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Flow of Generalized Burger's Fluid in Rayleigh Stokes Problem

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ABSTRACT

Generalized fractional Burgers fluid is analyzed for the flow of an edge in Rayleigh stokes problem. Newtonian and non-Newtonian expressions of velocity field have been established by the usage of Laplace and double Fourier Sine transforms along with their inverses. The constitutive and governing equations are solved by fractional calculus techniques. The general solutions are presented as the sum of non-Newtonian and Newtonian forms and expressed in terms of Fox-H Function. All imposed conditions (initial, natural and boundary) are fulfilled. The general solutions have been particularized for fractionalized as well as ordinary in six types of fluids such as Newtonian fluid, second grade fluid, Maxwell fluid, Oldroyd-B fluid, Burger fluid and generalized Burgers fluid. Finally the rheology is graphically described by various parameters on fluid flow.
KEY WORDS: Transforms, Generalized Burger's model, Stokes problem and graphical rheology.

1. INTRODUCTION

At present, there is no refusing realism that characteristics of non-Newtonian fluids cannot be described by Navier-Stokes equations, this is due to realism that several technological and industrial processes frequently take place in non-Newtonian fluids. If flows of non-Newtonian fluids are assumed between two side of plates then such flows turns into several processes of industry, for instance, paper production, drawing of copper wires, glass blowing, polymer suspension, extrusion of plastic sheets, continuous stretching of plastic films and various others. In the non-Newtonian fluids models, Burgers' fluid model [1-3], the Oldroyd-B fluid model [4-5], Maxwell fluid model [6-7] and the second grade fluid model [8-9] are known as viscoelastic models, because they recognize both characteristics viscosity as well as elasticity. The fractional Burgers' fluid for Couette and Poiseuille flows between two parallel plates is discussed by Hyder Ali Muttaqi Shah [10]. Exact analytical solutions for temperature distributions and velocity fields for magnetohydrodynamics generalized Burgers' fluid in the presence of radiation effects on the heat transfer is investigated in [11]. The fractional generalized Burgers' fluid for oscillating, accelerating and rotating flows with porous medium and magnetohydrodynamics have been obtained for exact solutions along with fractional calculus approach in [12-18]. In continuation, our full attention is on the study of fluid problem between two parallel plates, in last few years, several researchers and mathematicians have diverted their research directions towards the fluid problem related between two parallel plates. However, we mention here very latest research work between two parallel plates. Exact solutions for constantly accelerating plate between two side walls perpendicular to the plate in generalized Oldroyd-B fluid is persuaded by [19]. Using fractional derivative approach, flow between two side walls perpendicular to the plate in second grade fluid is investigated in [20]. Due to suddenly moved plate the flow between two side walls in Maxwell fluid is found out by T. Hayat and et al. [21]. A constant pressure gradient between two side walls perpendicular to the plate in generalized Oldroyd-B fluid is traced out in [22]. Influence of accelerated flows between two side walls perpendicular to the plate in generalized Oldroyd-B fluid is communicated by [23]. Taza Gul and et al. analyzed impacts of magnetic field without slip boundary suppositions for third grade fluid flow using vertical belt [24]. Kashif and et al. investigated fractionalized viscoelastic fluid for magnetohydrodynamic flow by employing transformation techniques [25]. We also include here few recent work related to viscoelastic fluids [26-34]. Observing the above motivations, we traced out generalized fractional Burgers fluid for the flow of an edge in Rayleigh stokes problem. Newtonian and non-Newtonian expressions of velocity field have been established by the usage of Laplace and double Fourier Sine transforms along with their inverses. The constitutive and governing equations are solved by fractional calculus techniques. The general solutions are presented as the sum of non-Newtonian and Newtonian forms and expressed in terms of Fox-H Function. All imposed conditions (initial, natural and boundary) are fulfilled. The general solutions have been particularized for fractionalized as well as ordinary in six types of fluids such as Newtonian fluid, second grade fluid, Maxwell fluid, Oldroyd-B fluid, Burger fluid and generalized Burgers fluid. The rheology

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is graphically described by various parameters on fluid flow and in order to discuss the effects of various parameters, we depicted figures to analyze the fluid flows.

2. Modeling of Governing Equations

The rheological equations for fractional generalized Burgers' fluid are

$$\boldsymbol{T} = -p\mathbf{I} + \boldsymbol{N}, \quad \mu\Lambda_3 D_t^{\psi}\boldsymbol{A} = \boldsymbol{N}\left(\Lambda_2 D_t^{2\phi} + \Lambda_1 D_t^{\phi} + 1\right), \tag{1}$$

Where, $-p\mathbf{I}, \boldsymbol{N}, \boldsymbol{T}, \boldsymbol{A}, \mu, (\Lambda_3 < \Lambda_1), \Lambda_1, \Lambda_2, \psi, \phi$ and D_t^{ψ} denotes indeterminate spherical stress, the extra stress tensor, Cauchy stress tensor, first Rivlin–Ericksen tensor, dynamic viscosity, the retardation time, relaxation time, new material parameter, fractionalized parameters and fractional derivative operator respectively [10-18].We assume velocity field for such problem related between two parallel plates and extra-stress tensor as

$$\boldsymbol{V} = \boldsymbol{V}(y,z,t) = W(y,z,t)i, \boldsymbol{N} = \boldsymbol{N}(y,z,t), \tag{2}$$

here i is identity vector along the x-co-ordinate direction. The limitation of incompressibility is consequently fulfilled for the velocity field. The fluid being at rest up to the moment *t* = 0as well as considering the initial condition

$$\boldsymbol{N}(y,z,0) = 0, \tag{3}$$

Employing equation (2) into equation (1) and considering equation (3) for initial condition, we get the following system of equations in fractionalized form

$$\tau_{zz} = \tau_{yy} = \tau_{yz} = 0, \tag{4}$$

$$\mu\left(1 + \Lambda_3 D_t^{\psi} + \Lambda_4 D_t^{2\psi}\right)\frac{\partial W(y,z,t)}{\partial y} = \left(1 + \Lambda_1 D_t^{\phi} + \Lambda_2 D_t^{2\phi}\right)\tau_1(y,z,t), \tag{5}$$

$$\mu\left(1 + \Lambda_3 D_t^{\psi} + \Lambda_4 D_t^{2\psi}\right)\frac{\partial W(y,z,t)}{\partial z} = \left(1 + \Lambda_1 D_t^{\phi} + \Lambda_2 D_t^{2\phi}\right)\tau_2(y,z,t), \tag{6}$$

