

Modified Algorithm for Solving Nonlinear Equations in Single Variable

Ayaz Ali Sial¹, Rafique Ahmed Memon², Nek Muhammad Katbar³, Farooq Ahmad⁴

¹Department of Basic Sciences and Related Studies, MUET, Jamshoro, Sindh, Pakistan.

^{2,3}Department of Basic Sciences and Related Studies, MUET, SZAB Campus Khairpur, Sindh, Pakistan.

⁴Punjab Higher Education Department, Govt. Islamia College Civil Lines, Lahore, Pakistan

Received: February 14, 2017

Accepted: April 29, 2017

ABSTRACT

In this research paper, algorithm is being developed for solving nonlinear equations in one variable. In closed methods, modified algorithm entails all the essential ingredients for fast convergence of any numerical technique. It has certainly got ascendancy in terms of accuracy, precision and convergence relatively to BISECTION METHOD and REGULA FALSE POSITION METHOD. Modified algorithm is characterized by its validity and applicability for all class of problems in nonlinear equations. Few examples are tested for effective implementation. MATLAB and EXCEL is executed for graphical representation.

KEYWORDS: closed method, accuracy, nonlinear

1. INTRODUCTION

There are boundless issues creating in science and engineering, which are past the areas of believability to the extent tolerable solutions are concerned. Thusly, there is zero chance to get out, rather than swing to numerical methods. Numerical security is a vital piece of the numerical method. Exactness, capability, soundness is considered as an essential motivational part behind the iterative technique. Regarding the as speedy rate of joining, respectably more close to a right game plan are concerned, it misses the mark. More sensibly, numeric-cum-exact solutions are taxing the brains of nation's authorities. An acceptably intense plan of the non-linear equation is fundamentally a mind-boggling undertaking inconsistent applications issues ascending out of physical wonders once in a while. Numerical precision is the matter of prime concern while making any numerical technique. The ebb and flow research is equipped to develop such change of a computation that must be instrumental in accelerating the path toward separation technique ie (BM+RFPM) for the arrangement of van der Waal condition. AS a generally, different alterations in iterative frameworks, for instance, Bisection, Regula and Newton procedures have been proposed in forming, which has either equivalent or best executions over these techniques. A near examination of different frameworks for finding root has been poked and tried by [Ehiwario, J. C. et al 2014]. [Biswa, 2012] has separated that wide collection of composing exist and are in similitude to this reality that slice is continually arranged to combining however slowly. [Iwetan, 2012] researched that the customary root-finding estimation contains distinctive systems i.e., false position, Newton-Raphson. [Srivastava, 2011] has made essential, but a relative examination of the union of division, Newton and Secant techniques in taking after demand Secant method>Newton method>Bisection. [Dr Farooq Ahmed et al 2012] new subordinate free iterative procedure for illuminating non-organize logical clarifications has been investigated. [Tanakan, 2013] the clearest root-discovering tally is the twofold chase method. It performs well when f is real and continuous. It requires previous values of two beginning appraisals, a and b with the true objective that $f(a).f(b)<0$ then discover midpoint of $[a, b]$ and after that pick whether the root lies in $[a, ((a+b)/2)]$ or $[((a+b)/2), b]$. Repeat until the between time is near nothing [Gyurhan ned Zhi OV, 2015] displayed an approach that can be material to the optional iterative process which is straightly or quadratically joined together. [D .K.R Babajee et al, 2016] differentiated dynamical direct on quadratic polynomial and one of the Newton's arrangement. This examination is described in all Cayley Quadratic Test (CQT) which can be used as a first test to check the profitability of such strategies. Additionally, they have given a short comprehension in cubic polynomials. [Elena Braverman et al, 2016] has given some game plan of the thickness subordinate Nagumo condition. [A. A. Sangah et al, 2016] upgraded the union of Bracketing Methods by using "improving Bracket Algorithm".

^a **Corresponding Author:** Ayaz Ali Sial, Department of Basic Sciences and Related Studies, MUET, Jamshoro, Sindh, Pakistan.
Email: ayaz.siyal83@gmail.com

2. METHODOLOGY OF MODIFIED ALGORITHM

Step: 01

For non-linear equation of the form $f(x)=0$ contained in $[a, b]$, whereas $f(a).f(b)<0$.first initial approximation is derived from

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Step: 02

Suppose new approximation is x_1 , then we find new interval which satisfies the general condition. $f(a).f(x_1)<0$ or $f(x_1).f(b)<0$. Now if it lies in $[x_1, b]$.

