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Radiation Effects and Chemical Reaction on Boundary Layer Flow of Electrically Conducting Casson Fluids Owing to Porous Shrinking Sheet

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ABSTRACT

This article considers radiative heat transfer and magneto hydrodynamic boundary layer f stagnation point flow of Casson fluids due to shrinking sheet. The set of model equations in the form of partial derivatives has been transformed to its ordinary differential form numerical solution of the finally governing equations is sought by using codes in computational software Mathematica. The physical behavior of the problem is revealed through plots of velocity field, temperature field and concentration field under the effects of emerging parameters namely Casson parameter β, Magnetic field parameter *M*, sheet shrinking parameter $\frac{b}{a}$, Schmidt number S_c, chemical reaction parameter γ, radiation parameter R_n, Heat source parameter *S* Prandtl number P_r, Porosity parameter *k* and Wall suction parameter f_w. The result are presented in graphical form.

KEYWORDS: Heat transfer, Boundary layer, stagnation point, Casson fluids, Mathematica, Prandtl number.

1. INTRODUCTION

Among the non-Newtonian fluid models, one of them is Casson fluid model which was introduced by Casson in 1959. It is based on the model structure and its behavior of both liquid and solid of a two-phase suspension that exhibits yield stress. Examples of Casson fluid are as follows: Jelly, tomato sauce, honey, soup, concentrated fruit juices. Human blood is also treated as Casson fluid. Many researchers have been more interest in Casson fluid model and explained the boundary layer problem. Many fluids in industries resemble non-Newtonian behavior. Non-Newtonian fluids are more appropriate than Newtonian fluids because of their varied industrial applications like petroleum drilling, polymer engineering, certain separation processes, food manufacturing etc. For non-Newtonian fluids, the relationship between stress and the rate of strain is not linear and it is difficult to express all these properties in a single constitutive equation. Consequently, these fluid models in [1-9] have been proposed depending on various physical characters. Casson fluid is one such type of such non-Newtonian fluid, which behaves like an elastic solid, with a yield shear stress existing in the constitutive equation. This fluid model has its origin in modelling of flow of many biological fluids especially blood. Examples of such fluids include foams, yoghurt, molten chocolate, cosmetics, nail polish, tomato puree etc. Casson [10] introduced this model to predict the flow behavior of pigment oil suspensions of the printing ink type. Later on, several researchers studied Casson fluid pertaining to different flow situations. The unsteady boundary layer flow and heat transfer of a Casson fluid over a moving flat plate with a parallel free stream was studied by Mustafa et al. [11]. The exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet with and without external magnetic field was discussed by Bhattacharyya et al. [12-13]. The Casson fluid has an infinite viscosity at zero rate of shear and a yield stress below which no flow occurs and a zero velocity at an infinite shear rate [14-15]. An excellent collection of articles can be found in [16-17]. Kameswarn [18] investigated Dual solutions of Casson fluid flow over a stretching sheet. Chamkha [19] described Hydromagnetic three dimensional free convection on a vertical stretching sheet with heat generation. Sulochana et al. [20] described the effect of heat source/sink on three dimensional Casson fluid with Soret and thermal radiation. Unsteady three dimensional flow of casson-carreau fluid is explained by Raju and Sandeep [21]. Similarity solution of three dimensional Casson nanofluid with convective conditions is explained by Sulochana et al. [22]. Animasaun Isaac Lare [23] explained Casson fluid flow over an exponential stretching sheet with heat generation. Khalid et al. [24] studied the unsteady free convection flow of Casson fluid past an oscillating

