J. Basic. Appl. Sci. Res., 1(10)1704-1708, 2011

ISSN 2090-424X
Journal of Basic and Applied Scientific Research
www.textroad.com

New Exact Solution for Creep Behavior of Rotating Thick-Walled Cylinders

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ABSTRACT

This paper describes a new analytical solution for the steady state creep in rotating thick cylindrical shells subjected to internal and external pressure. The creep response of the material is governed by Norton's law. Exact solutions for stresses are obtained under plane strain assumption. The values used in this study are arbitrary chosen to demonstrate the effect of angular velocity on stresses distributions.
KEY WORDS: Cylinder, Shell, Thick-Walled, Rotating, Creep.

1. INTRODUCTION

Cylinder is a commonly employed component in various structural and engineering applications such as pressure vessels, accumulator shells, cylinders for aerospace industries, nuclear reactors, and military applications, pressure vessel for industrial gases or a media transportation of high-pressurized fluids and piping of nuclear reactors. Creep analysis of thick-walled cylindrical shells is an active topic, which attracts a lot of research attention. Rotating cylinders play an important role in the machine design. The problem of a uniformly rotating thick-walled circular cylinder arises occasionally in the design of turbine rotors. Creep analyses of thick walled cylinders subjected to various types of loading have been carried out by many investigators for steady as well as non-steady state creep. Weir [1] investigated creep stresses in pressurized thick walled tubes. Considering large strains, Rimrott and Luke [2], obtained the creep stresses of a rotating hollow circular cylinder made of isotropic and homogeneous materials. Bhatnagar and Gupta [3] obtained solution for an orthotropic thick-walled internally pressurized cylinder by using constitutive equations of anisotropy creep and Norton's creep law. A range of thick-walled tubes operating under creep conditions is analyzed for loading conditions which include internal pressure, external surface loading and inertia loading by Sim and Penny [4]. Assuming that the plane strain condition, Bhatnagar et al. [5], analysis of an internally pressurised, homogeneous, orthotropic rotating cylinder subjected to a steady state creep condition are obtained. In the another study, with considering, the effect of anisotropy on stress and strain, creep analysis of thick-walled orthotropic rotating cylinders has been investigated by Bhatnagar et al. [6]. Creep damage simulation of thick-walled tubes using the theta projection concept investigated by Loghman and Wahab [7]. Gupta and Pathak [8] studied thermo creep analysis in a pressurized thick hollow cylinder.

2. ANALYSIS

Consider a thick-walled cylinder with an inner radius a, and an outer radius b, subjected to internal and external pressures P_a and P_b that are axisymmetric, and rotating at a constant angular velocity ω about its axis.

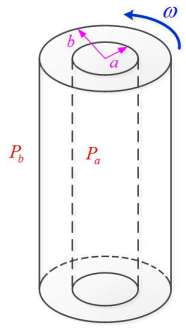

Fig. 1. Geometry and Boundary Conditions of the cylinder.

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The relations between radial, circumferential and axial strain rates $(\dot{\varepsilon}_r, \dot{\varepsilon}_\theta, \dot{\varepsilon}_z)$ and radial, circumferential and axial stresses $(\sigma_r, \sigma_\theta, \sigma_z)$ for an incompressible, isotropic material can be described with

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{\sigma_e}\left[\sigma_r - \frac{1}{2}(\sigma_\theta + \sigma_z)\right] \tag{1}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_e}{\sigma_e}\left[\sigma_\theta - \frac{1}{2}(\sigma_r + \sigma_z)\right] \tag{2}$$

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{\sigma_e}\left[\sigma_z - \frac{1}{2}(\sigma_r + \sigma_\theta)\right] \tag{3}$$

where $\dot{\varepsilon}_e$ and σ_e are the equivalent strain rate and equivalent stress, respectively.