Step:03

Decompose $[x_1, b]$ into subintervals by using harmonic mean $[x_1, \frac{2x_1b}{x_1+b}]$, or $[\frac{2x_1b}{x_1+b}, b]$ and choose the subinterval where lies root.

Step: 04

If it lies in $[\frac{2x_1b}{x_1+b}, b]$ second approximation can more likely to be refined by using regula falsi position

$x_2 = \frac{cf(d)-df(c)}{f(d)-f(c)}$ where as $c = \frac{2x_1b}{x_1+b}$ and $d=b$ similarly we find x_3, x_4 , upto x_n by the same process.

Table 1: Problem #1 $f(V)=10V^3-24.63V^2+1.3V-0.0039$ [2, 3]

ITERATIONS	BISECTION METHOD		FALSE POSITION METHOD		Modified algorithm	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	2.5		2.23366		2.386538	
2nd	2.25	25	2.341387	10.77275	2.406451	1.991345
3rd	2.375	12.5	2.384157	4.276951	2.408797	0.234527
4th	2.4375	6.25	2.400079	1.592234	2.409069	0.027276
5th	2.40625	3.125	2.405861	0.578247	2.409101	0.003168
6th	2.421875	1.5625	2.407942	0.208093	2.409105	0.000368
7th	2.414063	0.78125	2.408689	0.07464	2.409105	4.27E-05
8th	2.410156	0.390625	2.408956	0.02674	2.409105	4.96E-06
9th	2.408203	0.195313	2.409052	0.009576	2.409105	5.76E-07
10th	2.40918	0.097656	2.409086	0.003429	2.409105	6.68E-08
11th	2.408691	0.048828	2.409098	0.001228	2.409105	7.76E-09
12th	2.408936	0.024414	2.409103	0.00044	2.409105	9.01E-10
13th	2.409058	0.012207	2.409104	0.000157	2.409105	1.05E-10
14th	2.409119	0.006104	2.409105	5.63E-05	2.409105	1.22E-11
15th	2.409088	0.003052	2.409105	2.02E-05	2.409105	1.42E-12