In the above equations (5) and (6), we used the below definition so called Caputo fractional operator

$$\frac{\partial^{\psi} h(t)}{\partial t^{\psi}} = \begin{cases} \dfrac{1}{\Gamma(1-\psi)}\displaystyle\int_0^t \frac{h'(\chi)}{(t-\chi)^{\psi}}d\chi, & 0 < \psi < 1 \\ \dfrac{dh(t)}{dt}, & \psi = 1 \end{cases}, \tag{7}$$

In the absence of body force the balance of linear momentum diminishes to

$$\frac{\partial \tau_2(y,z,t)}{\partial z} + \frac{\partial \tau_1(y,z,t)}{\partial y} - \rho\frac{\partial W(y,z,t)}{\partial t} - \frac{\partial p}{\partial x} = 0, \qquad \frac{\partial p}{\partial z} = \frac{\partial p}{\partial y}, \tag{8}$$

Assuming that there is absenteeism of pressure gradient in the flow direction and eliminating $\tau_1(y,z,t)$ and $\tau_2(y,z,t)$ between equations $(5-6)$ and $(8)_1$. We investigate the governing equations

$$\left(1 + \Lambda_1 D_t^{\psi} + \Lambda_2 D_t^{2\psi}\right)\frac{\partial W(y,z,t)}{\partial t} = \nu\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2}\right)\left(1 + \Lambda_3 D_t^{\phi} + \Lambda_4 D_t^{2\phi}\right)\frac{\partial^2 W(y,z,t)}{\partial y^2}, \tag{9}$$

This model contains as special cases such as Newtonian when $(\Lambda_4 \to \Lambda_3 \to \Lambda_2 \to \Lambda_1 \to 0)$, when $(\Lambda_4 \to \Lambda_2 \to \Lambda_1 \to 0)$ second grade, when $(\Lambda_4 \to \Lambda_3 \to \Lambda_2 \to 0)$ Maxwell, when $(\Lambda_4 \to \Lambda_2 \to 0)$ Oldroyd-B, $(\Lambda_4 \to 0)$ Burger. It is also noted that when $\psi \to \phi \to 1$ then above all models are termed as ordinary fluid models. In order to state the problem, we assume an incompressible fractional generalized Burger fluid possesses the space of first dial of rectangular edge as $y, z \geq 0$; $-\infty < x < \infty$. When $t = 0^{+}$ the fluid fetched to constant velocity Ω_0 in the x direction. Owing to shear, fluid slowly moved and its velocity of the form $(2)_1$. The initial and boundary conditions along with governing equations (5,6)and (9) as

$$\text{For, } z, y > 0; \quad \frac{\partial^2 W(y,z,0)}{\partial t^2} = \frac{\partial W(y,z,0)}{\partial t} = W(y,z,0) = 0, \tag{10}$$

$$\text{For, } t > 0; \quad W(y,0,t) = W(0,z,t) = \Omega_0, \tag{11}$$

$$\text{For, } z^2 + y^2 \to 0; \quad as \quad \frac{\partial W(y,z,t)}{\partial z} \to \frac{\partial W(y,z,t)}{\partial y} \to W(y,z,t) \to 0. \tag{12}$$

Equation (12) represents that there is no shear in the free stream when the fluid is at rest at infinity

3. Solution of the problem

3.1 Solution of Generalized Burger fluid for velocity field

Applying double Fourier sine transform to equation (9-11), we attain

$$\frac{\partial W_s(\lambda,\varepsilon,t)}{\partial t}\left(1 + \Lambda_1 D_t^{\psi} + \Lambda_2 D_t^{2\psi}\right) + \nu\left(1 + \Lambda_3 D_t^{\phi} + \Lambda_4 D_t^{2\phi}\right)(\lambda^2 + \varepsilon^2)W_s(\lambda,\varepsilon,t) - \frac{2\,\Omega_0 \nu\,(\lambda^2 + \varepsilon^2)}{\pi\,\lambda\,\varepsilon}$$
$$\times\left(1 + \Lambda_3 D_t^{\phi} + \Lambda_4 D_t^{2\phi}\right)W_s(\lambda,\varepsilon,t) = 0, \tag{13}$$

Where $W_s(\lambda,\varepsilon,t)$ is the Fourier sine transform of $W(y,z,t)$ illustrated as

$$W_s(\lambda,\varepsilon,t) = 2/\pi \int_0^\infty \int_0^\infty W(y,z,t)\, Sin(y\lambda)\, Sin(y\varepsilon)\, d\lambda\, d\varepsilon, \tag{14}$$

has to justify imposed conditions equation (10-11),

$$\frac{\partial^2 W_s(\lambda,\varepsilon,0)}{\partial t^2} = \frac{\partial W_s(\lambda,\varepsilon,0)}{\partial t} = W_s(\lambda,\varepsilon,0) = 0, \qquad \lambda,\varepsilon > 0, \tag{15}$$

Applying Laplace transform to equation (13), we obtain

$$\bar{W}_s(\lambda,\varepsilon,\xi) = \frac{2\,\Omega_0\,\nu\,(\lambda^2+\varepsilon^2)}{\pi\,\lambda\,\varepsilon} \frac{\left(1+\Lambda_3\xi^{\phi}+\Lambda_4\xi^{2\phi}\right)}{\xi[(\xi+\Lambda_1\xi^{\psi+1}+\Lambda_2\xi^{2\psi+1})+\nu\,(\lambda^2+\varepsilon^2)(1+\Lambda_3\xi^{\phi}+\Lambda_4\xi^{2\phi})]}, \tag{16}$$

Our aim is to express the solutions as a sum of Newtonian and non-Newtonian Part $W_N(y,z,t)+W_{Non}(y,z,t)$, hence$\Lambda_1 \to \Lambda_2 \to \Lambda_3 \to \Lambda_4 \to 0$ in equation (16), we have get Newtonian part as

$$\bar{W}_{s(N)}(\lambda,\varepsilon,,\xi) = \frac{2\,\Omega_0\,\nu\,(\lambda^2+\varepsilon^2)}{\pi\,\lambda\,\varepsilon\,\xi\,(\xi+\,\nu\lambda^2+\nu\varepsilon^2)}\ , \tag{17}$$

