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Buoyancy and the Chemical Reaction Effects on MHD Flow of Casson Fluids through a Porous Medium Due to a Porous Shrinking Sheet

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ABSTRACT

A computational study for the effects of buoyancy and chemical diffusion on heat and mass transfer of Casson fluids flow due to a permeable shrinking surface has been considered. The fluids flows through a porous medium in the presence of a magnetic field. The similar mathematical model of the problem is obtained by employing suitable similarity functions. The resulting non-linear equation coupled ordinary differential form are then solved by employing coding in Mathematica. Rigorous computational work has been carried out to observe the effects of emerging parameters on the physical equation namely thermal function $\theta(\eta)$, velocity function $f'(\eta)$ and concentration function $\phi(\eta)$. Representative results of these equations are presented in form of plots of these functions.
KEYWORDS: Bouyancy effect, chemical diffusion, stagnation point, Casson fluids, porous medium.

1. INTRODUCTION

The magnetohydrodynamic (MHD) flow of non-Newtonian fluids has relevance to the alloys, optimization of solidification processes of metals, and nuclear fuel debris treatment. Several researchers are occupied in the investigation of Bouyancy effects with chemical reaction in the flow of Casson fluids, when the flow is due to a sheet with moving boundary. Mustafa et al. [1] studied the unsteady boundary layer flow and of Casson fluid over a moving flat plate. Recently, Hassan et al and Nadeem and Sajjad [2-3] investigated the unsteady magnetic hydrodynamic (MHD) stagnation point flow of Casson fluids with radiation heat transfer has been investigated. The fluid flows past porous shrinking sheet. The exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet with and without external magnetic field was discussed by Bhattacharyya et al. [4-5]. The Casson fluid has an infinite viscosity at zero rate of shear and a yield stress below which no flow occurs and a zero velocity at an infinite shear rate [6-7]. An excellent collection of articles can be found in [8-9]. Kameswarn [10] investigated Dual solutions of Casson fluid flow over a stretching sheet. Chamkha [11] described Hydromagnetic three dimensional free convection on a vertical stretching sheet with heat generation. stretching sheet. The solution they obtained is by a power series method analystically, further Nandeppanavar [12-14] investigated the heat transfer analysis of Casson fluid due to stretching sheet with convective heating condition both Numerical and analytical results in terms of Kummer's function and RungeKutta flurth order method with shooting technique. Attia and Ahmed[15] studied the transient Coutte flow analysis of Casson fluid between parallel plates with heat transfer analysis. Bhattacharyya et.al[16] have given an analytical solution for magnetohydrodynamic boundary layer flow of Casson fluid, they also studied the effect of wall mass transfer analysis too. Swati[17] studied the effect of thermal radiation on the flow and heat transfer analysis of Casson fluid over an unsteady stretching sheet with effect of suction and blowing. Shehzad et.al[18] investigated the mass transfer of magnetohydrodyanic flow of Casson fluid with an chemical reaction.

2. MATHEMATICAL ANALYSIS

The steady flow of Casson fluid is investigated in the presence of magnetic field of strength B_0 .The fluid is incompressible that flows fluid due to a permeable sheet which stretches /shrinks. u and v are respectively the two

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dimensional velocity components in x and y directions. The fluid temperature is T and species concentration is C. The permeability of porous medium is K, where in the external flow temperature is T_∞ in and concentration of species is C_∞.

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2(x)u}{\rho}+g\gamma(T-T_\infty)-\left(1+\frac{1}{\beta}\right)\frac{\upsilon}{K}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0(T-T_\infty)}{\rho C_p}+\frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2+\frac{\beta^* u}{\rho C_p}(T-T_\infty)+\frac{16\alpha}{3k^*\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma B_0^2}{\rho C_p}u^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 u}{\partial y^2}-R(C-C_\infty) \tag{4}$$

where ρ is density, σ is the electrical conductivity, c_p is the specific heat capacity at constant pressure, μ is dynamic viscosity, $\beta^* u\ (T-T_\infty)$ and $Q_0(T-T_\infty)$ are heat generated or absorbed per unit volume, B_0 is the applied magnetics induction, g is the acceleration due to gravity, other symbols have usual meanings as described in the relevant literature.

The boundary conditions are:

$$u(x,0)=U_w=-cx,\quad v=-v_w, T=T_w, C=C_w \quad \text{at } y=0$$
$$u\to 0, T\to T_\infty, C\to C_\infty \quad \text{at } y\to\infty \tag{5}$$

Using similarity transformations:

The velocity components are described in terms of the stream function $\psi(x,y)$:

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

$$\psi(x,y)=x\sqrt{c\upsilon}f(\eta),\ \eta=y\sqrt{\frac{c}{\upsilon}}$$

$$u=xaf',\quad v=-\sqrt{\upsilon a}f,$$
$$T=T_\infty+(T_w-T_\infty)\theta(\eta),$$
$$C=C_\infty+(C_w-C_\infty)\phi(\eta),$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4) we get

$$\left(1+\frac{1}{\beta}\right)f'''-Mf'+Gr\theta=f'^2-ff''+\left(1+\frac{1}{\beta}\right)K_p u \tag{6}$$

$$(4+3Rn)\theta''+3Rn\Pr(f\theta'+Q\theta-(1+B)f'\theta+Ecf''^2+ME_c f'^2)=0 \tag{7}$$

$$\phi'' + S_c f \theta' - K_r \phi) = 0 \tag{8}$$

and the boundary conditions are

$$f'(0) = 1,\ f(0) = f_w\ ,\ \theta(0) = 1, \phi(0) = 1$$
$$f'(\infty) = 0,\ \theta(\infty) = 0, \phi(\infty) = 0 \tag{9}$$

where **a**s $M = \left(\frac{\sigma}{\rho c}\right)^{\frac{1}{2}} B_0$ is the magnetic parameter, $G_r = g\gamma\left(\frac{T_w - T_\infty}{c^2 x}\right)$ is Grashof number, $P_r = \frac{\upsilon}{\alpha}$ is Prandtl number, $R_n = \frac{16\sigma^* T^3{}_\infty}{3k^* k}$ is thermal radiation, $B = \frac{\beta^* x}{\rho C_p}$ is heat generation coefficient and $Q = \frac{Q_0}{\rho C_p c}$ heat source or sink parameter, E_c the Eckert number, S_c is the Schmidt number, K_r is the chemical reaction parameter.