For long open-end cylindrical shell, $\dot{\varepsilon}_z$ is zero. Then

$$\sigma_z = 0.5(\sigma_r + \sigma_\theta) \tag{4}$$

Substituting Eq. (4) into Eqs. (1) and (2), yields

$$\dot{\varepsilon}_r = \frac{3\dot{\varepsilon}_e}{4\sigma_e}[\sigma_r - \sigma_\theta] \tag{5}$$

$$\dot{\varepsilon}_\theta = \frac{-3\dot{\varepsilon}_e}{4\sigma_e}[\sigma_r - \sigma_\theta] \tag{6}$$

The Norton equation gives a relation between the equivalent strain rate and equivalent stress that is suggested for steady state creep in the form

$$\dot{\varepsilon}_e = B\sigma_e^{\,n} \tag{7}$$

where B and n are material parameters describing the creep performance in the cylindrical shells. Substituting Eq. (7) into Eqs. (5) and (6) as follows

$$\dot{\varepsilon}_r = \frac{-3}{4}B\sigma_e^{\,n-1}(\sigma_\theta - \sigma_r) \tag{8}$$

$$\dot{\varepsilon}_\theta = \frac{3}{4}B\sigma_e^{\,n-1}(\sigma_\theta - \sigma_r) \tag{9}$$

In the plane strain condition, von Mises effective stress is as

$$\sigma_e = \frac{\sqrt{3}}{2}(\sigma_\theta - \sigma_r) \tag{10}$$

The strain rate $(\dot{\varepsilon}_r, \dot{\varepsilon}_\theta)$, and displacement rate $(\dot{u})$, relations can be written as

$$\dot{\varepsilon}_r = \frac{d\dot{u}}{dr} \tag{11}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}}{r} \tag{12}$$

By eliminating $\dot{u}$ from Eqs. (11) and (12), the equation of compatibility can be obtained as

$$r\frac{d\dot{\varepsilon}_\theta}{dr} + \dot{\varepsilon}_\theta - \dot{\varepsilon}_r = 0 \tag{13}$$

Substituting Eqs. (8) to Eq. (10) into Eq. (13), equation of compatibility can be rewritten as

$$2(\sigma_\theta - \sigma_r) + nr\left(\frac{d\sigma_\theta}{dr} - \frac{d\sigma_r}{dr}\right) = 0 \tag{14}$$

Due to the fact that a rotating thick cylinder is an axisymmetric problem, its equilibrium equation is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \rho r\omega^2 = 0 \tag{15}$$

where ρ and r are density and radial direction, respectively. By substituting Eq. (14) into Eq. (15), we have

$$\sigma_r'' + \frac{n+2}{nr}\sigma_r' + \frac{2n+2}{n}\rho\omega^2 = 0 \tag{16}$$

Total solution of Eq. (16) is as follows

$$\sigma_r = -\frac{\rho\omega^2}{2}r^2 + C_1 r^{-\frac{2}{n}} + C_2 \tag{17}$$

Substituting σ_r from Eq. (17) in Eq. (15) and solving for σ_θ

$$\sigma_\theta = -\frac{\rho\omega^2}{2}r^2 + \frac{n-2}{n}C_1 r^{-\frac{2}{n}} + C_2 \tag{18}$$

Using Eqs. (17), (18) and (4), axial stress is obtained as

$$\sigma_z = -\frac{\rho\omega^2}{2}r^2 + \frac{n-1}{n}C_1 r^{-\frac{2}{n}} + C_2 \tag{19}$$

Substituting Eq. (10) into Eq. (7), and then using Eqs. (17) and (18), effective strain rate is obtained

$$\dot{\varepsilon}_e = B(-\frac{\sqrt{3}}{n}C_1)^n r^{-2} \tag{20}$$

The boundary condition for stresses are as follows

$$\sigma_r\Big|_{r=a} = -P_a \quad \& \quad \sigma_r\Big|_{r=b} = -P_b \tag{21}$$

