 1973) performed tests that their results are not consistent with the Treynor-Sharp-Lintner-Mossin CAPM, however, could be explained by the zero-beta version of the CAPM. Roll’s (1977) (Roll 1977) critique demonstrates that: The market, as defined in the theoretical CAPM, is not a single equity market, but an index of all wealth. The market index must include bonds, property, foreign assets, human capital and anything else, tangible or intangible that adds to the wealth of mankind. Roll points out that "the portfolio used by Black, Jensen and Scholes was certainly not the true portfolio". Roll shows that unless this market portfolio is known with certainty then the CAPM never could be tested. So the testing of CAPM is impossible. Cox, Ingersoll and Ross (1985) (Cox, Ingersoll et al. 1985) derived an explicit rational expectations equilibrium model that endogenously calculates the price of assets in an economy with production. Their works linked the interest rate and the yield curve to the fundamentals of the economy. The CAPM states that the expected return on all assets is a function of covariance of the market portfolio. The CAPM implies that the expected return on any risky assets follows the equation:

\[ E(R_i) = \beta_i \left[ E(R_m) - r_f \right] + r_f \]

(1)

where, \( (R_i) \) is the return on asset \( i \), \( R_f \) is the risk-free rate, \( R_m \) is the return on the market portfolio, and

\[ \beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} = \frac{\sigma_{im}}{\sigma_{mm}} \]

(2)

is the asset’s beta. The variance of the return on asset \( i \) would be:
\[ \sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\epsilon i}^2 \]  
(3)

which says the total risk of an asset (variance) can be divided into systematic risk \( \beta_i^2 \sigma_m^2 \) and unsystematic risk \( \sigma_{\epsilon i}^2 \).

2.2. Wavelets analysis

Wavelet transformation provides an effective way to deal with some non-stationary properties of stochastic processes. The basic idea of wavelets method is to decompose a time series into a series of coefficients through a group of wave-like functions. The group of wave-like functions is dyadic translated from a single mother wavelet, which has certain required mathematic properties, e.g. usually they are orthogonal functions with unity energy. Each set of wavelet coefficients represent a piece of the time series in different time-scale and frequency-scale adaptively and the series of wavelet coefficients preserves the energy of original time series. Wavelet analysis is a refinement of Fourier analysis. Fourier Transform (FT) processes the raw signal (time series) by using a mathematic transformation, which transforms the signal from time domain into frequency spectrum. Processed signal tells us how many frequencies and how much (energy) of each frequency exists in the raw signal but it does not give us the time information. If the signal is stationary, information is not needed but in real world most of our data sets, especially in financial and economic data, are non-stationary. One way to treat non stationary time series before wavelets is Windowed Fourier Transform (WFT). WFT can locate the time, which only transform part of a signal and that segment of signal is small enough that we can assume that portion of signal is stationary. By using a particular window function and shifting the window along the time dimension of the signal we can localize the frequency in the signal, and then we obtained a time-frequency representation of the signal. The transformation coefficients are the amplitudes of different frequencies at different time. Continuous Wavelet Transform (CWT) uses a particular wavelet, which has some required properties, as the window function (same logic in WFT). There are two main different between CWT and WFT. First one is that in CWT we use a wavelet to replace the cosine in WFT, which will give us much spikes information in the decomposed signal. Second one, the most significant characteristic, is the width of the window is changed for different frequencies. As in WFT, the windowed signal multiplied with window function then integral across time continuously. Continuous transformation contains highly redundant information because we decompose the time series from time domain into frequency and time domain adding one dimension. Discrete Wavelet Transform (DWT) discards the signal sample by two each time according to Nyquist’s rule, which tells us that at lower frequencies (scale) the sampling rate can be decreased, and then orthogonal DWT decomposes the signal at different frequency bands (scale) with different resolution (shifting) and in each frequency band the existing frequency localized in a particular time according to that band’s resolution. The orthogonal wavelet series approximation to a continuous time signal \( f(t) \) is given by

\[ f_j(t) = \sum_k S_{j,k} \Phi_{j,k}(t) + \sum_k d_{j,k} \Psi_{j,k}(t) + \cdots + \sum_k d_{J,k} \Psi_{J,k}(t) \]  
(4)