3. RESULTS AND DISCUSSION

The physical insight of this work is examined through numerical computation of the system of equations (6) to (9). The plots for chemical space concentration temperature distribution and velocity function are mapped under the influence of dimensionless parameters standing for buoyancy, magnetic fields strength, suction/injection, Casson fluid behavior, porosity of medium, Radiation heat generation chemical diffusion. Results have been computed for some representative values of the influencing parameters by using codes in Mathematica.

The Grashof number G_r and magnetic field strength Casson reduction in flow speed as presented respectively in fig.1 and fig.2.Fig.3 also shows that injection decreases the speed of flow.Fig.4 and fig.5 respectively demonstrate the effects of Casson parameter and porosity parameter on flow velocity f'. Both of these parameters reduced the magnitude of velocity.

The thermal distribution $\theta(\eta)$ decreases with the increase in the value of magnetic parameter *M* and the Casson parameter β as shown as shown respectively in fig.6 and fig.7, but the fig.8 shows that radiation causes increase in $\theta(\eta)$. The Prandtl number P_r and suction parameter f_w cause sufficient reduction in $\theta(\eta)$ as demonstrated respectively in fig.9 and fig.10.The increase in heat source parameter *B* increases $\theta(\eta)$ bur increase in G_r and the porosity parameter K_p decreases $\theta(\eta)$ as shown respectively in fig.11, fig.12 and fig.13.

Fig.14, fig.15 and fig.16 respectively show that the concentration function $\phi(\eta)$

decreases in magnitude with increase in the value of f_w, K_r and S_c

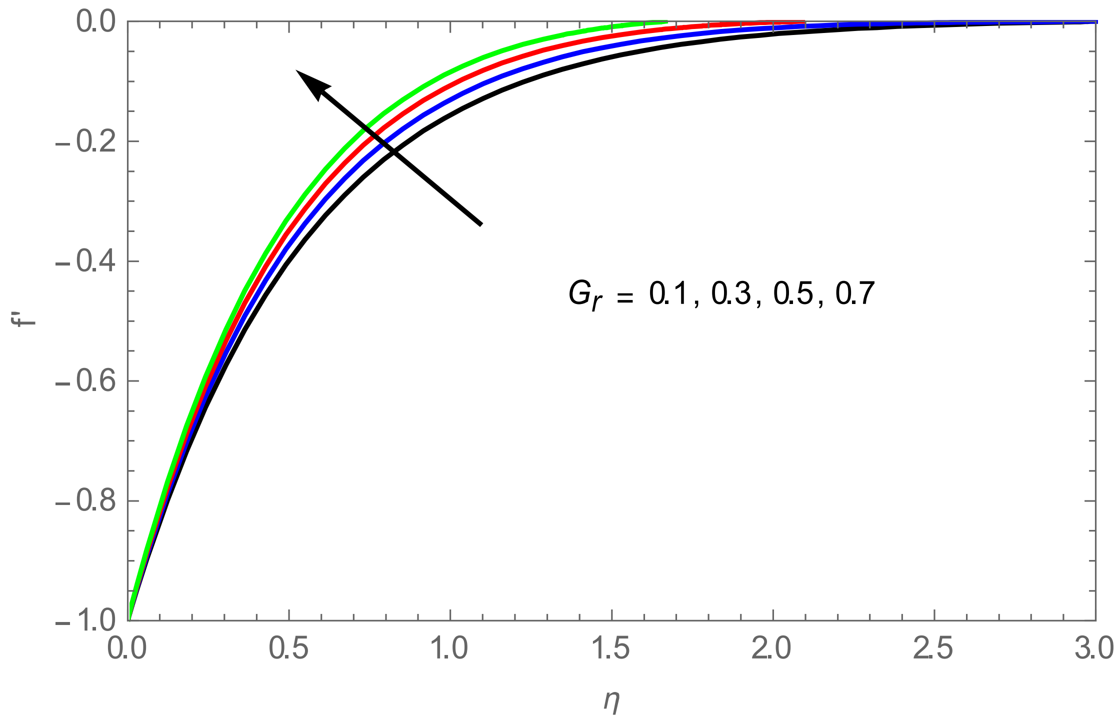

Fig.1: The plot for curves of f' under the effect of Gr.