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vertical plate with constant wall temperature. Three-dimensional MHD boundary layer flow of Casson nanofluid through a linearly stretching surface with convective boundary condition was depicted by Nadeem et al. [25]. Akbar [26] studied the exact solutions of the magnetic field effect on peristaltic flow of Casson fluid in an asymmetric channel in presence of crude oil refinement. MHD flow of Casson fluid over a stretching surface in presence of Dufour and Soret effects was analyzed by Hayat et al. [27]. Khalid et al. [28] discussed an unsteady free convection MHD flow of Casson fluid through an oscillating vertical plate embedded in a porous medium with constant wall temperature. The stagnation-point flow of non-Newtonian incompressible Casson fluid past a stretching surface in presence of Dufour and Soret effects was depicted by Kameswarani et al. [29].
In this study they showed that shrinking case reduces the velocity boundary layer thickness and enhances the concentration boundary layer thickness. The effects of Casson parameters, the suction at surface, radiation parameters extends the previous study of the problem considered by Dash et al [30] MHD flow heat and mass diffusion of electrically conducting stagnation point flow past a stretching/shrinking sheet and chemical reaction.

2. MATHEMATICAL ANALYSIS

We consider the steady two-dimensional flow of an incompressible Casson fluid over an linearly shrinking sheet. The fluid is electrically conducting. A uniform magnetic field is applied normal to the sheet, and the induced magnetic field is neglected under the approximation of small Reynolds number. We also assume the rheological equation of Casson fluid, reported by Mustafa et al. [31], is:

$$\tau_{ij} = \left[\mu_B + \left(\frac{P_y}{\sqrt{2\pi}} \right)^{\frac{1}{n}} \right]^n 2e_{ij}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} denotes the $(i,j)^{th}$ component of the deformation rate, π be the product of the component of deformation rate itself, π_c be a critical value of this product based on the non-Newtonian model, μ_B be the plastic dynamic viscosity of the non-Newtonian fluid and P_y be the yield stress of the fluid.

The governing equation of the motion become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + U\frac{dU}{dx} - \frac{\upsilon}{K_1}\left(1+\frac{1}{\beta}\right)u - \frac{\sigma B_0^{\,2}(u-U)}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}\left(T - T_\infty\right) + \frac{16\alpha}{3\beta\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R(C - C_\infty) \tag{4}$$

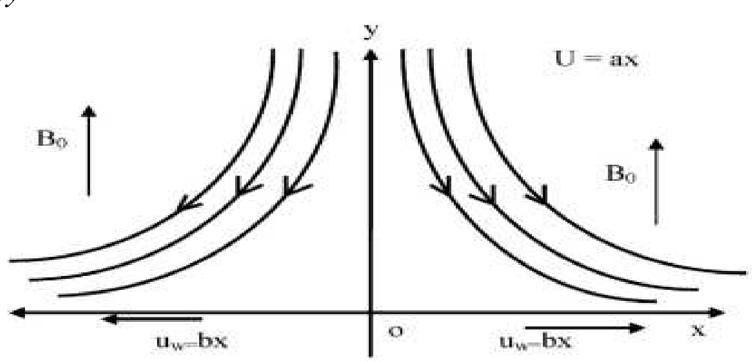

Fig. 1. Flow geometry.

$$u = bx, \quad v = 0, \quad T = T_w, C = C_w \qquad \text{at } y=0$$

$$u \to U = ax, \quad T \to T_\infty, C \to C_\infty \qquad \text{at } y \to \infty \tag{5}$$

Using similarity transformations:
The velocity components are described in terms of the stream function $\psi(x, y)$ as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \psi(x, y) = x\sqrt{a\upsilon} f(\eta), \ \eta = y\sqrt{\frac{a}{\upsilon}}$$

$$u = xaf', \ v = -\sqrt{\upsilon a} f, \ T = T_\infty + (T_w - T_\infty)\theta(\eta), C = C_\infty + (C_w - C_\infty)\phi(\eta) \tag{6}$$

$$\left(1 + \frac{1}{\beta}\right) f''' - M(f' - 1) + 1 - \left(1 + \frac{1}{\beta}\right) Kf' = f'^2 - ff'' \tag{7}$$

$$(4 + 3R_n)\theta'' + 3R_n \Pr(f\theta' + P_r S\theta) = 0 \tag{8}$$

$$\phi'' + S_c(f\theta' - \gamma\phi) = 0 \tag{9}$$

The associated boundary conditions (6) are:

$$f = f_w, f' = b/a, \theta = 1, \phi = 1 \quad as \quad \eta = 0$$
$$f' = 1, \theta = 0, \phi = 0 \qquad as \quad \eta \to \infty \tag{10}$$

where $P_r = \frac{\upsilon}{k}$ is the Prandtl number, $K = \frac{\upsilon}{aK_1}$ is the permeability parameter, $M = \sqrt{\frac{\sigma}{\rho b}} B_0$ is the magnetic parameter, $\gamma = \frac{R}{a}$ is non-dimensional rate of solutal. $Sc = \frac{\upsilon}{D}$ Schmidt number. Non dimensional temperature $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$, non dimensional concentration $\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$,

The set of resulting equations (7) to (10) is highly non-linear differential equation. Numerical solution of the problem has been sought out by ND solve command and codes in computational software Mathematica. The physical insight of the problem has been explored through several computations with sufficient ranges of the parameters involved in the study. The values of the parameter are chosen arbitrary. The fixed values are used as $M = 1, R = 0.5, \frac{b}{a} = -1.24, K = 0.1, \beta = 1$. The results have been presented in the form of plots for the curves of f', θ and ϕ.

Fig. 2 shows the effect of shrinking parameter $\frac{b}{a}$ on the horizontal velocity f'. The magnitude of the velocity increases with increase of shrinking parameter. The result corresponds to the boundary layer effects of viscous fluids and the usual back flow due shrinking phenomenon has been controlled with function and magnetic field.

The effect of magnetic parameter on flow field is demonstrated through curves of f' as presented in fig.3. It is to note that increase in the value of magnetic parameter causes increase in $|f'|$.Fig.4 is denoted to reveal that velocity f' decreases with increase in the porosity parameter but reverse ? effect for casson parameter is observed as shown in fig.5.

The radiation parameter R_n and that source parameter S $(S > 0)$ show increasing effect on temperature distribution as depicted respectively in fig.6 and fig.7. But Prandtl number and shrinking parameters have decreasing effect on temperature function $\theta(\eta)$ as shown respectively in fig.8 and fig.9.

The increase in chemical reaction parameter γ causes increase in concentration $\theta(\eta)$ as presented in fig.10. Fig. 11 reveal that Schmidt number S_c causes a significant decrease in concentration and shrinking parameter also causes a small decrease in concentration as demonstrate in fig.12.

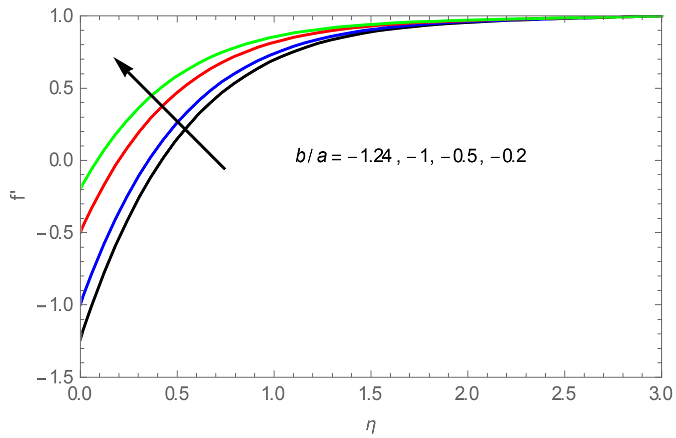

Fig.2: The plot for curves of f' for different values of $\frac{b}{a}$ $f_w = 2, M = 1, K = 0.1, \beta = 1$