Switching equation (17) into equation (16), we obtain equivalently,

$$\begin{aligned}\bar{W}_s(\lambda,\varepsilon,\xi) = {} & \frac{2\,\Omega_0}{\pi\,\lambda\,\varepsilon}\left\{\frac{1}{\xi}-\frac{1}{(\xi+\,\nu\lambda^2+\nu\varepsilon^2)}\right\}+\frac{2\,\Omega_0\,\nu\,(\lambda^2+\varepsilon^2)}{\pi\,\lambda\,\varepsilon} \\ & \times\frac{\left(1+\Lambda_3\xi^{\phi}+\Lambda_4\xi^{2\phi}\right)-\left(1+\Lambda_1\xi^{\psi}+\Lambda_2\xi^{2\psi}\right)}{(\xi+\,\nu\lambda^2+\nu\varepsilon^2)[\xi(1+\Lambda_1\xi^{\psi}+\Lambda_2\xi^{2\psi})+\nu\,(\lambda^2+\varepsilon^2)(1+\Lambda_3\xi^{\phi}+\Lambda_4\xi^{2\phi})]},\end{aligned} \tag{18}$$

expressing equation (18) in terms of series form as,

$$\begin{aligned}\bar{W}_s(\lambda,\varepsilon,\xi) = {} & \frac{2\,\Omega_0}{\pi\,\lambda\,\varepsilon}\left\{\frac{1}{\xi}-\frac{1}{(\xi+\,\nu\lambda^2+\nu\varepsilon^2)}\right\}+\frac{2\,\Omega_0\,\nu\,(\lambda^2+\varepsilon^2)}{\pi\,\lambda\,\varepsilon}\frac{1}{(\xi+\,\nu\lambda^2+\nu\varepsilon^2)} \\ & \times\left[\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(\Lambda_1)^{\beta}}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^{\gamma}\frac{1}{\gamma!}\sum_{\theta=0}^{\infty}\frac{(-\Lambda_3)^{\theta}}{\theta!}\sum_{\varphi=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^{\varphi}\frac{1}{\varphi!}\right. \\ & \times\frac{\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\theta+1)\Gamma(\alpha+\theta)}{\Gamma(\alpha-\beta+1)\Gamma(\beta-\gamma+1)\Gamma(\alpha)\Gamma(\theta-\varphi+1)}\frac{1}{\xi^{\psi\gamma-\psi\beta-2\psi\gamma-\alpha-\phi\theta+\phi\varphi-2\phi\varphi}} \\ & -\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^{\beta}}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^{\gamma}\frac{1}{\gamma!}\sum_{\theta=0}^{\infty}\frac{(-\Lambda_1)^{\theta}}{\theta!}\sum_{\varphi=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^{\varphi}\frac{1}{\varphi!} \\ & \left.\times\frac{\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\theta+1)\Gamma(\alpha+\theta)}{\Gamma(\alpha-\beta+1)\Gamma(\beta-\gamma+1)\Gamma(\alpha)\Gamma(\theta-\varphi+1)}\frac{1}{\xi^{\phi\gamma-\phi\beta-2\phi\gamma+\alpha-\psi\theta+\psi\varphi-2\psi\varphi+1}}\right],\end{aligned} \tag{19}$$

Applying inverse Laplace transform on equation (19), we get

$$\begin{aligned}W_s(\lambda,\varepsilon,t) = {} & \frac{2\,\Omega_0}{\pi\,\lambda\,\varepsilon}-\frac{2\,\Omega_0}{\pi\,\lambda\,\varepsilon}Exp(-\nu\lambda^2-\nu\varepsilon^2)t+\frac{2\,\Omega_0\,\nu\,(\lambda^2+\varepsilon^2)}{\pi\,\lambda\,\varepsilon}Exp(-\nu\lambda^2-\nu\varepsilon^2)t\,* \\ & \times\left[\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(\Lambda_1)^{\beta}}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^{\gamma}\frac{1}{\gamma!}\sum_{\theta=0}^{\infty}\frac{(-\Lambda_3)^{\theta}}{\theta!}\sum_{\varphi=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^{\varphi}\frac{1}{\varphi!}\right. \\ & \times\frac{\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\theta+1)\Gamma(\alpha+\theta)t^{\psi\gamma-\psi\beta-2\psi\gamma-\alpha-\phi\theta+\phi\varphi-2\phi\varphi}}{\Gamma(\alpha-\beta+1)\Gamma(\beta-\gamma+1)\Gamma(\alpha)\Gamma(\theta-\varphi+1)\Gamma(\psi\gamma-\psi\beta-2\psi\gamma-\alpha-\phi\theta+\phi\varphi-2\phi\varphi+1)} \\ & -\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^{\beta}}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^{\gamma}\frac{1}{\gamma!}\sum_{\theta=0}^{\infty}\frac{(-\Lambda_1)^{\theta}}{\theta!}\sum_{\varphi=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^{\varphi}\frac{1}{\varphi!} \\ & \left.\times\frac{\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\theta+1)\Gamma(\alpha+\theta)t^{\phi\gamma-\phi\beta-2\phi\gamma+\alpha-\psi\theta+\psi\varphi-2\psi\varphi+1}}{\Gamma(\alpha-\beta+1)\Gamma(\beta-\gamma+1)\Gamma(\alpha)\Gamma(\theta-\varphi+1)\Gamma(\phi\gamma-\phi\beta-2\phi\gamma+\alpha-\psi\theta+\psi\varphi-2\psi\varphi+2)}\right],\end{aligned} \tag{20}$$

Inverting equation (20) by means of Fourier sine transform, we get

$$\begin{aligned}W(y,z,t) = {} & \Omega_0-\Omega_0 erf\left(z/2\sqrt{t\nu}\right)erf\left(y/2\sqrt{t\nu}\right)+\frac{4\Omega_0\nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty\frac{(\lambda^2+\varepsilon^2)}{\lambda\,\varepsilon}\sin(z\varepsilon)\sin(y\lambda) \\ & \times Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)\,d\omega\,d\lambda\,d\varepsilon\left[\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(\Lambda_1)^{\beta}}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^{\gamma}\frac{1}{\gamma!}\right. \\ & \times\sum_{\theta=0}^{\infty}\frac{(-\Lambda_3)^{\theta}}{\theta!}\sum_{\varphi=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^{\varphi}\frac{1}{\varphi!}\frac{\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\theta+1)\Gamma(\alpha+\theta)}{\Gamma(\alpha-\beta+1)\Gamma(\beta-\gamma+1)\Gamma(\alpha)\Gamma(\theta-\varphi+1)}\end{aligned}$$

$$\times\frac{t^{\psi\gamma-\psi\beta-2\psi\gamma-\alpha-\phi\theta+\phi\varphi-2\phi\varphi}}{\Gamma(\psi\gamma-\psi\beta-2\psi\gamma-\alpha-\phi\theta+\phi\varphi-2\phi\varphi+1)}-\frac{4\Omega_0\nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty\frac{(\lambda^2+\varepsilon^2)}{\lambda\,\varepsilon}\sin(z\varepsilon)\sin(y\lambda)$$
$$\times Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)\,d\omega\,d\lambda\,d\varepsilon\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^\alpha\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^\beta}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^\gamma\frac{1}{\gamma!}$$
$$\times\sum_{\theta=0}^{\infty}\frac{(-\Lambda_1)^\theta}{\theta!}\sum_{\varphi=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^\varphi\frac{1}{\varphi!}\frac{\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\theta+1)\Gamma(\alpha+\theta)}{\Gamma(\alpha-\beta+1)\Gamma(\beta-\gamma+1)\Gamma(\alpha)\Gamma(\theta-\varphi+1)}$$
$$\times\left.\frac{t^{\phi\gamma-\phi\beta-2\phi\gamma+\alpha-\psi\theta+\psi\varphi-2\psi\varphi+1}}{\Gamma(\phi\gamma-\phi\beta-2\phi\gamma+\alpha-\psi\theta+\psi\varphi-2\psi\varphi+2)}\right], \tag{21}$$