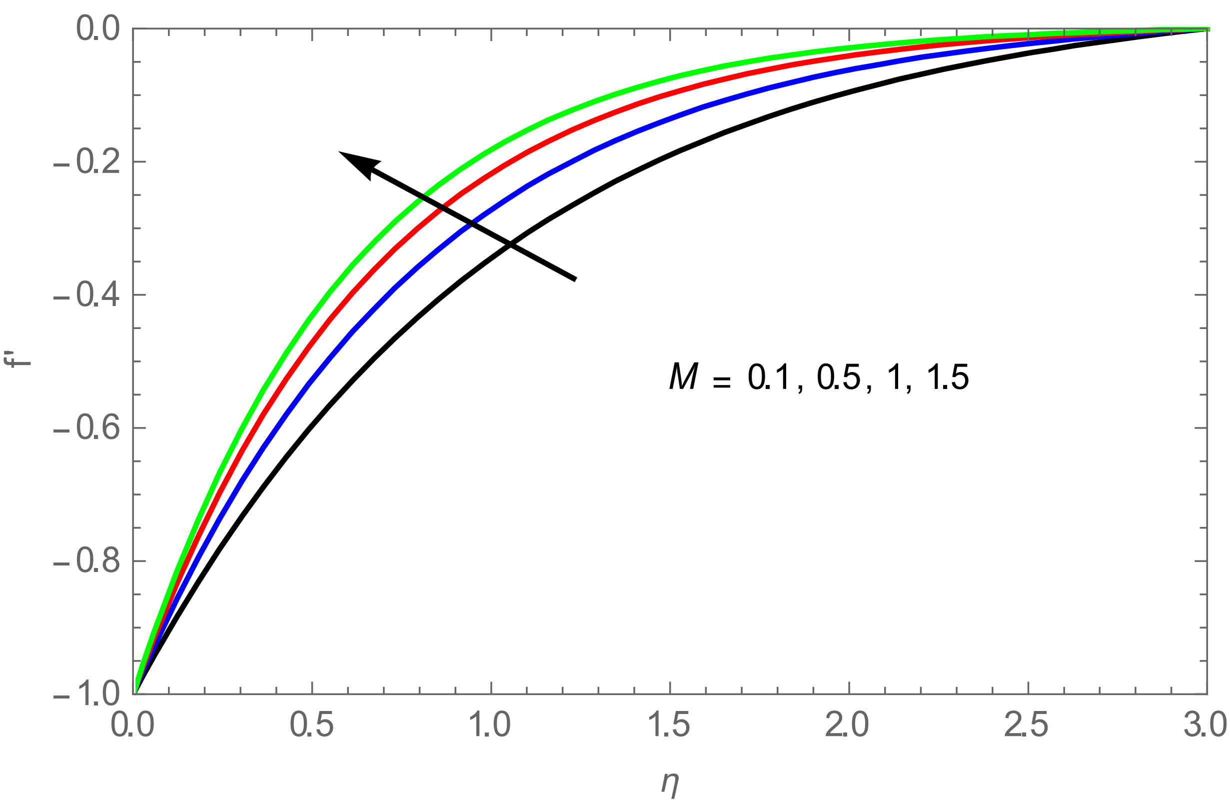

Fig.2: The plot for curves of f' under the effect of M

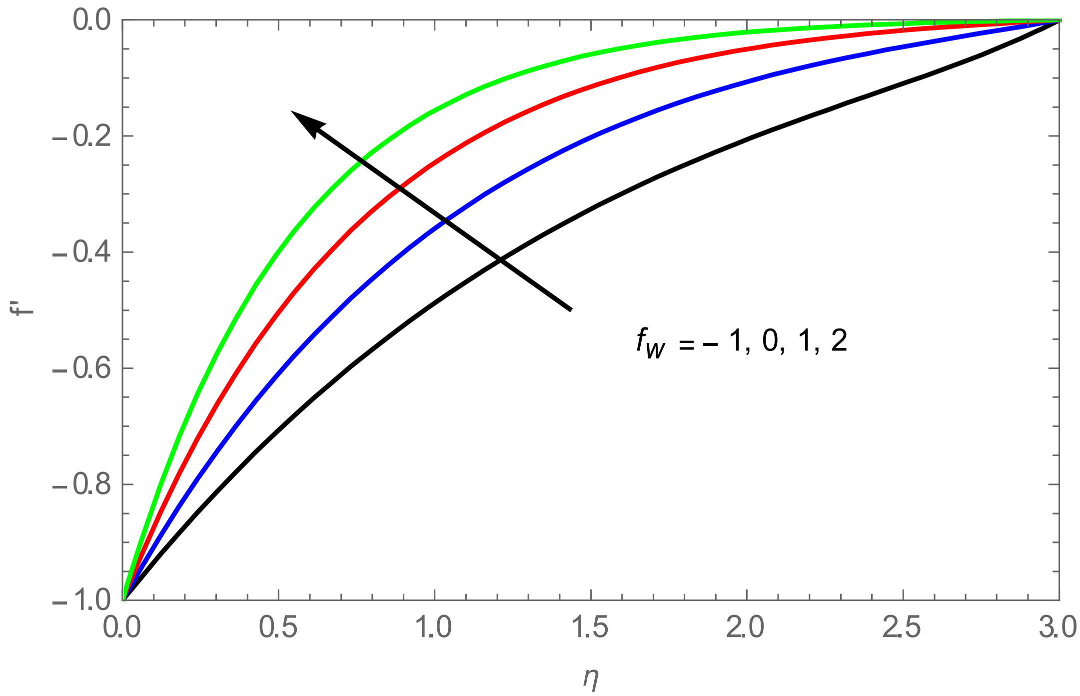

Fig.3: The plot for curves of f' under the effect of suction/injection f_w.