\( J \) is the number of multiresolution components, levels (scales), and \( k \) (shift) ranges from 1 to the number of coefficients in the specified component. \( \Phi \) and \( \Psi \) are the approximating orthogonal wavelet functions given by

\[ \Phi_{j,k}(t) = 2^{-j/2} \Phi\left(\frac{t - 2^j k}{2^j}\right) \]  
(5)

\[ \Psi_{j,k}(t) = 2^{-j/2} \Psi\left(\frac{t - 2^j k}{2^j}\right) \]

Wavelet coefficients are given by

\[ d_{j,k} = \int \Psi_{j,k}(t) f(t) dt; \quad j = 1, \ldots, J \]

\[ s_{j,k} = \int \Phi_{j,k}(t) f(t) dt \]  
(6)

2.3. Wavelet variance and covariance

The analysis of wavelet variance includes the decomposition of the variance of series that are continuous at different time scales. This substantiates the fact that the scales play an important role in the overall fluctuations of a time series (Percival and Walden, 2000). If \( (X_1, X_2, \cdots, X_n) \) is the considered time series with a stationary process and variance of \( \sigma^2 \), and if \( v^2(\tau_j) \) is the decomposition of the wavelet variance at scale \( \tau_j = 2^{j-1} \), then relation (7) is established:
\[ \sigma^2_X = \sum_{j=1}^{\infty} \sigma^2_X(\tau_j) \] (7)

The unbiased estimator of the wavelet variance at scale \( \tau_j \) for the market return \( R_{mt} \) is expressed as relation (8):

\[ \hat{\sigma}^2_{R_m}(\tau_j) \equiv \frac{L}{(n_j - L_j')2^j} \sum_{t=L_j'}^{n_j-1} d^2_{jt} \] (8)

In relation (8), \( L \) is the wavelet filter width. \( n_j = n/2^j \) is the number of transformation coefficients of the discrete wavelet at level \( j \). \( n \) is the sample size. \( L_j' = (I - \frac{L}{2^j})(L - 2) \) is the number of boundary coefficients of the discrete wavelet transformation, provided that \( n_j > L_j' \) (Gencay, Selcuk and Whitcher, 2002).

The unbiased wavelet covariance between the market return of \( R_m \) and stocks return of \( R_i \) at scale \( \tau_j \) is expressed as relation (9) (Gencay, Selcuk and Whitcher, 2002):

\[ \hat{\sigma}^2_{R_m R_i}(\tau_j) \equiv \frac{L}{(n_j - L_j')2^j} \sum_{t=L_j'}^{n_j-1} d^R_m d^R_i \] (9)

The beta wavelet estimator for the asset of \( i \) at scale \( \tau_j \) is defined as relation (10) (Gencay, Whitcher and Selcuk, 2003):

\[ \hat{\beta}_i(\tau_j) = \frac{\hat{\sigma}^2_{R_m R_i}(\tau_j)}{\hat{\sigma}^2_{R_m}(\tau_j)} \] (10)

3- Data and variables
3.1- Stock return

In this study, the data related to the daily return of stocks \( R_i \) of 15 active companies in the Tehran Stock Exchange, for a period of 1211 work days of the exchange (June 14, 2004 to June 14, 2009) have been calculated and used for the estimation of the beta of the noted companies at different time scales. The criterion for the selection of companies has been their volume of trade and the number of trade days for those companies, and it has been tried to select different companies for this purpose. The percent change of the price of a stock on a specific day has been considered as the criterion for the return of that stock in the noted period, which is calculated via relation (11):

\[ R_{it} = \frac{P_{it} - P_{it-1}}{P_{it-1}} \times 100 \] (11)

3.2- Risk-free return rate

To calculate the risk-free rate of return, the annual interest rate of the investment bonds issued by the central bank is used. These bonds are almost risk-free, because the payments of their principal and interest are guaranteed by the central bank.