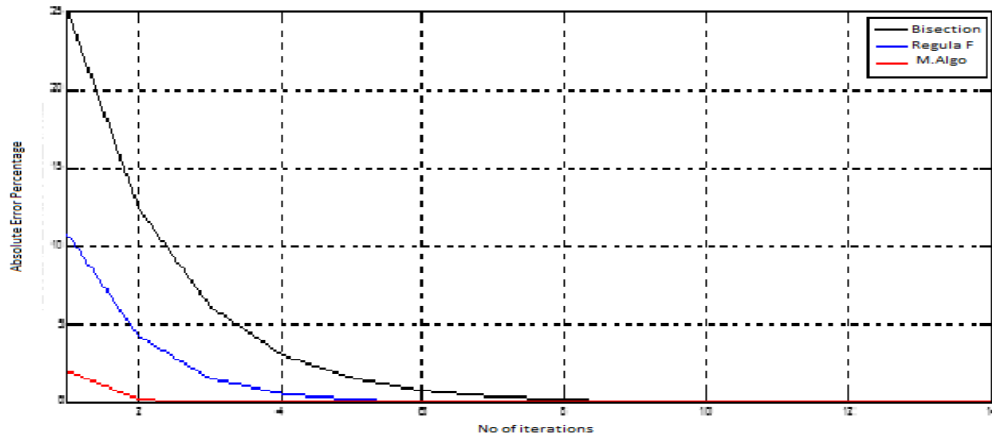


Fig 1: Absolute error graph of algebraic by Modified Algorithm

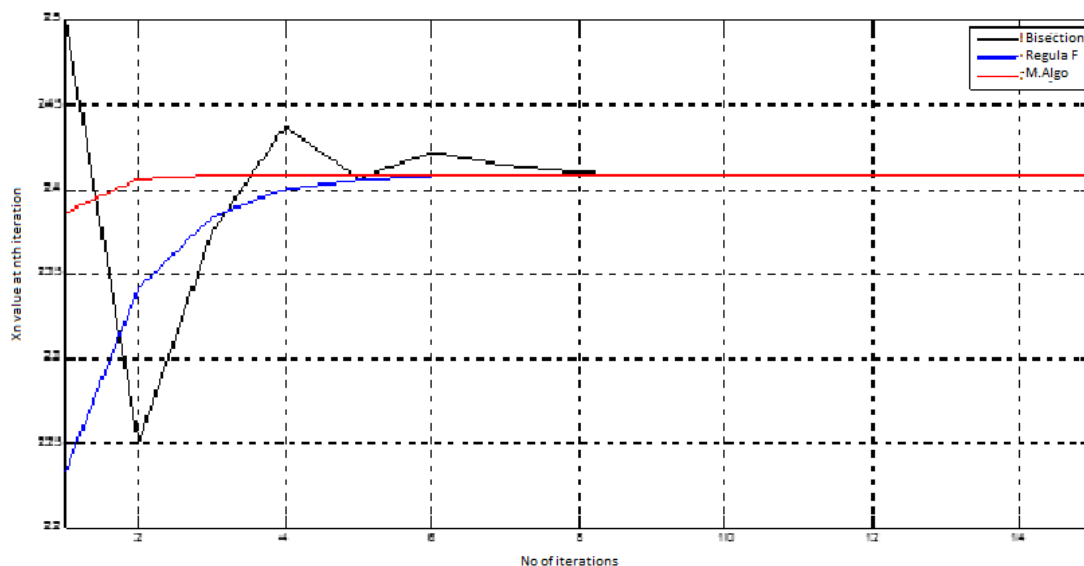
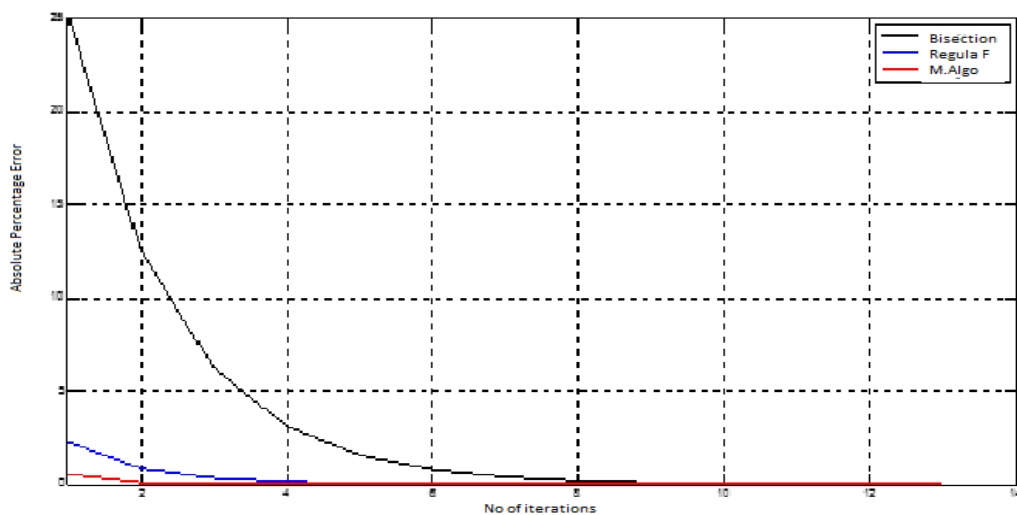


Fig 2: X_n value graph of algebraic equation by modified algorithm

Table 2: Problem#2 (ALGEBRAIC) $x^3-2x-5=0$ [2, 3]