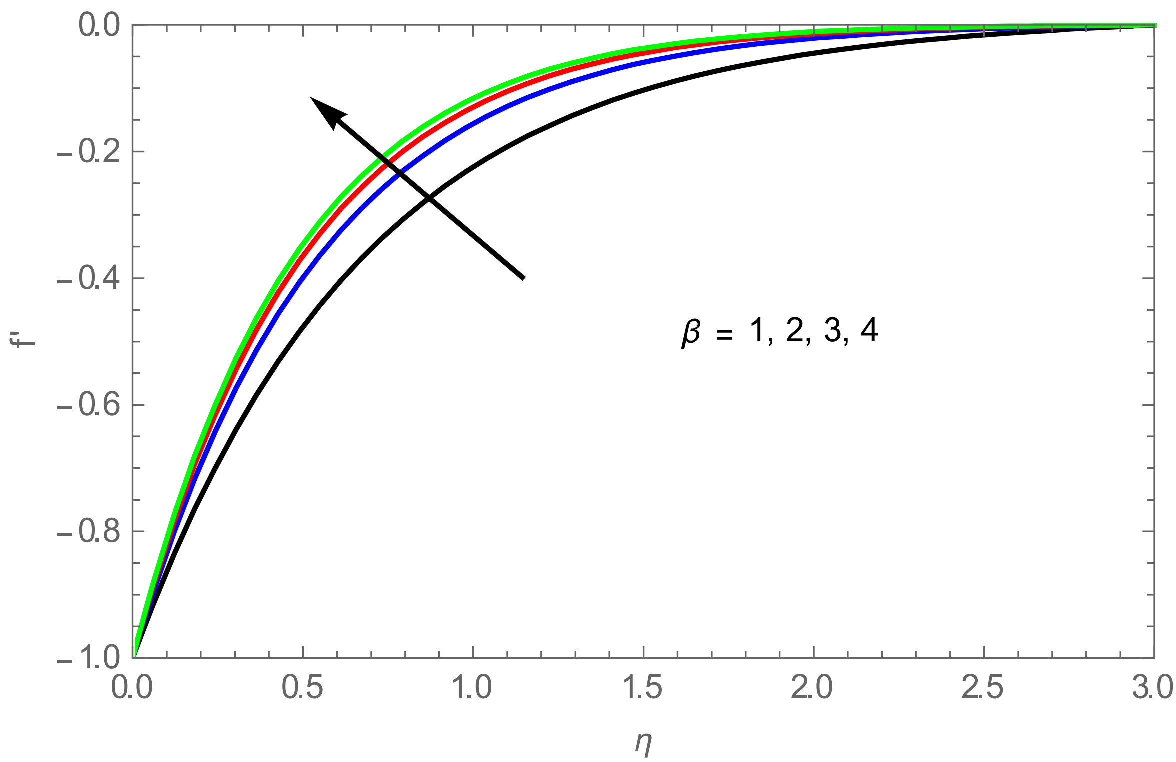

Fig.4: The plot for curves of f' under the effect of β

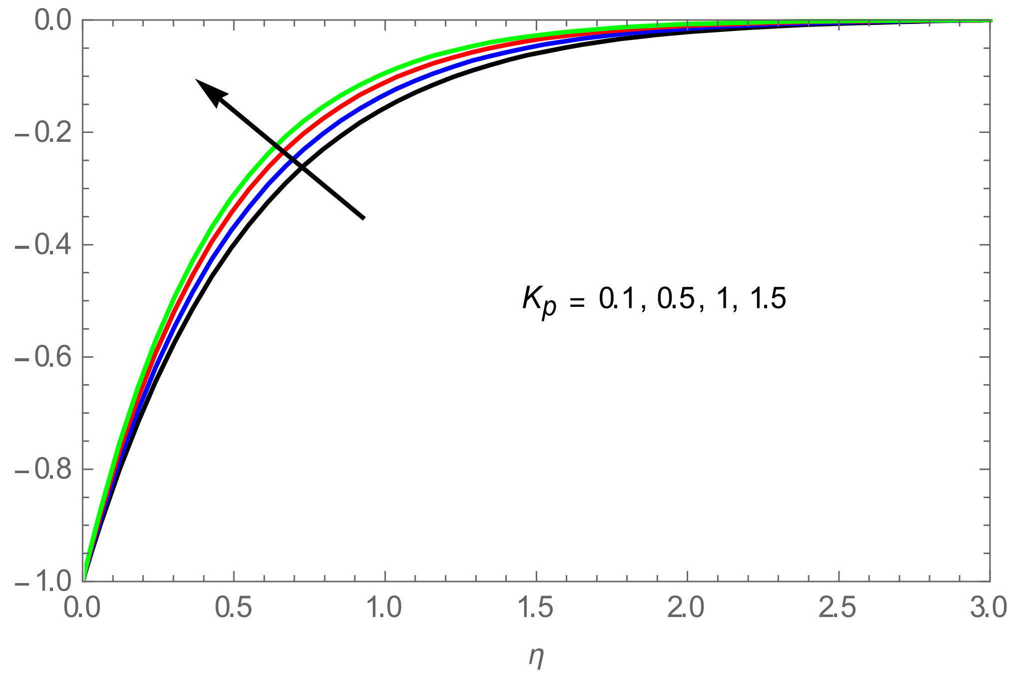

Fig.5: The plot for curves of f' under the effect of Porosity parameter K_p

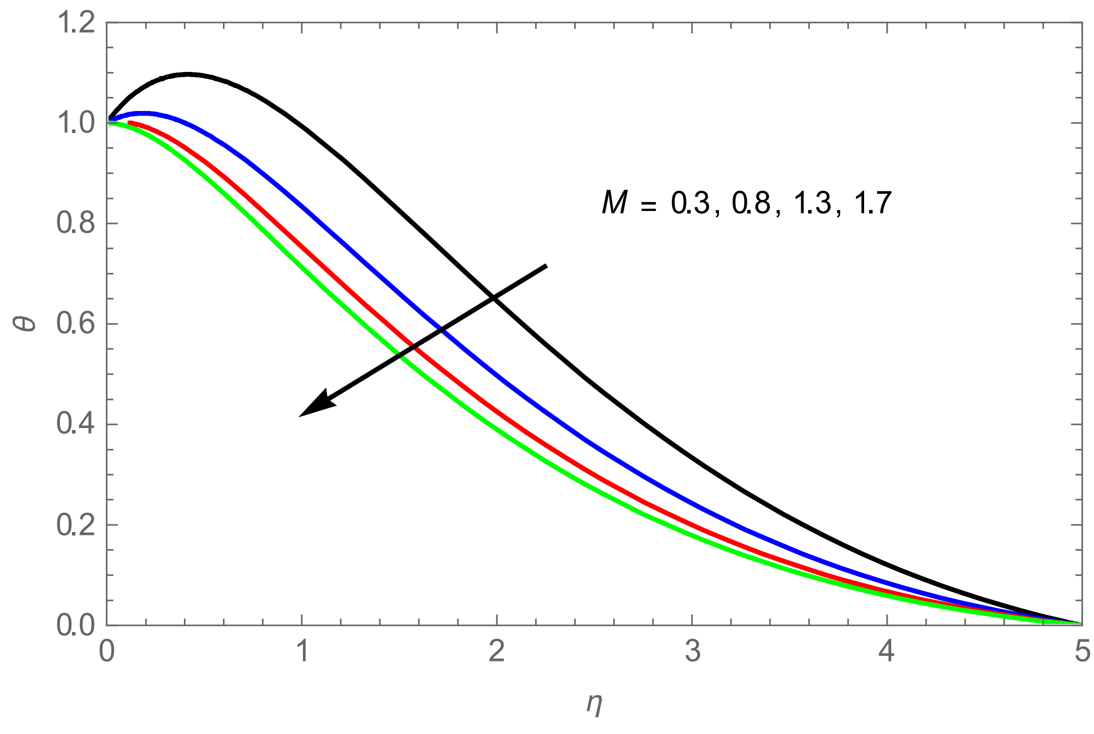