Finally velocity field is expressed in compact form in terms of Fox-H function as,

$$W_{GB}(y,z,t)=W_N(y,z,t)+\frac{4\Omega_0\nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty\frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}\;Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^\alpha\sum_{\beta=0}^{\infty}\frac{(\Lambda_1)^\beta}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^\gamma\frac{1}{\gamma!}\sum_{\theta=0}^{\infty}\frac{(-\Lambda_3)^\theta}{\theta!}t^{\psi\gamma-\psi\beta-2\psi\gamma-\alpha-\phi\theta+\phi\varphi-2\phi\varphi}$$
$$\times\boldsymbol{H}_{4,5}^{1,4}\left[\frac{\Lambda_4}{\Lambda_3}\middle|\begin{matrix}(-\alpha,0),(-\beta,0),(-\theta,0),(-\theta-\alpha,0)\\(0,1),(\beta-\alpha,0),(\gamma-\beta,0),(1-\alpha,0),(-\theta,-1),(\psi\gamma-\psi\beta-2\psi\gamma-\alpha-\phi\theta,-\phi)\end{matrix}\right]d\omega\,d\lambda\,d\varepsilon$$
$$-\frac{4\Omega_0\nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty\frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^\alpha\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^\beta}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^\gamma\frac{1}{\gamma!}\sum_{\theta=0}^{\infty}\frac{(-\Lambda_1)^\theta}{\theta!}t^{\phi\gamma-\phi\beta-2\phi\gamma+\alpha-\psi\theta+\psi\varphi-2\psi\varphi+1}$$
$$\times\boldsymbol{H}_{4,5}^{1,4}\left[\frac{\Lambda_2}{\Lambda_1}\middle|\begin{matrix}(-\alpha,0),(-\beta,0),(-\theta,0),(-\theta-\alpha,0)\\(0,1),(\beta-\alpha,0),(\gamma-\beta,0),(1-\alpha,0),(-\theta,-1),(\phi\gamma-\phi\beta-2\phi\gamma-\psi\theta+2,-\psi)\end{matrix}\right]d\omega\,d\lambda\,d\varepsilon. \tag{22}$$

Where, Newtonian part of velocity field is $W_N(y,z,t)=\Omega_0-\Omega_0 erf\left(z/2\sqrt{t\nu}\right)erf\left(y/2\sqrt{t\nu}\right)$ and relation of Fox-H function is [35]

$$\sum_{k}^{\infty}\frac{(-R)^k\prod_{j=1}^{p}\Gamma\left(f_j+F_jk\right)}{k!\,\prod_{j=1}^{q}\Gamma\left(g_j+G_jk\right)}=\boldsymbol{H}_{p,q+1}^{1,p}\left[R\middle|\begin{matrix}(1-f_1,F_1),(1-f_2,F_2),\dots,\left(1-f_p,F_p\right)\\(0,1),(1-g_1,G_1),(1-g_2,G_2),\dots,\left(1-g_q,G_q\right)\end{matrix}\right]. \tag{23}$$

Now we can investigate shear stresses $\tau_1(y,z,t)$ and $\tau_2(y,z,t)$ by substituting equation (22) into equations (5) and (6) as obtained by Fetecau and et al. for Oldroyd-B fluid [19].It is also noted that one can have ordinary solutions for even six models namely (Newtonian, second grade, Maxwell, Oldroyd-B, Burger and generalized Burgers fluid) if we put $\phi=\psi=1$ in equation (22).

3.2 Solution of Simple Burger fluid for velocity field when $\Lambda_4\to 0$

Employing similar methodology, we retrieve the solution for simple Burger fluid for velocity field as

$$W_B(y,z,t)=W_N(y,z,t)+\frac{4\Omega_0\nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty\frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}\;Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^\alpha\sum_{\beta=0}^{\infty}\frac{(\Lambda_1)^\beta}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_2}{\Lambda_1}\right)^\gamma\frac{1}{\gamma!}t^{\psi\theta+2\alpha-\psi\gamma-2\psi\theta-\phi\beta}$$
$$\times\boldsymbol{H}_{3,4}^{1,3}\left[\Lambda_3\middle|\begin{matrix}(-\alpha,0),(-\beta,0),(1-\alpha,1)\\(0,1),(\beta-\alpha,0),(\gamma,0),(-\gamma,-1),(2\alpha-\psi\gamma-\phi\beta+1,-\psi)\end{matrix}\right]d\omega\,d\lambda\,d\varepsilon$$
$$-\frac{4\Omega_0\nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty\frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^\alpha\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^\beta}{\beta!}\sum_{\gamma=0}^{\infty}\left(\frac{-\Lambda_4}{\Lambda_3}\right)^\gamma\frac{1}{\gamma!}t^{\psi\theta-\gamma+2\alpha-\phi\beta-2\psi\theta}$$
$$\times\boldsymbol{H}_{3,4}^{1,3}\left[\frac{\Lambda_2}{\Lambda_1}\middle|\begin{matrix}(-\alpha,0),(-\alpha-\gamma,0),(0,1)\\(0,1),(\beta-\alpha,0),(-\gamma,0),(-\gamma,-1),(2\alpha-\psi\gamma-\phi\beta+1,-\psi)\end{matrix}\right]d\omega\,d\lambda\,d\varepsilon. \tag{24}$$

3.3 Solution of Oldroyd-B fluid for velocity field when $\Lambda_4\to\Lambda_2\to 0$

Employing similar methodology, we retrieve the solution for Oldroyd-B fluid for velocity field as

$$W_{OB}(y,z,t) = W_N(y,z,t) + \frac{4\Omega_0 \nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty \frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}\;Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(\Lambda_1)^\beta}{\beta!}\boldsymbol{H}_{2,3}^{1,2}\left[\Lambda_3\left|\begin{matrix}(-\alpha,0),(1-\alpha,1)\\(0,1),(\beta-\alpha,0),(1-\alpha,0),(\psi\beta+\alpha,-\phi)\end{matrix}\right.\right]t^{-\psi\beta-\phi\gamma-\alpha}\,d\omega\,d\lambda\,d\varepsilon$$
$$-\frac{4\Omega_0 \nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty \frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^{\alpha}$$
$$\times\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^\beta}{\beta!}\boldsymbol{H}_{2,3}^{1,2}\left[\Lambda_1\left|\begin{matrix}(-\alpha,0),(1-\alpha,1)\\(0,1),(\beta-\alpha,0),(1-\alpha,0),(\alpha-\phi\beta+2,-\psi)\end{matrix}\right.\right]t^{\alpha-\phi\beta-\psi\gamma+1}\;d\omega\,d\lambda\,d\varepsilon. \quad (25)$$