3.3- Return of market portfolio

For the calculation of the daily return of market portfolio \( R_{mt} \) within a certain period, the percent change of the total price index of the Tehran stock exchange (TEPIX) for a period of 1211 work days of the exchange (June 14, 2004 to June 14, 2009) has been considered, and expressed as relation (12):

\[ R_{mt} = \frac{TEPIX_{t,1} - TEPIX_{t-1,1}}{TEPIX_{t-1,1}} \times 100 \] (12)

4- Analysis of the results

When the time series is decomposed at \( J \) different scales, for the daily-obtained data, the wavelet scale is such that the first scale \( (D_1(t)) \) shows fluctuations with 2- to 4-day dynamics, the second scale \( (D_2(t)) \) shows fluctuations with 4- to 8-day dynamics, the third scale \( (D_3(t)) \) shows fluctuations with 8- to 16-day dynamics, and . . . , and the \( j \)th scale \( (D_j(t)) \) indicates fluctuations with \( 2^j \) to \( 3^j \) day dynamics, so that ultimately, after subtracting the fluctuations (at \( j \) different scales) from the original time series, the \( S_j(t) \) is obtained at the \( j \)th scale, which shows the fluctuations of the mentioned time series at higher than \( 3^j \) day dynamics, and indicates the trend of the noted
time series. Since the data are acquired on a daily basis, each stock’s beta is only calculated for 6 scales, because the research data are for a 5-year period, and the 7th scale not only includes \( D_1(t) \) fluctuations with an almost one-year dynamics, but also \( S_\alpha(t) \) the ones with higher than one-year dynamics; thus, during the decomposition of the time series, the \( D_\gamma(t) \) cannot be separated from the \( S_\gamma(t) \) (Gencay, Selcuk and Whitcher, 2002). By using the 8th order Dubchase wavelet, a multi-analytic wavelet transformation of the six scales of daily fluctuations of the market portfolio return \( (R_{ma}) \) has been estimated and presented as Fig. 1. In Fig. 1, the \( S \) shows the daily fluctuations of the market portfolio return, whose long-term trend has been calculated as \( S^\alpha \) after decomposition at six different scales. On this basis, in order to evaluate the effect of the fluctuations of market portfolio return on the fluctuations of stocks return at different time scales, a multi-analytic wavelet transformation of the six scales of daily fluctuations of the daily return of each stock \( (R_g) \) has also been estimated by means of the 8th order Dubchase wavelet, and then, the wavelet variance of the daily fluctuations of market portfolio return and the wavelet covariance between the daily fluctuations of market portfolio return and the daily fluctuations of a particular stock have been calculated at each scale, and finally, by using relation (14), the beta of the particular stock has been calculated at that scale (Gencay, Whitcher and Selcuk, 2003). The results of the calculations are listed in Table (1).

In Table (1), the effect of the fluctuations of market portfolio return on the fluctuations of stocks return is mostly positive, at different time scales, and depending on the type of stock, this influence could be larger at some scales and smaller at the others. The results of Table (1) present interesting guidelines for the investors in the Tehran Stock Exchange, and the study and comparison of these financial trends at different time scales provide detailed analysis for daily, short-term and long-term investment decisions. Since day by day trading practices with short-term investment horizons generally consider quick changes in the daily and short-term fluctuations, and long-term trading practices with long-term investment horizons (like those of large financial institutions) mostly consider fluctuations of the capital market, then the prospective investors in the Tehran stock market should decide on the allocation of their funds for this purpose, depending on their degree of risk-taking or risk-avoiding characteristics for long-term investment in the market, and their own investment horizons. For example, although the Tehran Cement Company has a larger systematic investment risk at the 2 – 4 day decision scale, compared to the Paksan Company, but at the 128 – 256 day decision scale, it has a lower systematic investment risk. Therefore, to the risk-taking individuals, the Tehran Cement Company will be more attractive in the short-term horizon; while the Paksan Company will be more appealing to the investors in the long-term horizon. Figure (2) clearly shows the positive association between the fluctuations of market portfolio return and the fluctuations of the return of stocks of Pars Oil Company at the 6th scale (with 64 – 128 day dynamics). Figure (3) demonstrates the positive association between the fluctuations of market portfolio return and the fluctuations of the return of stocks of Tehran Cement Company at the 5th scale (with 32 – 64 day dynamics). Figure (4) illustrates the positive association between the fluctuations of market portfolio return and the fluctuations of the return of stocks of Melli Bank Investment Company at the 4th scale (with 64 – 128 day dynamics).