Fig.6: The plot for curves of $\theta(\eta)$ under the effect of M

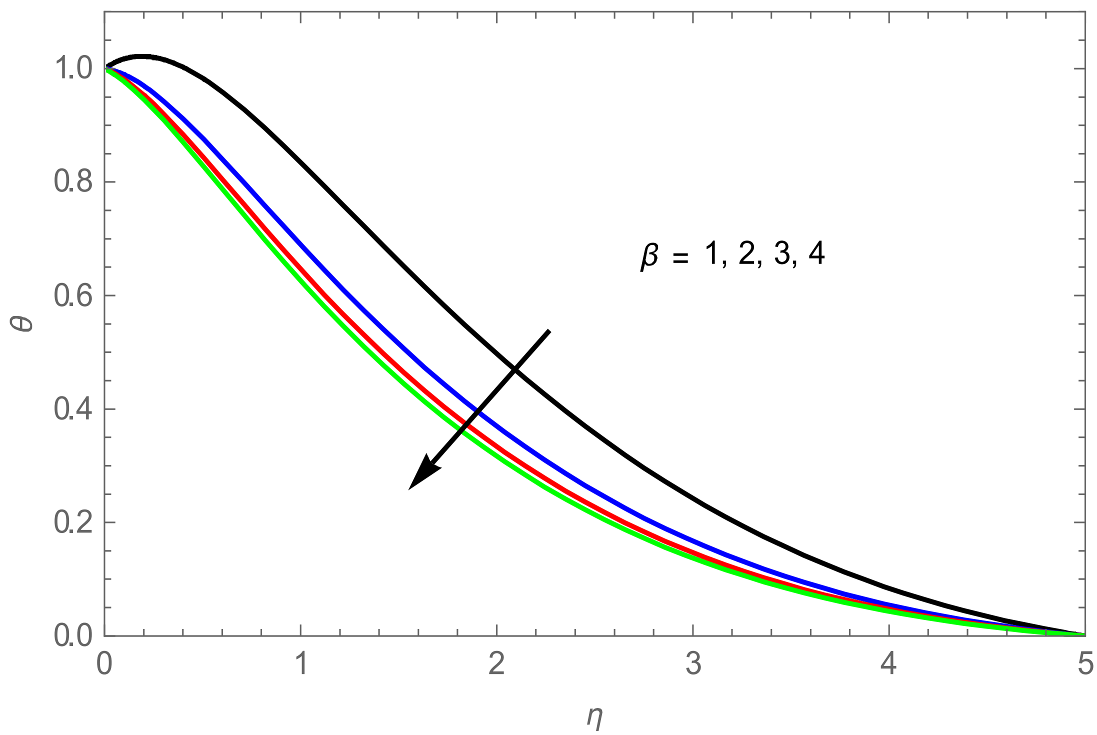

Fig.7: The plot for curves of $\theta(\eta)$ under the effect of β

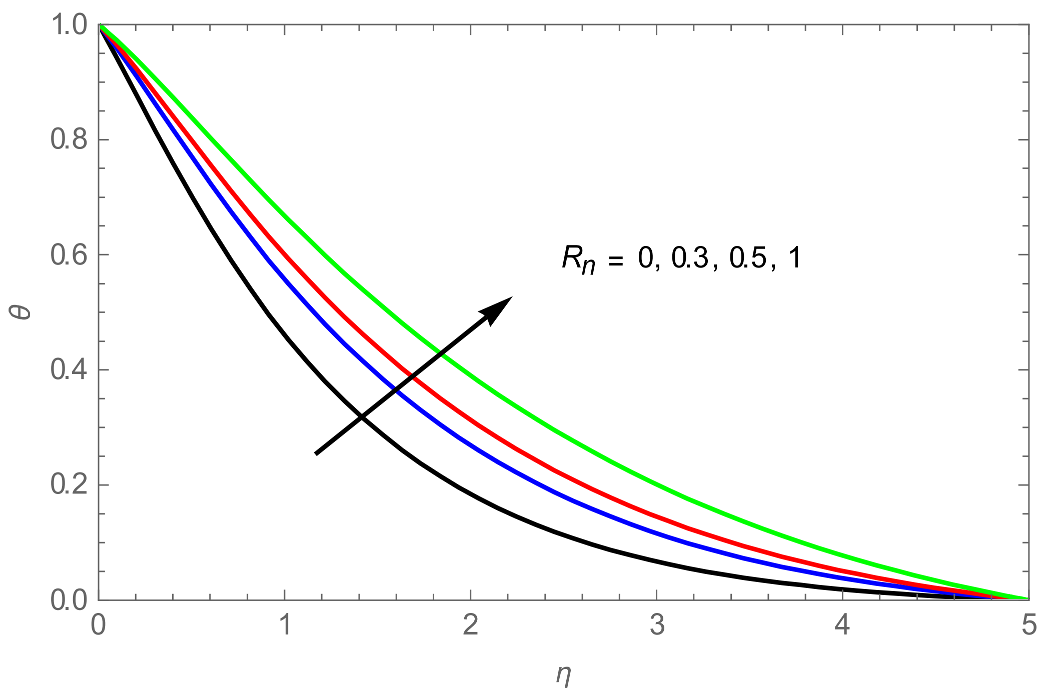

Fig.8: The plot for curves of $\theta(\eta)$ under the effect of R_n