3.4 Solution of Maxwell fluid for velocity field when $\Lambda_4 \to \Lambda_3 \to \Lambda_2 \to 0$

Employing similar methodology, we retrieve the solution for Maxwell fluid for velocity field as

$$W_M(y,z,t) = W_N(y,z,t) + \frac{4\Omega_0 \nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty \frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}\;Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^{\alpha}\boldsymbol{H}_{1,2}^{1,1}\left[\Lambda_1\left|\begin{matrix}(-\alpha,0)\\(0,1),(-\alpha,-1),(\alpha,\psi)\end{matrix}\right.\right]t^{-\psi\beta-\alpha}\;d\omega\,d\lambda\,d\varepsilon-\frac{4\Omega_0 \nu}{\pi^2}$$
$$\times\int_0^t\int_0^\infty\int_0^\infty \frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^\beta}{\beta!}\boldsymbol{H}_{1,2}^{1,1}\left[\Lambda_1\left|\begin{matrix}(-\alpha,0)\\(0,1),(-\alpha,-1),(1,-\psi)\end{matrix}\right.\right]t^{-\psi\beta+1}\;d\omega\,d\lambda\,d\varepsilon. \quad (26)$$

3.5 Solution of Second Grade fluid for velocity field when $\Lambda_4 \to \Lambda_2 \to \Lambda_1 \to 0$

Employing similar methodology, we retrieve the solution for Maxwell fluid for velocity field as

$$W_{SC}(y,z,t) = W_N(y,z,t) + \frac{4\Omega_0 \nu}{\pi^2}\int_0^t\int_0^\infty\int_0^\infty \frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}\;Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}\left(\frac{1}{\nu\lambda^2+\nu\varepsilon^2}\right)^{\alpha}\boldsymbol{H}_{1,2}^{1,1}\left[\Lambda_3\left|\begin{matrix}(1-\alpha,1)\\(0,1),(1-\alpha,0),(\alpha,-\phi)\end{matrix}\right.\right]t^{-\psi\beta-\alpha}\;d\omega\,d\lambda\,d\varepsilon-\frac{4\Omega_0 \nu}{\pi^2}$$
$$\times\int_0^t\int_0^\infty\int_0^\infty \frac{(\lambda^2+\varepsilon^2)\,sin(z\varepsilon)\,sin(y\lambda)}{\lambda\,\varepsilon}Exp(-\nu\lambda^2-\nu\varepsilon^2)(t-\omega)$$
$$\times\sum_{\alpha=0}^{\infty}(-\nu\lambda^2-\nu\varepsilon^2)^{\alpha}\sum_{\beta=0}^{\infty}\frac{(-\Lambda_3)^\beta}{\beta!}\boldsymbol{H}_{1,2}^{1,1}\left[\Lambda_3\left|\begin{matrix}(1-\alpha,1)\\(0,1),(1-\alpha,0),(-\psi-1,-\phi)\end{matrix}\right.\right]t^{-\psi\beta+\alpha+1}\;d\omega\,d\lambda\,d\varepsilon. \quad (27)$$

4. Concluding Remarks

The main purpose of this article is to investigate generalized fractional Burger fluid for the flow of an edge in Rayleigh stokes problem in Newtonian and non-Newtonian form. Velocity field have been developed by the usage of Laplace and double Fourier Sine transforms along with their inverses. The general solutions are presented in the compact form as the sum of Newtonian and non-Newtonian forms and expressed in terms of $\boldsymbol{H}_{\delta,\vartheta+1}^{\delta,\vartheta}(Z)$ (Fox-H Function). All compulsory conditions (initial, natural and boundary) are fulfilled. The general solutions have been particularized for fractionalized as well as ordinary in five types of fluids such as Newtonian, second grade, Maxwell, Oldroyd-B, Burger and generalized Burgers fluid. In order to analyze the rheology of parameter for fluid flows, Comparison through graphical illustration is presented for five models namely second grade, Maxwell, Oldroyd-B, Burger and generalized Burgers fluid with and without ordinary and fractional effects at two different times. However major consequences and outcomes are:

- The general solutions (equations 22-27) for velocity field are presented in the compact form as the sum of Newtonian and non-Newtonian fluid and expressed in terms of $\boldsymbol{H}_{\delta,\vartheta+1}^{\delta,\vartheta}(Z)$ (Fox-H Function).
- By fixing all rheological parameters and enhancing the values of viscosity(ν) and time(t) in Fig. 1(a) and 1(b), velocity field for generalized Burger fluid has vivid influence for small values of viscosity(ν) and large values of time(t) parameters. It is a physical fact that the stability of fluid flows for smaller values of viscosity(ν) have greater than higher values.
- The effects of retardation(Λ_2) and relaxation (Λ_1)time parameters are depicted in Fig. 2(a) and 2(b) in which the fluid flows is identical in both cases to what we expect between both plates. Fig. 2(a) and

2(b) also reveal that the decline is produced in velocity field, resulting the fluid is sequestrating on the both region of plates.

- Fig. 3(a) and 3(b) are drawn for the impacts in fluid flow of material parameters $(\Lambda_3 < \Lambda_1)$ decreases boundary layer thickness between two plates over the boundary. It is also pointed out that retardation (Λ_2) and relaxation (Λ_1) time parameters have reciprocal behavior withmaterial parameters $(\Lambda_3 < \Lambda_1)$, see also Figs. 2(a), 2(b), 3(a) and 3(b).
- Contrast among five models namely second grade, Maxwell, Oldroyd-B, Burger and generalized Burgers fluid with and without ordinary and fractional effects at fix time $t = 0.5$ are illustrated in Fig. 4. In this figure, we have depicted behavior of fluid flow with and without ordinary and fractional effects. Fig. 4(a) demonstrates the fluid flow with ordinary effects which describes flow in extremely thickness situation; on the other hand Fig. 4(b) has scattering behavior of fluid flow. Meanwhile, Fig. 5 is drawn at time $t = 5.0$, the influence of five models on fluid flow with and without ordinary and fractional effects for thickness situation and scattering behavior is good in agreement. This is due to the fact that integer-order models are not more adequate than fractional-order models, because description of the memory in fluid flow is identified by fractional derivatives due singular kernel.