These observations verify the primary results. At lower scales, due to smaller dynamics, observational comparison is not possible. \( S_\alpha(t) \) is obtained after subtracting the fluctuations at six different scales from the original time series, which shows the original time series with fluctuations that only have higher that 128-day dynamics, and in fact is indicative of the trend of the mentioned time series in the long term. Therefore, to examine the long-term correlation of the market portfolio return and the return of stocks, the stocks’ beta for the smoothened series of \( S_\alpha(t) \) market portfolio returns and the smoothened series of \( S_\alpha(t) \) returns of stocks have been calculated and shown in Table (2). As is obvious, the estimate of the beta of stocks at long-term dynamics, with fluctuations with higher than 128-day dynamics, is positive for all the companies, and this indicates the positive association of the trend of market portfolio return and the trend of stocks return in the Tehran Stock Exchange. In Figure (5), the positive correlation between the trend of market portfolio return and the trend of return of Sepahan Cement Company stocks, at fluctuations with long-term dynamics can be observed. By using relation (12) and the obtained betas, the average risk premium of market at every scale can be obtained. So, in another investigation, the association between the average risk premium of a stock and its corresponding beta at different scales has been shown in Table (3). This table outlines the estimate of OLS average risk premium of stocks (as the dependent variable) relative to the beta of stocks (as the independent variable), at different time scales. The results indicate that, at the 10% level, the slope is significant at the 1st to 4th scales; thus, the CAPM model is a multi-scale representation, and the short-term and medium-term investment periods can more appropriately explain the association between the return of a stock and its beta in the Tehran Stock Exchange. On this basis, the following results have been obtained:
1. In the investigation by Fernandez (2006) to estimate the systematic risk of 24 stocks in the Santiago Stock Exchange, he concluded that the CAPM model only supports medium-term investment periods. Moreover, the results of the research conducted by Rhaei, Ben Ammou and Ben Mabrouk (2007) showed that, at short-term and long-term scales, there is a stronger association between the excess return of stocks and the systematic risk level, and consequently the French stock market is more efficient in the short-term and long-term periods. On this basis, the Tehran stock market is different from the stock markets of France and Santiago. The results obtained from the multi-scale CAPM model indicate that, due to the existing association between the excess return of stocks and the systematic risk level at the short-term and long-term time scales, the Tehran stock market is more efficient for short-term and medium-term horizons. The long-term investment in the Tehran Stock Exchange is affected by factors that contradict the assumptions of the CAPM model; for example, the contradiction of the assumptions pertaining to an efficient market (e.g. symmetrical information, non-frictional market, total competitiveness, etc.) has led to the inefficiency of the Tehran stock market for investments with long-term horizons. Therefore, for the Tehran Stock Exchange to become efficient again for long-term investment horizons, it is recommended that the above problems be solved.

2. Unfortunately, one of the major problems of the capital markets of most countries with newly-emerging economies is the inappropriateness of financial resource allocation. At the present, the capital market of Iran faces a similar drawback. To eliminate such a problem, it is necessary to get to know the right investment opportunities by using more accurate methods that help predict the vital decision variables. Based on the prediction by the multi-scale CAPM model, the relationship between the return of a stock and its beta is more significant at investment horizons with 2 – 32 day dynamics, and the major part of the market influence is on the price of personal assets in the range of fluctuations of the 1\textsuperscript{st} to 4\textsuperscript{th} scales. Therefore, since the prediction of the CAPM model in explaining the correlation between the return of a stock and its beta and in determining the return expected by the investors in long-term investment horizons (longer than 32 days) is not a significant and reliable prediction, it is recommended that the investors consider the prediction of the above model only for investment scales with short-term and medium-term horizons, before they allocate their financial resources.