Using the above boundary conditions, the constants C_1 and C_2 are obtained

$$C_1 = \frac{2(P_b - P_a) + \rho\omega^2(a^2 - b^2)}{2\left(a^{-\frac{2}{n}} - b^{-\frac{2}{n}}\right)} \tag{22}$$

$$C_2 = \frac{\rho\omega^2}{2}b^2 - \frac{2(P_b - P_a) + \rho\omega^2(a^2 - b^2)}{2\left(a^{-\frac{2}{n}} - b^{-\frac{2}{n}}\right)}b^{-\frac{2}{n}} - P_b \tag{23}$$

Hence, the radial, circumferential, axial and equivalent stresses, and effective strain rate are as follows

$$\sigma_r = \frac{\rho\omega^2 a^2}{2}\left(k^2 - R^2\right) + \frac{2(P_b - P_a) + \rho\omega^2 a^2\left(1-k^2\right)}{2\left(1-k^{-\frac{2}{n}}\right)}\left(R^{-\frac{2}{n}} - k^{-\frac{2}{n}}\right) - P_b \tag{24}$$

$$\sigma_\theta = \frac{\rho\omega^2 a^2}{2}\left(k^2 - R^2\right) + \frac{2(P_b - P_a) + \rho\omega^2 a^2\left(1-k^2\right)}{2\left(1-k^{-\frac{2}{n}}\right)}\left(\frac{n-2}{n}R^{-\frac{2}{n}} - k^{-\frac{2}{n}}\right) - P_b \tag{25}$$

$$\sigma_z = \frac{\rho\omega^2 a^2}{2}\left(k^2 - R^2\right) + \frac{2(P_b - P_a) + \rho\omega^2 a^2\left(1-k^2\right)}{2\left(1-k^{-\frac{2}{n}}\right)}\left(\frac{n-1}{n}R^{-\frac{2}{n}} - k^{-\frac{2}{n}}\right) - P_b \tag{26}$$

$$\sigma_e = \left(-\frac{\sqrt{3}}{n}\right)\frac{2(P_b - P_a) + \rho\omega^2 a^2\left(1-k^2\right)}{2\left(1-k^{-\frac{2}{n}}\right)}R^{-\frac{2}{n}} \tag{27}$$

$$\dot{\varepsilon}_e = B\left[\left(-\sqrt{3}\right)\frac{2(P_b - P_a) + \rho\omega^2 a^2\left(1-k^2\right)}{2n\left(1-k^{-\frac{2}{n}}\right)}\right]^n R^{-2} \tag{28}$$

where $R = r/a$ and $k = b/a$.

3. RESULTS AND DISCUSSION

In the previous section, the exact solution of creep stresses for rotating thick hollow cylinder subjected to uniform pressures on the inner and outer surfaces have been obtained. In this section, some graphs of distribution of the radial, circumferential, axial, and von Mises equivalent stresses along the radial direction are presented. Consider a cylinder under the internal and external pressure of 60 MPa and 40 MPa, respectively. The cylinder has the inner and outer radius of 0.3 m and 0.45 m. The constants are taken as $n = 10$, and $\rho = 7860$ kg/m^3. In addition, angular velocity varies from 300 rad/s to 420 rad/s.

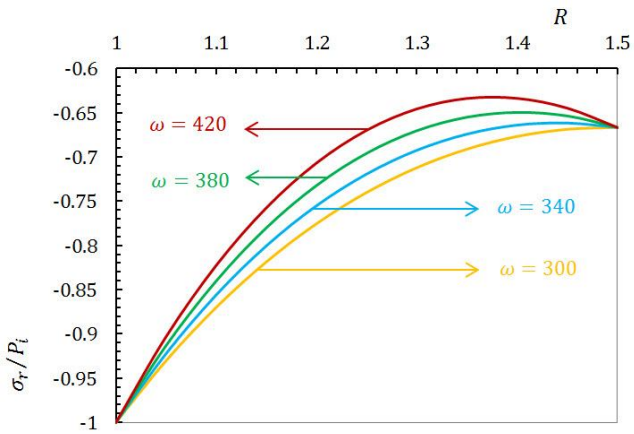

Fig. 2. Distribution of Radial Stress.