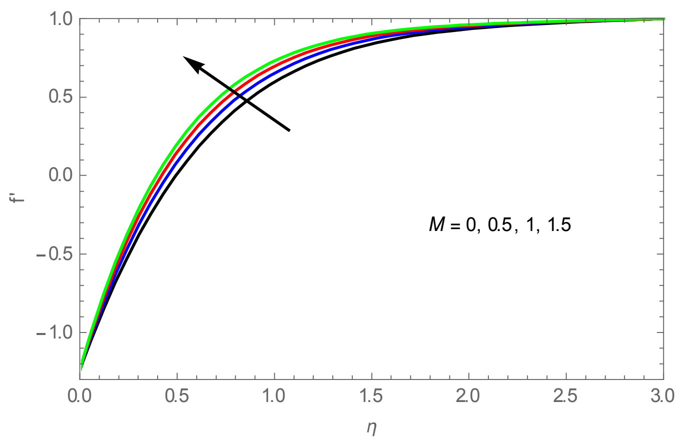

Fig.3: The plot for curves of f' under the effect of magnetic parameter *M*. $f_w = 2, \frac{b}{a} = -1.24, K = 0.1, \beta = 1$

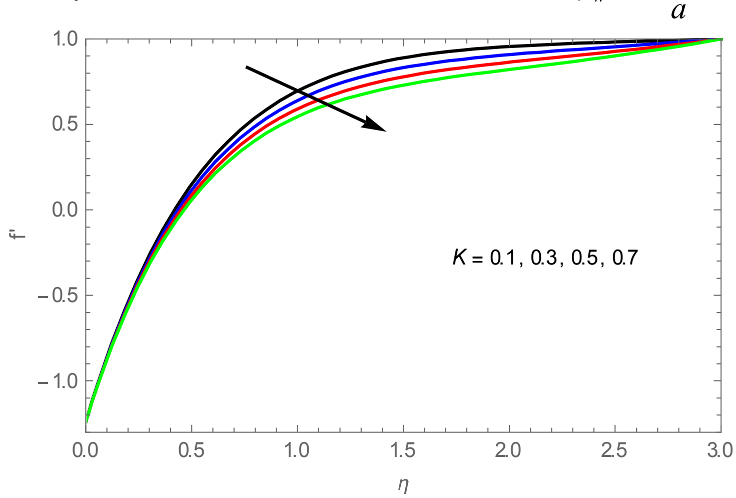

Fig.4: The plot for curves of f' under the effect of porosity parameter *K*. $f_w = 2, \frac{b}{a} = -1.24, M = 1, \beta = 1$

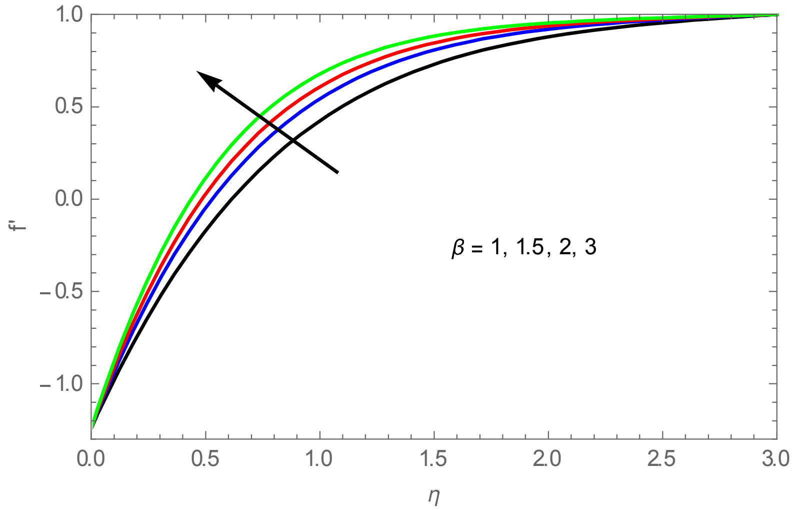

Fig.5: The plot for curves of f' under the effect of caisson parameter β. $f_w = 2, \frac{b}{a} = -1.24, M = 1, K = 0.1,$