The capital assets pricing model explains that the risk premium of personal asset is equal to the risk premium of market multiplied by its beta. The beta parameter measures the degree of concurrent movement of the asset return and the return of market portfolio. In this study, the wavelet method was tested for the Tehran Stock Exchange as a new analysis technique in the financial sector, with the focus on the estimation of CAPM at different time scales. According to the obtained results, the relationship between risk and return in the Tehran stock market at the 1\textsuperscript{st} to 4\textsuperscript{th} scales (2 – 32 day horizons) is significant; therefore, the prediction for this market, in a multi-scale framework, is more inclined towards short-term and medium-term investment periods. These results showed that the stock market of Iran is different from the stock markets of France and Santiago. On this basis, the prediction of the CAPM model in explaining the correlation between the return of a stock and its beta and in determining the return expected by the investors in long-term investment horizons (longer than 32 days) will not be a significant and reliable prediction. Moreover, the results obtained from the multi-scale model demonstrate that the long-term investment behavior in the Tehran Stock Exchange is influenced by factors that contradict the assumptions of the noted model. As an example, the contradiction of the assumptions pertaining to an efficient market (e.g. symmetrical information, non-frictional market, perfect competitiveness, etc.) has led to the inefficiency of the Tehran stock market for investments with long-term horizons. Therefore, for the Tehran Stock Exchange to become efficient again for long-term investment horizons, it is recommended that the above problems be solved.

5. Final remarks

In this study, Wavelet method as a new analysis technique in finance were tested by focusing on the estimation of the CAPM at different time scale for Tehran Stock Exchange as an emerging marketplace. According to the results, the relationship between risk and return in the Tehran stock market at the 1\textsuperscript{st} to 4\textsuperscript{th} scales (2 – 32 day horizons) is significant; therefore, the prediction for this market, in a multi-scale framework, is more inclined towards short-term and medium-term investment periods. The results demonstrate that the long-term investment behavior in the Tehran Stock Exchange is influenced by factors such as asymmetrical information, frictional market, imperfect competitiveness, etc. Therefore, for the TSE to become efficient again for long-term investment horizons, it is recommended that the above problems be solved.
Appendix