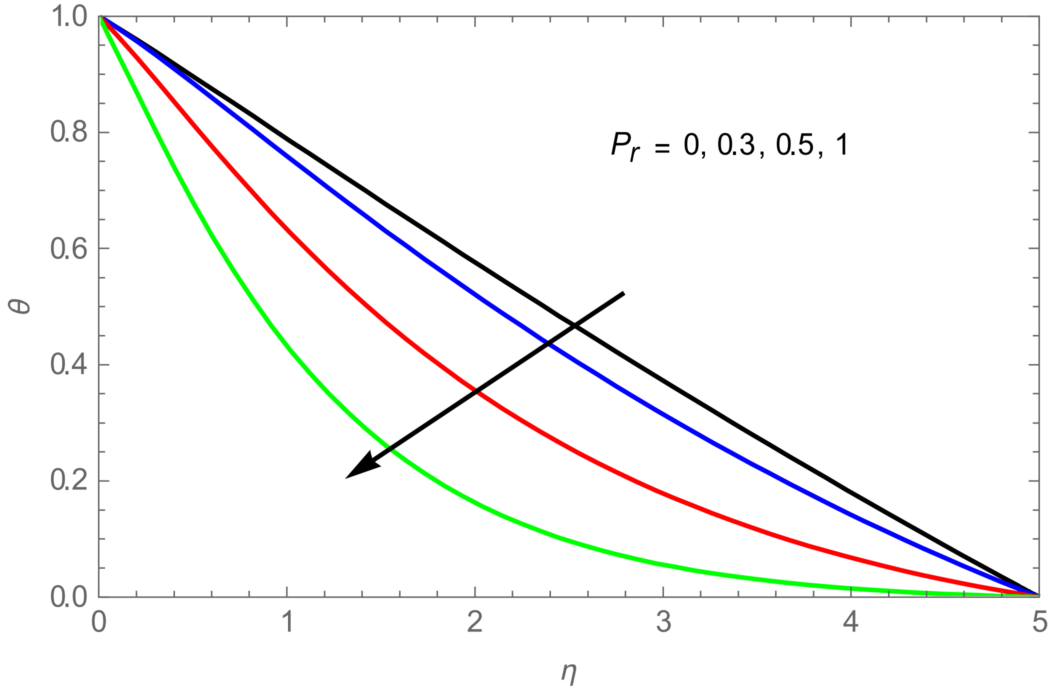

Fig.9: The plot for curves of $\theta(\eta)$ under the effect of P_r

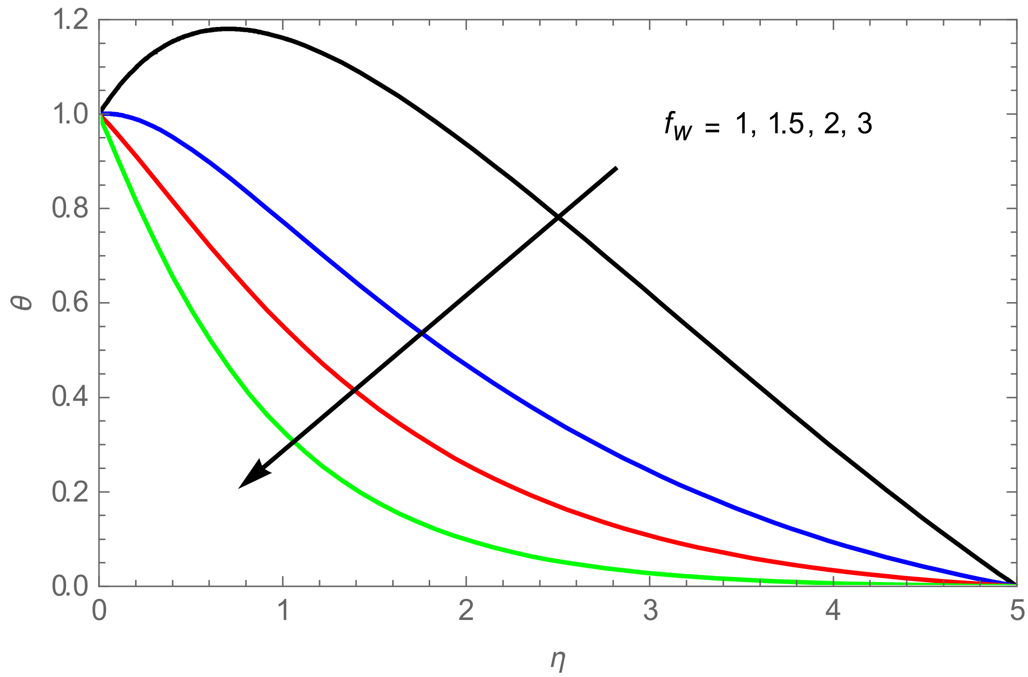

Fig.10: The plot for curves of $\theta(\eta)$ under the effect of suction/injection f_w