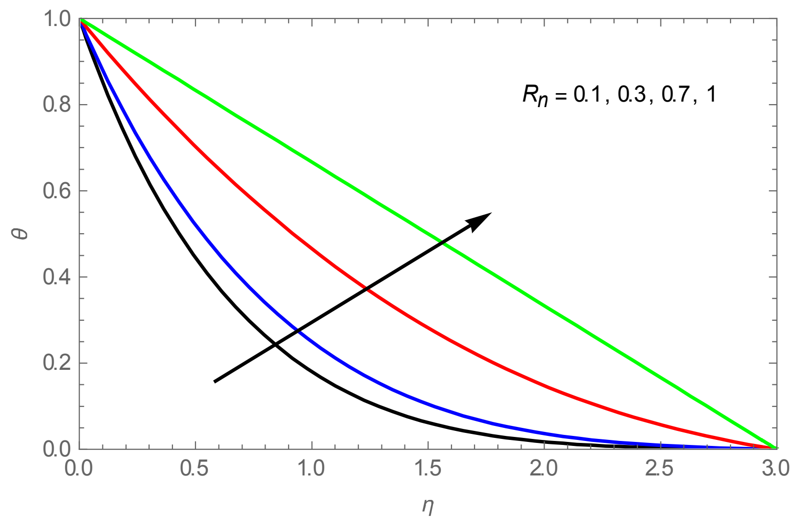

Fig.6: The plot for curves of θ under the effect of radiation parameter R_n.
$f_w = 2, M = 1, K = 0.1, \beta = 1, S = 0.2, \Pr = 1,$

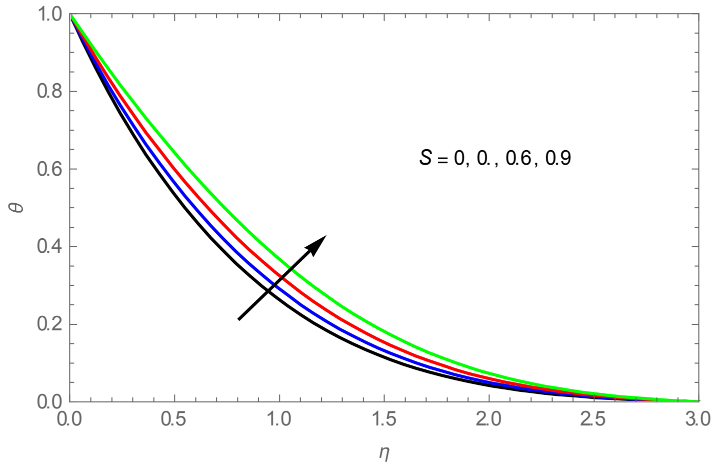

Fig.7: The plot for curves of θ under the effect of S. $f_w = 2, M = 1, K = 0.1, \beta = 1, Rn = 0.1, \Pr = 1,$