Table 1: Beta for each scale

<table>
<thead>
<tr>
<th>Scale</th>
<th>Sepahan Cement</th>
<th>Tehran Cement</th>
<th>Saipa</th>
<th>Amir Kabir Steel</th>
<th>Pars Oil &amp; Petrochemical Investment</th>
<th>Melli Bank Investment</th>
<th>Pakhsan</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETA1</td>
<td>1.208</td>
<td>0.5321</td>
<td>0.2762</td>
<td>0.3578</td>
<td>0.9810</td>
<td>0.5886</td>
<td>0.971</td>
</tr>
<tr>
<td>BETA2</td>
<td>0.9004</td>
<td>0.248</td>
<td>0.9139</td>
<td>0.4819</td>
<td>0.4157</td>
<td>0.5539</td>
<td>1.0551</td>
</tr>
<tr>
<td>BETA3</td>
<td>1.6136</td>
<td>0.4084</td>
<td>0.08223</td>
<td>0.2833</td>
<td>0.1678</td>
<td>0.4418</td>
<td>1.2987</td>
</tr>
<tr>
<td>BETA4</td>
<td>0.5908</td>
<td>0.57701</td>
<td>0.6556</td>
<td>0.4326</td>
<td>0.2956</td>
<td>0.6859</td>
<td>0.6056</td>
</tr>
<tr>
<td>BETA5</td>
<td>0.9433</td>
<td>1.2824</td>
<td>0.6916</td>
<td>0.5981</td>
<td>-0.0279</td>
<td>0.4483</td>
<td>1.1412</td>
</tr>
<tr>
<td>BETA6</td>
<td>0.6727</td>
<td>0.2301</td>
<td>0.5501</td>
<td>-0.0153</td>
<td>0.1113</td>
<td>0.4020</td>
<td>0.9894</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale</th>
<th>Behshahr Industrial Investment</th>
<th>Khodro Electric</th>
<th>Arak Petrochemical</th>
<th>Saipa Diesel</th>
<th>Pars Daro</th>
<th>Eqtesad Novin Bank</th>
<th>Kar Afarin Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETA1</td>
<td>0.605</td>
<td>0.7286</td>
<td>0.4612</td>
<td>0.0515</td>
<td>0.0615</td>
<td>-0.001</td>
<td>-0.1624</td>
</tr>
<tr>
<td>BETA2</td>
<td>0.8221</td>
<td>0.5363</td>
<td>1.1493</td>
<td>0.7071</td>
<td>0.2826</td>
<td>0.7248</td>
<td>0.0434</td>
</tr>
<tr>
<td>BETA3</td>
<td>1.0111</td>
<td>0.1604</td>
<td>-0.013</td>
<td>0.0253</td>
<td>-0.068</td>
<td>0.349</td>
<td>-0.9189</td>
</tr>
<tr>
<td>BETA4</td>
<td>1.1288</td>
<td>0.6805</td>
<td>0.7730</td>
<td>0.3941</td>
<td>0.3001</td>
<td>1.347</td>
<td>0.1882</td>
</tr>
<tr>
<td>BETA5</td>
<td>1.0502</td>
<td>0.0001</td>
<td>0.8688</td>
<td>0.1397</td>
<td>-0.275</td>
<td>0.7573</td>
<td>0.4718</td>
</tr>
<tr>
<td>BETA6</td>
<td>1.0698</td>
<td>0.2847</td>
<td>0.3435</td>
<td>0.9512</td>
<td>0.0411</td>
<td>1.2534</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Figure 1: Level 6 Wavelet transformation for the daily market portfolio return
Table 2: estimation of beta at long-term dynamics

<table>
<thead>
<tr>
<th>Stock</th>
<th>Beta</th>
<th>Stock</th>
<th>Beta</th>
<th>Stock</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saipa Diesel</td>
<td>0.602</td>
<td>Petrochemical Investment</td>
<td>0.199</td>
<td>Sepahan Cement</td>
<td>1.078</td>
</tr>
<tr>
<td>Pars Daro</td>
<td>0.462</td>
<td>Melli Bank Investment</td>
<td>0.57</td>
<td>Tehran Cement</td>
<td>0.939</td>
</tr>
<tr>
<td>Eqtesad Novin Bank</td>
<td>1.052</td>
<td>Behshahr Industrial Development</td>
<td>0.855</td>
<td>Saipa</td>
<td>0.114</td>
</tr>
<tr>
<td>Karafarin Bank</td>
<td>0.476</td>
<td>Khodro Electric</td>
<td>0.035</td>
<td>Amir Kabir Steel</td>
<td>0.68</td>
</tr>
<tr>
<td>Paksan</td>
<td>0.451</td>
<td>Arak Petrochemical</td>
<td>0.762</td>
<td>Pars Oil</td>
<td>0.311</td>
</tr>
</tbody>
</table>

Figure 2: the fluctuations of market portfolio return ($D_M(t)$) and the fluctuations of the return of stocks of Pars Oil Company ($D_{5}(t)$)

Figure 3: the fluctuations of market portfolio return ($D_M(t)$) and the fluctuations of the return of stocks of Tehran Cement Company ($D_{5}(t)$)
Figure 4: the fluctuations of market portfolio return and the fluctuations of the return of stocks of Melli Bank Investment Company

Figure 5: the trend of market portfolio return and the trend of return of Sepahan Cement Company stocks

- the fluctuations of the return of stocks of Pars Oil Company
- the fluctuations of market portfolio return
REFERENCES


Hubbard, B. B., 1998, “The world according to wavelets: The story of a mathematical technique in the making”, 2e, Ak Peters Ltd., MA


11265