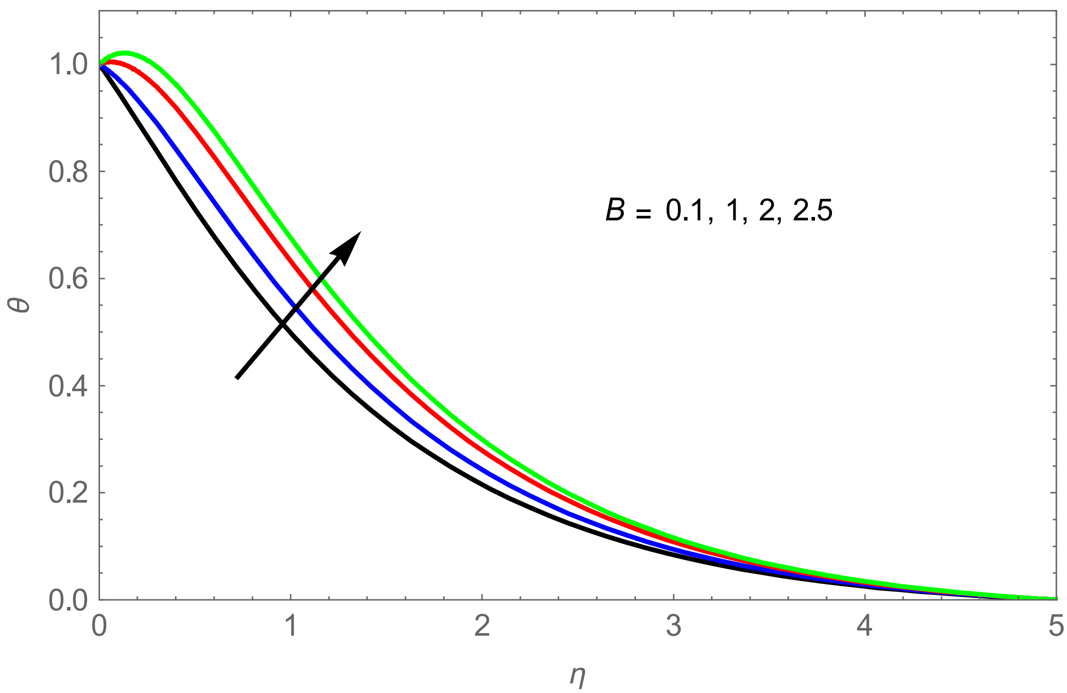

Fig.11: The plot for curves of $\theta(\eta)$ under the effect of parameter *B*

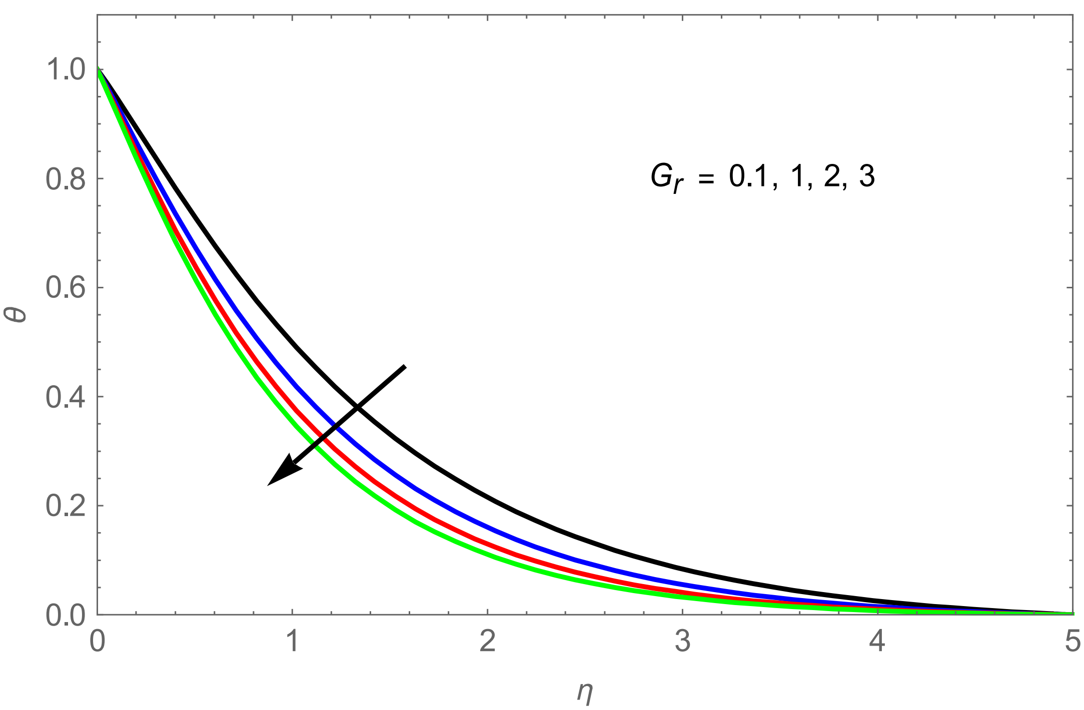

Fig.12: The plot for curves of $\theta(\eta)$ under the effect of Gr.

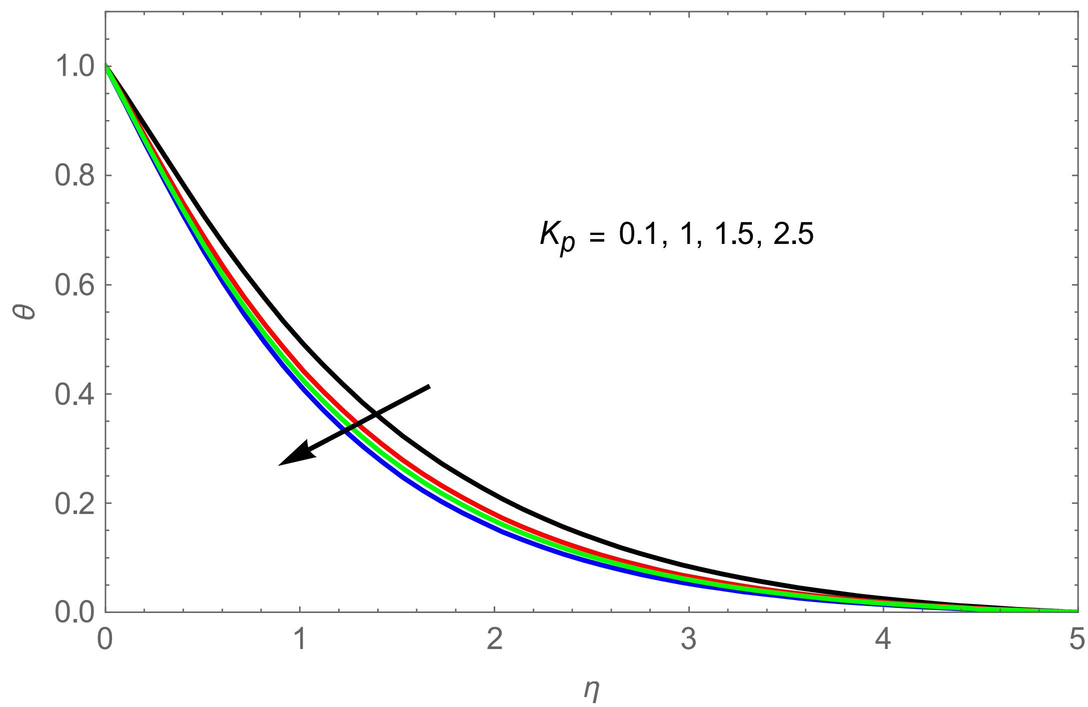

Fig.13: The plot for curves of $\theta(\eta)$ under the effect of Porosity parameter K_p