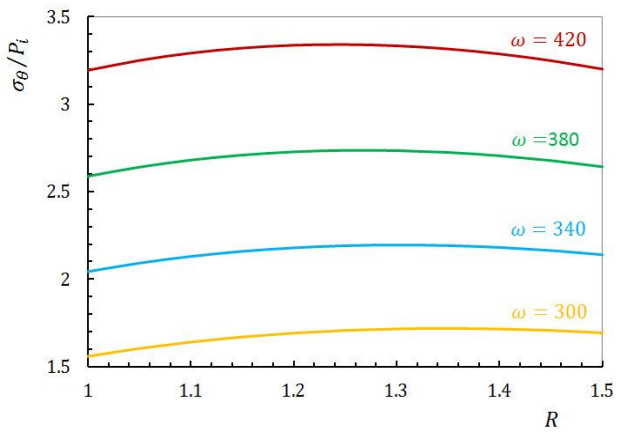

Fig. 3. Distribution of Circumferential Stress.

Figure 2 show the distribution of radial stresses in the radial direction. The radial stress decreases as ω increases.

Distributions of circumferential, axial, and von Mises equivalent stresses are shown in Figs. 3-5. In all these figures, for higher values of ω , the stresses increase.

According to Figs. 2-5, radial stress for all values of ω is compressive whereas circumferential, axial, and von Mises equivalent stresses are tensile.

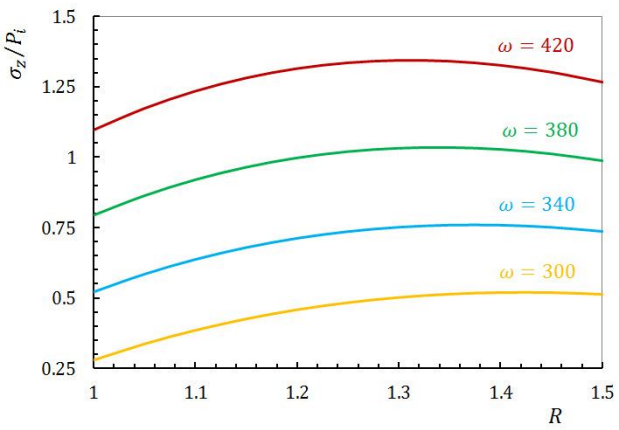

Fig. 4. Distribution of Axial Stress.

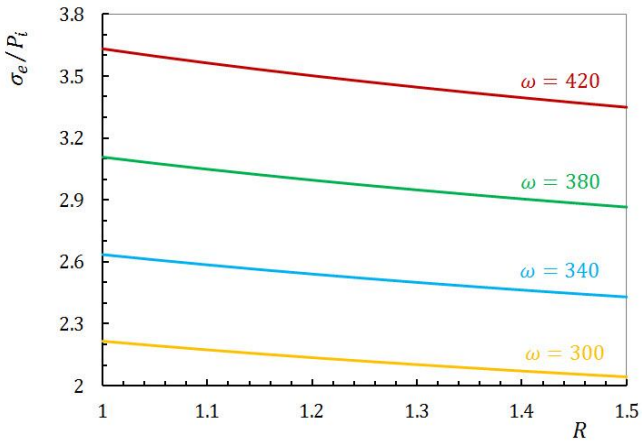

Fig. 5. Distribution of Equivalent Stress.

4. CONCLUSION

It is apparent that closed form solutions to simplified versions of real engineering problems are important. In the present study, a new exact solution procedure has been developed for the analysis of an internally and externally pressurized, homogeneous rotating thick-walled cylinder subjected to a steady state creep condition. Norton's power law of creep is employed to derive general expressions for stresses and strain rates in the rotating thick cylinder.

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