Iterations	Bisection method		False position method		Modified algorithm	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	2.5		2.058824		2.088365	
2nd	2.25	25	2.081264	2.244013	2.093491	0.51259
3rd	2.125	12.5	2.089639	0.837555	2.09437	0.087892
4th	2.0625	6.25	2.09274	0.310036	2.09452	0.01504
5th	2.09375	3.125	2.093884	0.114413	2.094546	0.002573
6th	2.109375	1.5625	2.094305	0.042174	2.094551	0.00044
7th	2.101563	0.78125	2.094461	0.015539	2.094551	7.53E-05
8th	2.097656	0.390625	2.094518	0.005725	2.094551	1.29E-05
9th	2.095703	0.195313	2.094539	0.002109	2.094551	2.2E-06
10th	2.094727	0.097656	2.094547	0.000777	2.094551	3.77E-07
11th	2.094238	0.048828	2.09455	0.000286	2.094551	6.44E-08
12th	2.094482	0.024414	2.094551	0.000105	2.094551	1.1E-08
13th	2.094604	0.012207	2.094551	3.88E-05	2.094551	1.88E-09
14th	2.094543	0.006104	2.094551	1.43E-05	2.094551	3.22E-10
15th	2.094543	0.006104	2.094551	1.43E-05	2.094551	3.22E-10



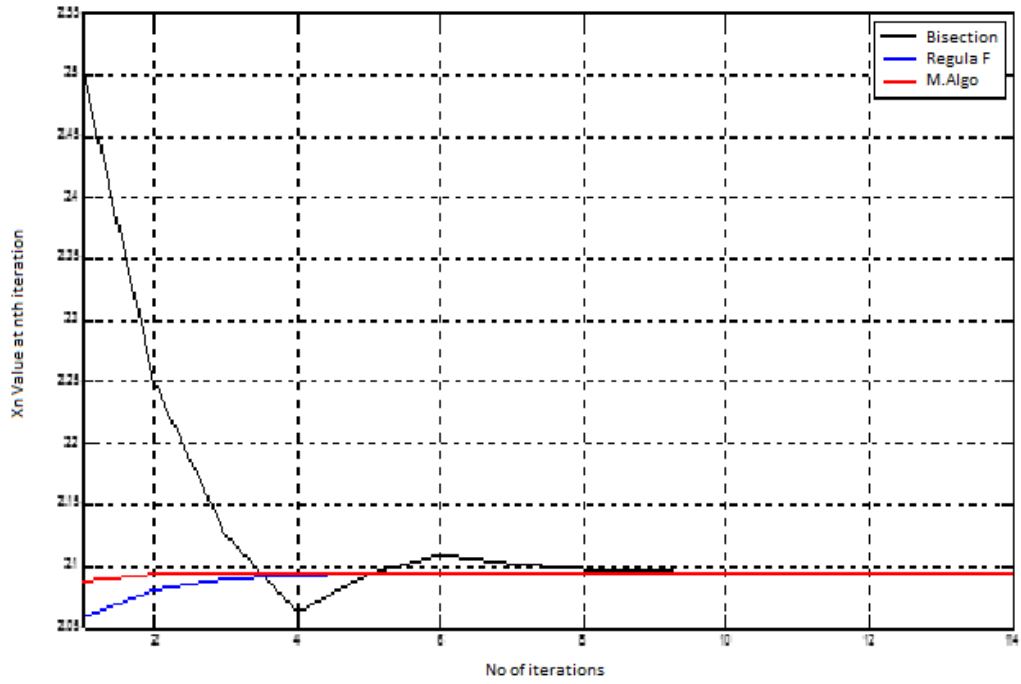
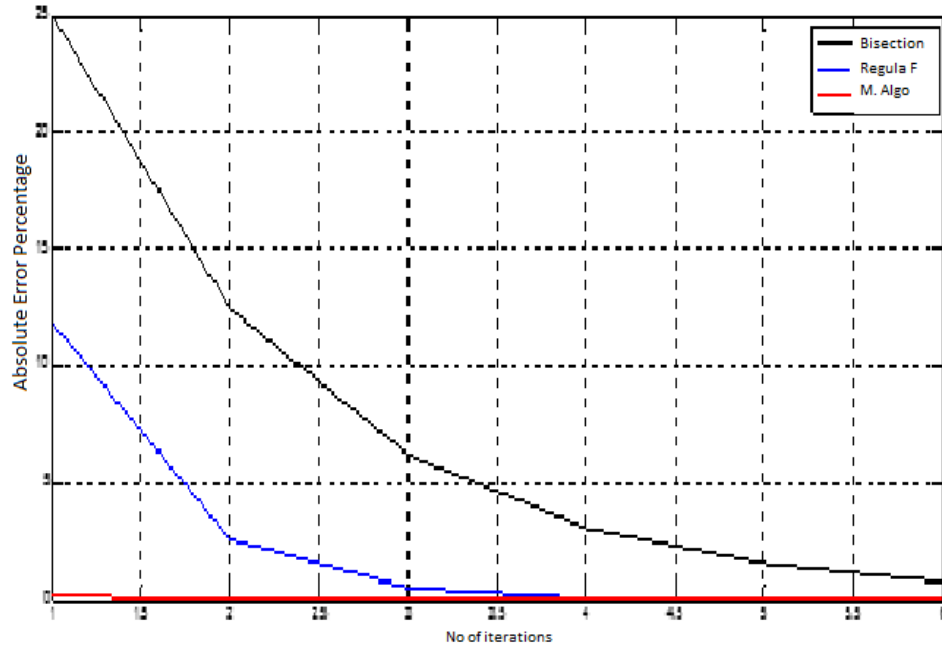
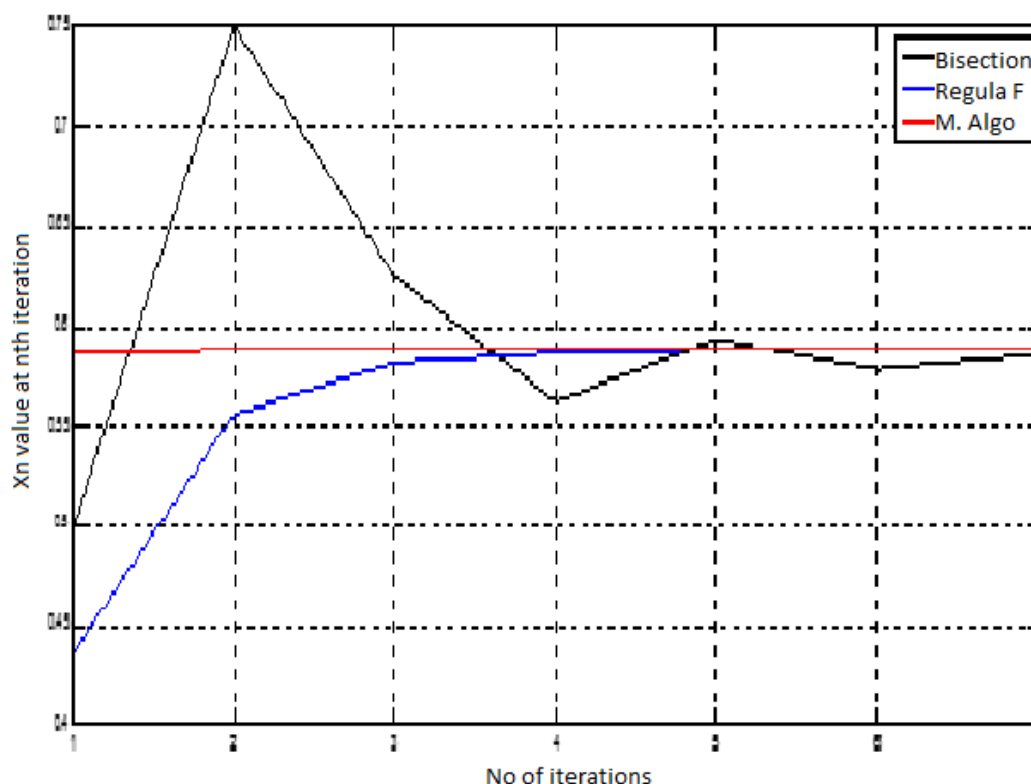