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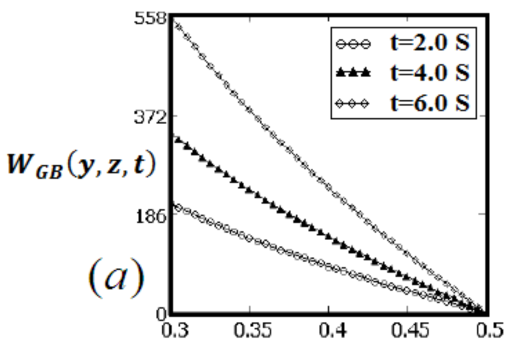

Figure 1(a): Profiles of velocity field for Generalized fractional Burger fluid given by equation (22) for $\Omega_0 = 1, \nu = 0.6, \mu = 1.2, \lambda = 3, \varepsilon = 4.7, \beta = 0.35, \gamma = 0.11,$ $\psi = 0.9, \phi = 0.002, \theta = 0.61, \alpha = 0.1, \omega = 1, \Lambda_1 = 1, \Lambda_2 = 0.3,$ $\Lambda_3 = 2.2, \Lambda_4 = 2.7$ and different values of t .

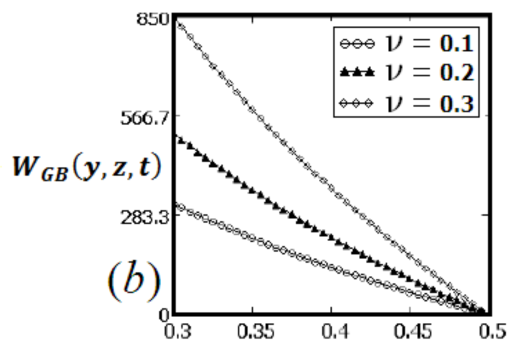

Figure 1(b): Profiles of velocity field for Generalized fractional Burger fluid given by equation (22) for $\Omega_0 = 1, t = 2\ s, \mu = 1.2, \lambda = 3, \varepsilon = 4.7, \beta = 0.35, \gamma = 0.11,$ $\psi = 0.9, \phi = 0.002, \theta = 0.61, \alpha = 0.1, \omega = 1, \Lambda_1 = 1, \Lambda_2 = 0.3,$ $\Lambda_3 = 2.2, \Lambda_4 = 2.7$ and different values of ν .

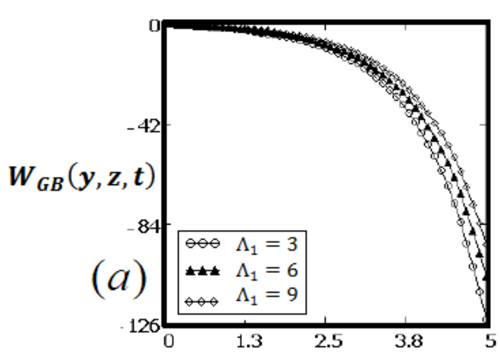

Figure 2(a): Profiles of velocity field for Generalized fractional Burger fluid given by equation (22) for $\Omega_0 = 5, t = 1\ s, \mu = 11.2, \lambda = 2, \varepsilon = 2.4, \beta = 0.7, \gamma = 0.1,$ $\psi = 0.1, \phi = 0.31, \theta = 0.7, \alpha = 0.1, \omega = 1, \nu = 11.5, \Lambda_2 = 0.3,$ $\Lambda_3 = 7.9, \Lambda_4 = 5.9$ and different values of Λ_1 .

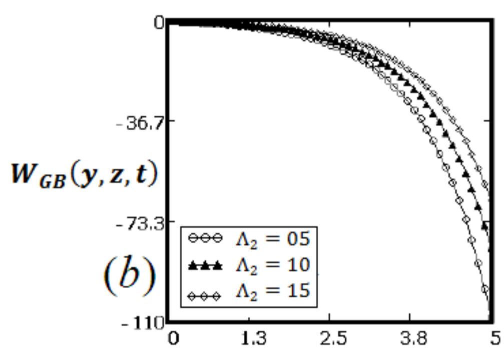

Figure 2(b): Profiles of velocity field for Generalized fractional Burger fluid given by equation (22) for $\Omega_0 = 3, t = 1\ s, \mu = 14.8, \lambda = 1, \varepsilon = 1.1, \beta = 0.7, \gamma = 0.1,$ $\psi = 0.5, \phi = 0.77, \theta = 0.11, \alpha = 0.19, \omega = 4, \nu = 18.5, \Lambda_1 = 21,$ $\Lambda_3 = 9.9, \Lambda_4 = 9.9$ and different values of Λ_2.

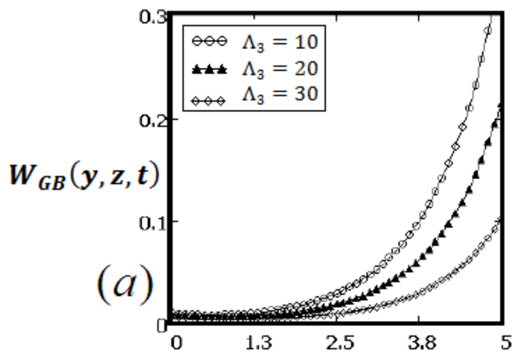

Figure 3(a): Profiles of velocity field for Generalized fractional Burger fluid given by equation (22) for $\Omega_0 = 2, t = 3\ s, \mu = 5.8, \lambda = 2.5, \varepsilon = 7.6, \beta = 9.11, \gamma = 8.16,$ $\psi = 0.8, \phi = 0.8, \theta = 3.6, \alpha = 6.9, \omega = 2.7, \nu = 5.7, \Lambda_1 = 3,$ $\Lambda_2 = 8, \Lambda_4 = 3.9$ and different values of Λ_3.

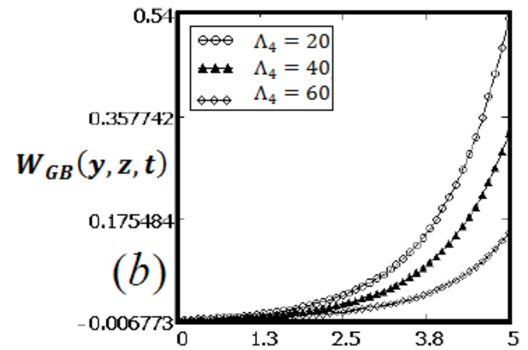

Figure 3(b): Profiles of velocity field for Generalized fractional Burger fluid given by equation (22) for $\Omega_0 = 2, t = 3\ s, \mu = 2.8, \lambda = 2.5, \varepsilon = 7.6, \beta = 9.11, \gamma = 8.16,$ $\psi = 0.8, \phi = 0.8, \theta = 3.6, \alpha = 6.9, \omega = 2.7, \nu = 11, \Lambda_1 = 3,$ $\Lambda_2 = 8, \Lambda_3 = 3.9$ and different values of Λ_4.