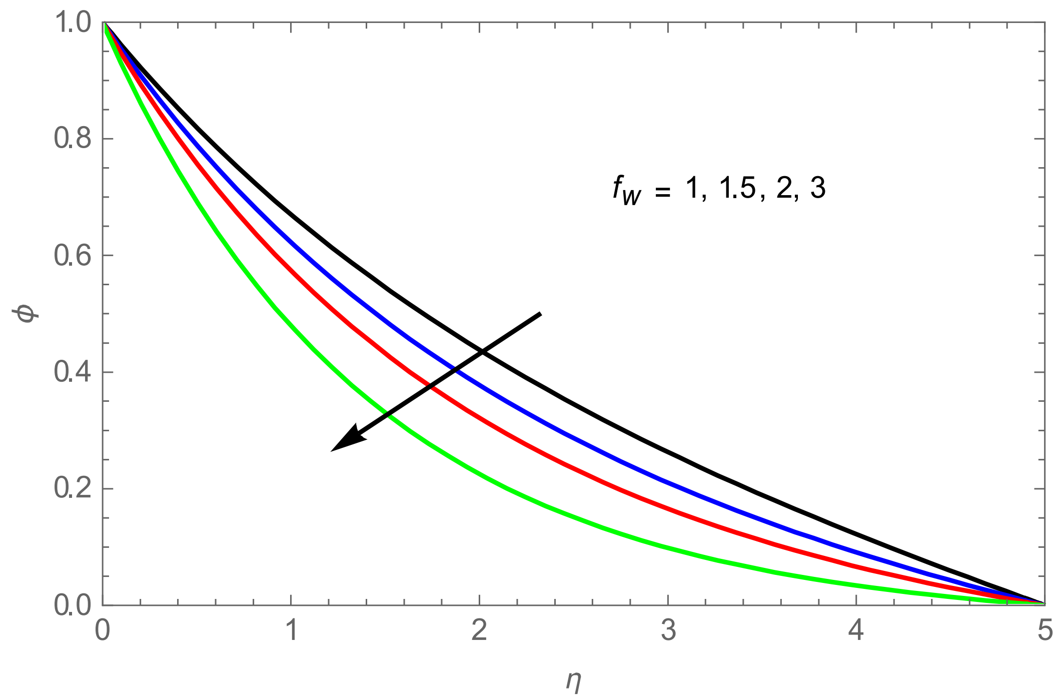

Fig.14: The plot for curves of $\phi(\eta)$ under the effect of suction/injection f_w

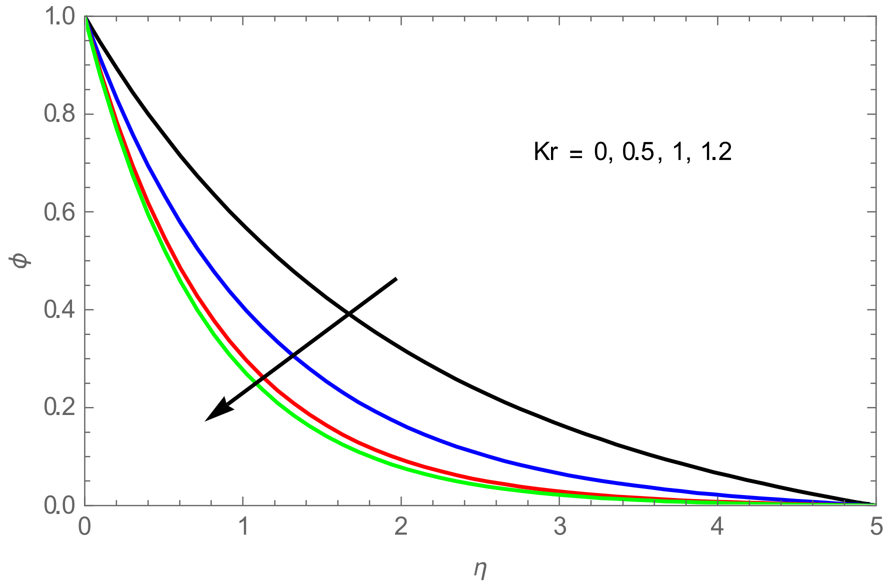

Fig.15: The plot for curves of $\phi(\eta)$ under the effect of chemical reaction parameter K_r

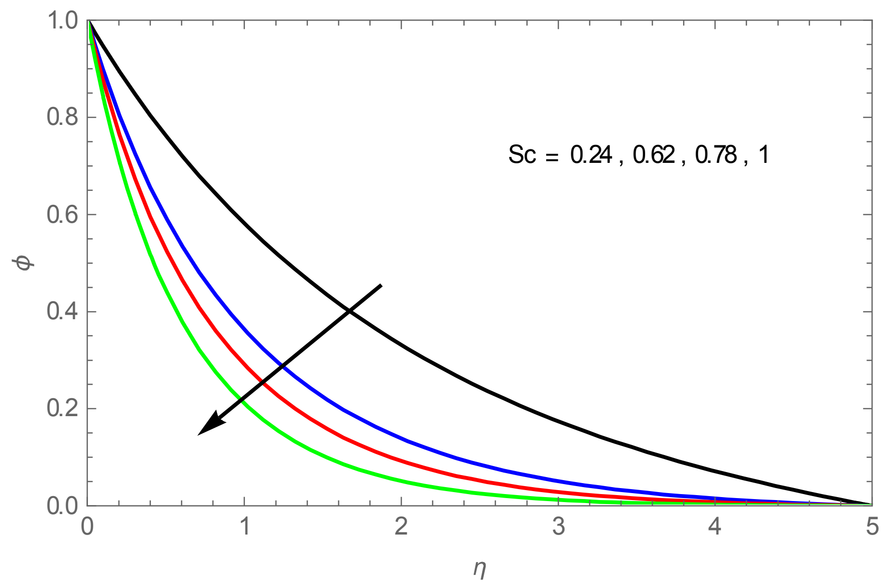

Fig.16: The plot for curves of $\phi(\eta)$ under the effect of Schmidt number S_c

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