Table 3: Example #3 $F(x)=\text{exsin}(x)-1$ $[0, 1]$

Iteration	Bisection method		False position method		Modified algorithm	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	0.5		0.437186		0.586612	
2nd	0.75	25	0.555986	11.87999	0.58851	0.189836
3rd	0.625	12.5	0.581881	2.589524	0.588532	0.002241
4th	0.5625	6.25	0.587189	0.530731	0.588533	2.64E-05
5th	0.59375	3.125	0.588262	0.107313	0.588533	3.12E-07
6th	0.578125	1.5625	0.588478	0.021638	0.588533	3.68E-09
7th	0.585938	0.78125	0.588522	0.004361	0.588533	4.34E-11





3. RESULTS AND DISCUSSION

In the present review, to justify the modified algorithm, couple of representations are tried on different issues. The findings are compared with existing closed methods. It is observed that modified algorithm is working quicker than other conventional closed methods. Additionally, time component and the number of iterations on modified algorithm and existing separating/sectioning methods are justifiable from the above graphical portrayal and data representation. For the van der Waal condition, the prevailing closed methods i.e. bisection method and regula false position take 9 and 7 iterations with $AE\%=0.19$, $AE\%=0.074$ separately, while modified algorithm takes 3 number of cycles with same precision i.e. $AE\%=0.23$ (Table.1, Fig.1(a) and Fig.1(b)). However, as it is basically examined that modified algorithm comes to 10^{-8} error. Basically, for algebraic function, Bisection and False Position methods take 14th and 10th number of iterations with $AE\%=0.0061$, $AE\%=0.0007$ individually, while modified algorithm brings in 5 cycles with a similar precision. Table.2, (Fig.2(a) and Fig.2(b)) lastly, for transcendental equation, bisection and false position method and modified algorithm take 7th, 4th and 1st number of iterations with a similar precision having $AE\%=0.78$, $AE\%=0.53$, and $AE\%=0.02$ respectively. Table.3, Fig.3(a) and Fig.3(b)), for which guarantee the best execution and noteworthy execution of the modified algorithm.

4. CONCLUSION

Finally, it has been concluded that modified algorithm performs better than anything existing closed methods i.e. Bisection and False Position Method from exactness and iterative point of view, consequently, developed algorithm is a decent achievement to determine roots of nonlinear conditions emerging out of different wonders in engineering and sciences.

REFERENCES

- Ali Asghar Sangah, Asif Ali Shaikh and Syed Feroz Shah (2016), Comparative Study of Existing Bracketing Methods with Modified Bracketing Algorithm for solving Nonlinear Equations in single variable, Sindh University Research Journal, Vol 48, No 1.
- Mamta. et al. (2005). On some third order iterative methods for solving non-linear equations. Appl. Math. Comput. Vol. 171. p. 272-280.
- Noor. M. A. et. al. (2006). Two-steps iterative methods for solving non-linear equations. Appl. Math. Comput. Vol. 181. No. 2. p. 1068-1075.
- A.Gaudin, Boumahart, Applied Numerical Methods, Prentice Hall, Englewood Cliffs, New Jersey, 1989.
- Biswa Nath Datta (2012), Lecture Notes on Numerical Solution of root Finding Problems.
- Burden. R. I &Faires. J. D. (2001). Numerical Analysis. Boston. PWS publishing company.
- Ehiwario, J.C., Aghamie, S.O. (2014) IOSR Journal of Engineering (IOSRJEN).Vol. 04, ||V1|| PP 01-07.
- Frontini. M. &Sormani. E. (2003). Some variants of Newton's method with third-order convergence and multiple roots. J. Comput. Appl. Math. Vol. 156. p. 345-354.
- Iwan, C.N, Fuwape, TanakanI.A, Olajide, M.S and Adenodi, R.A(2012), Comparative Study of the Bisection and Newton Methods in solving for Zero and Extremes of a Single-Variable Function. J. of NAMP vol.21 pp 173-176.
- Kanwar. M. V. et. al. (2004). On a class of quadratically convergent iteration formulae. Appl. Math. Comput. Vol. 166. No. 3. p. 633-6 Tanaka S. : A New Algorithm of Modified Bisection Method for Nonlinear Equation. Applied Mathematical Sciences", Vol. 7, 2013, no. 123, 6107 - 6114 HIKARI Ltd.
- T. Yamamoto. Historical development in convergence analysis for Newton's and Newton-like methods. J. Comput Appl. Math. 124 (2000) 123.
- X. Wu, Z. Shen and J. Xia: An improved regula falsi method with quadratic convergence of both diameter and point for enclosing simple zeros of nonlinear equations. Applied Mathematics and Computation, 144 : 381–388, (2003).
- A.A. Siyal, A.A. Shaikh, A.H. Shaikh, "Hybrid Closed Algorithm for Solving Nonlinear Equations in one Variable", Sindh Univ. Res. Jour. (Sci. Ser.) Vol. 48 (4) pp. 779-782 (2016).