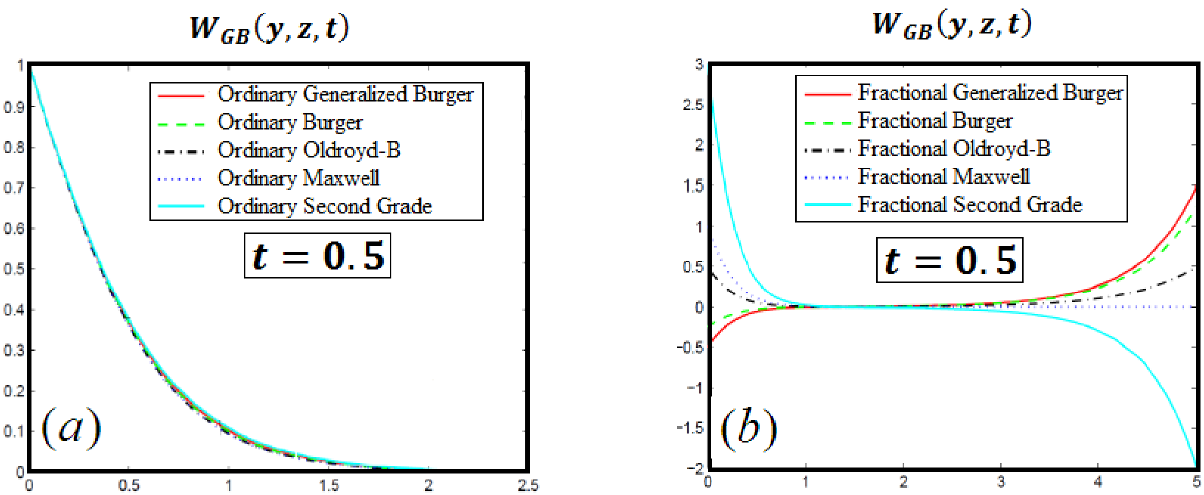

Figure 4: Comparison of velocity field for five models of fluid given by equations (22, 24, 25, 26, 27) for $\Omega_0 = 2, \mu = 2.8, \lambda = 2.5,$ $\varepsilon = 7.6, \beta = 9.11, \gamma = 8.16, \psi = 0.8, \phi = 0.8, \theta = 3.6, \alpha = 6.9, \omega = 2.7, \nu = 11, \Lambda_1 = 3, \Lambda_2 = 8, \Lambda_3 = 3.9, \Lambda_4 = 1.2$.

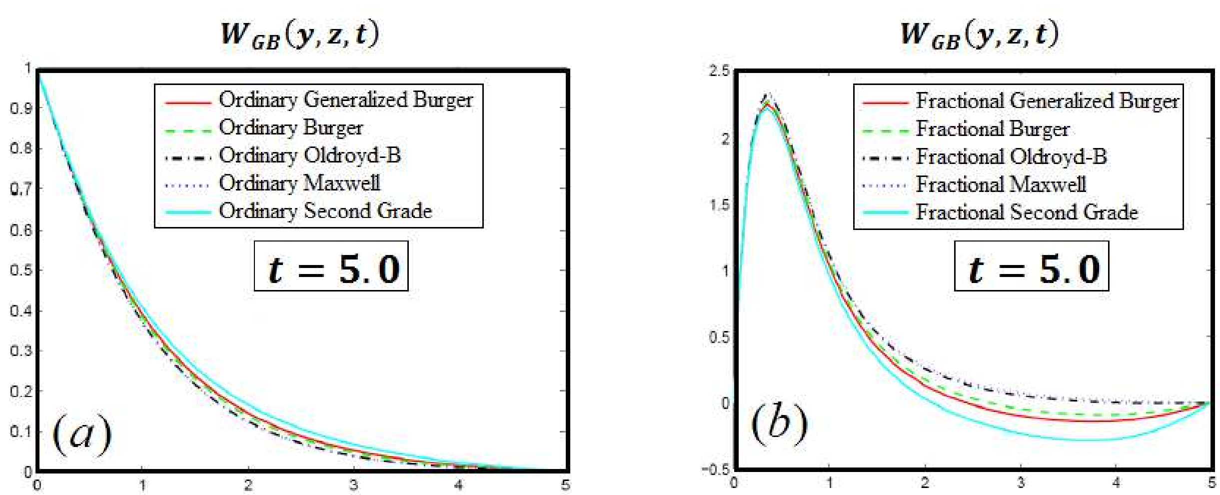

Figure 5: Comparison of velocity field for five models of fluid given by equations (22, 24, 25, 26, 27) for $\Omega_0 = 2, \mu = 2.8, \lambda = 2.5,$ $\varepsilon = 7.6, \beta = 9.11, \gamma = 8.16, \psi = 0.8, \phi = 0.8, \theta = 3.6, \alpha = 6.9, \omega = 2.7, \nu = 11, \Lambda_1 = 3, \Lambda_2 = 8, \Lambda_3 = 3.9, \Lambda_4 = 1.2$.

REFERENCES

[1] T. Hayat, C. Fetecau, S. Asghar, Some simple flows of a Burgers' fluid, Int. J. Eng. Sci. 44 (2006) 1423–1431.

[2] D.K. Tong, L.T. Shan, Exact solutions for generalized Burgers' fluid in an annular pipe, Meccanica 44 (2009) 427–431.

[3] I. Khan, K. Fakhar, S. Sharidan, Magnetohydrodynamic rotating flow of a generalized Burgers' fluid in a porous medium with hall current, Transp. Porous Med. 91 (2012) 49–58.

[4] S. Hyder Ali Muttaqi Shah, M. Khan, H.T. Qi, Exact solutions for a viscoelastic fluid with the generalized Oldroyd-B model, Nonlinear Anal. RWA 10 (2009) 2590–2599.

[5] H.B.H. Mohamed, B.D. Reddy, Some properties of models for generalized Oldroyd-B fluids, Int. J. Eng. Sci. 48 (2010) 1470–1480.

[6] J. Zierep, C. Fetecau, Energetic balance for the Rayleigh–Stokes problem of a Maxwell fluid, Int. J. Eng. Sci. 45 (2007) 617–627.

[7] T. Hayat, M. Sajid, Homotopy analysis of MHD boundary layer flow of an upper convected Maxwell fluid, Int. J. Eng. Sci. 45 (2007) 393–401.

[8] M. Khan, S. Nadeem, T. Hayat, A.M. Siddiqui, Unsteady motions of a generalized second-grade fluid, Math. Comput. Modell. 41 (2005) 629–637.

[9] F. Shen, W.C. Tan,Y.H. Zhao, T.Masuoka, The Rayleigh–Stokes problem for a heated generalized second grade fluid with fractional derivative model, Nonlinear Anal. RWA 7 (2006) 1072–1080.

[10] S. Hyder Ali Muttaqi Shah, Unsteady flows of a viscoelastic fluid with the fractional Burgers' model, Nonlinear Anal. RWA 11 (2010) 1714–1721.

[11] Y.Q. Liu, L.C. Zheng, X.X. Zhang, MHD flow and heat transfer of a generalized Burgers' fluid due to an exponential accelerating plate with the effect of radiation, Comput. Math. Appl. 62 (2011) 3123–3131.

[12] C.F. Xue, J.X. Nie, W.C. Tan, An exact solution of start-up flow for the fractional generalized Burgers' fluid in a porous halfspace, Nonlinear Anal. 69 (2008) 2086–2094.

[13] C.F. Xue, J.X. Nie, Exact solutions of Stokes' first problem for heated generalized Burgers' fluid in a porous half-space, Nonlinear Anal. RWA 9 (2008) 1628–1637.

[14] M. Khan, T. Hayat, Some exact solutions for fractional generalized Burgers' fluid in a porous space, Nonlinear Anal. RWA 9 (2008) 1952–1965.

[15] T. Hayat, S.B. Khan, M. Khan, Exact solution for rotating flows of a generalized Burgers' fluid in a porous space, Appl. Math. Modell. 32 (2008) 749–760.

[16] M. Khan, S.H. Ali, H.T. Qi, On accelerated flows of a viscoelastic fluid with the fractional Burgers' model, Nonlinear Anal. RWA 10 (2009) 2286–2296.

[17] M. Khan, S.H. Ali, H.T. Qi, Exact solutions of starting flows for a fractional Burgers' fluid between coaxial cylinders, Nonlinear Anal. RWA 10 (2009) 1775–1783.

[18] M. Khan, A. Anjum, C. Fetecau, H.T. Qi, Exact solutions for some oscillating motions of a fractional Burgers' fluid, Math. Comput. Modell. 51 (2010) 682–692.

[19] C. Fetecau, Corina Fetecau, M. Kamran, D. Vieru, Exact solutions for the flow of a generalized Oldroyd-B fluid induced by a constantly accelerating plate between two side walls perpendicular to the plate, J. Non-Newtonian Fluid Mech. 156 (2009) 189–201.

[20] M. Khan, S.W. Wang, Flow of a generalized second-grade fluid between two side walls perpendicular to a plate with a fractional derivative model, Nonlinear Anal. RWA 10 (2009) 203–208.

[21] T. Hayat, C. Fetecau, Z. Abbas, N. Ali, Flow of a Maxwell fluid between two side walls due to a suddenly moved plate, Nonlinear Anal. RWA 9 (2008) 2288–2295.

[22] L.C. Zheng, Z.L. Guo, X.X. Zhang, 3D flow of a generalized Oldroyd-B fluid induced by a constant pressure gradient between two side walls perpendicular to a plate, Nonlinear Anal. RWA 12 (2011) 3499–3508.

[23] S. Hyder Ali Muttaqi Shah, Some accelerated flows of generalized Oldroyd-B fluid between two side walls perpendicular to the plate, Nonlinear Anal. RWA 10 (2009) 2146–2150.

[24] Taza Gul, S. Islam, Kamran khan, Muhammad Altaf Khan, Rehan Ali Shah, Aaiza Gul, Murad Ullah (2014). Thin Film Flow Analysis of a MHD Third Grade Fluid on a Vertical Belt With no-slip Boundary Conditions, J. Appl. Environ. Biol. Sci., 4: 71-84.

[25] Kashif Ali Abro, Mukarram Hussain, Mirza Mahmood Baig (2016). Impacts of Magnetic Field on Fractionalized Viscoelastic Fluid,J. Appl. Environ. Biol. Sci., 6: 84-93.

[26] Sidra Abid, S. Islam, Taza Gul, Muhammad Altaf Khan, Saleem Nasir, Aaiza Gul(2014). Magnetichydrodynamic flow of unsteady second grade fluid between two vertical plates with oscillating boundary conditions, *J.* Appl. Environ. Biol. Sci., 4:0-01.

[27] H. Rasheed, Taza Gul, S.Islam, Saleem Nasir, Muhammad Altaf Khan, Aaiza Gul (2014). Study of coquette and poiseuille flows of an unsteady MHD third grade fluid, J. Appl. Environ. Biol. Sci., 4: 12-21.

[28] Taza Gul, M. Tariq Imran, Muhammad Altaf Khan, S. Islam, Haroon Rashid (2015).ADM Solution of Variable Viscosity of a MHD Thin Layer on a Lubricating Vertical Belt,J. Appl. Environ. Biol. Sci., 5: 176-185.

[29] Zeeshan, Saeed Islam, Rehan Ali Shah, Ilyas Khan, Taza Gul (2015). Exact Solution of PTT Fluid in Optical Fiber Coating Analysis using Two-layer Coating Flow, J. Appl. Environ. Biol. Sci., 5: 96-105.

[30] Taza Gul, Imran Khan, Muhammad Altaf Khan, S.Islam, Tanveer Akhtar, S.Nasir (2015).Unsteady Second order Fluid Flow between Two Oscillating Plates,J. Appl. Environ. Biol. Sci., 5: 52-62.

[31] Kashif Ali Abro (2016). Porous Effects on Second Grade Fluid in Oscillating Plate,J. Appl. Environ. Biol. Sci., 6: 71-82.

[32] Kashif Ali Abro, Asif Ali Shaikh and Ishtiaque Ahmed Junejo (2015). Analytical Solutions Under No Slip Effects for Accelerated Flows of Maxwell Fluids, Sindh Univ. Res. Jour. (Sci. Ser.), 47: 613-618.

[33] Kashif Ali Abro and Asif Ali Shaikh (2015). Exact analytical solutions for Maxwell fluid over an oscillating plane, Sci.Int.(Lahore), 27: 923–929.

[34] Muhammad Jamil, Kashif Ali Abro and Najeeb Alam Khan (2015). Helices of fractionalized Maxwell fluid, Nonlinear Engineering,4(4): 191–201.

[35] A. M. Mathai, R. K. Saxena, H. J. Haubold, The H-Functions: Theory and Applications, Springer, New York, (2010).