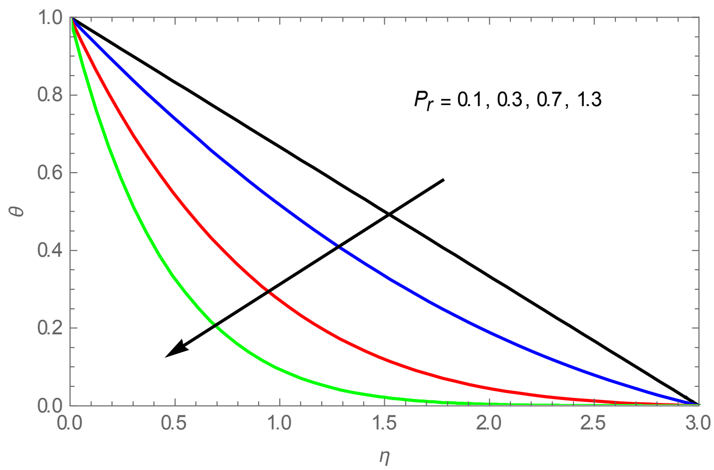

Fig.8: The plot for curves of θ under the effect of Prandtl number P_r

$$f_w = 2, M = 1, \frac{b}{a} = -1.24, K = 0.1, \beta = 1, Rn = 0.1$$

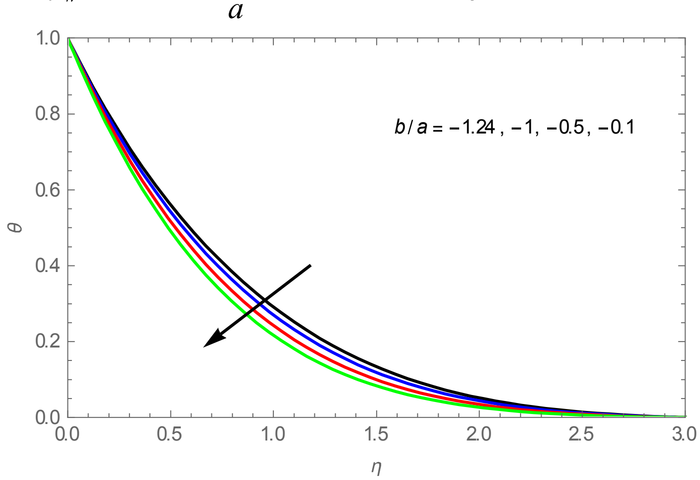

Fig.9: The plot for curves of θ under the effect for different values of $\frac{b}{a}$

$$f_w = 2, M = 1, K = 0.1, \beta = 1, S = 0.2, Rn = 0.1, \Pr = 1,$$

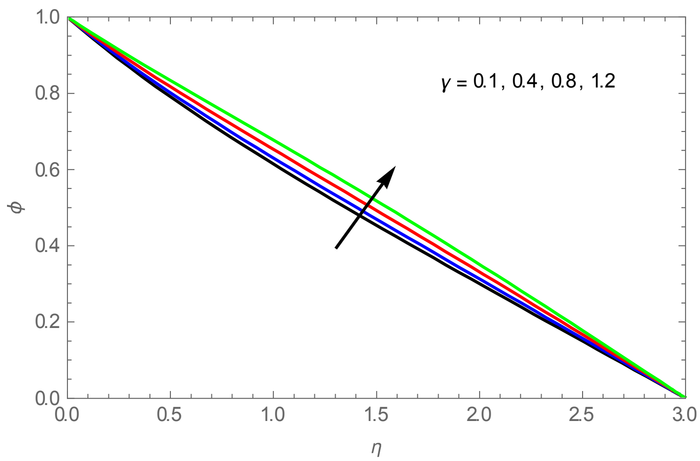

Fig.10: The plot for curves of ϕ under the effect of

γ $f_w = 2, M = 1, K = 0.1, \beta = 1, Rn = 0.1, \Pr = 1, Sc = 0.5$

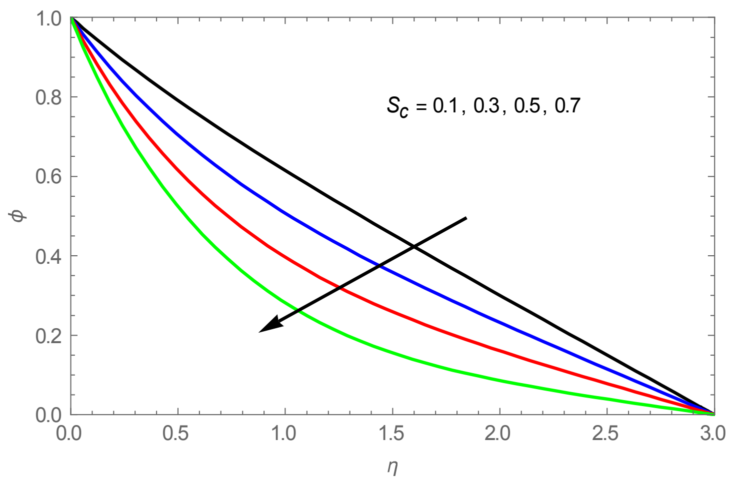

Fig.11: The plot for curves of ϕ under the effect of

S_c . $f_w = 2, M = 1, K = 0.1, \beta = 1, Rn = 0.1, \Pr = 1, \gamma = 0.1$

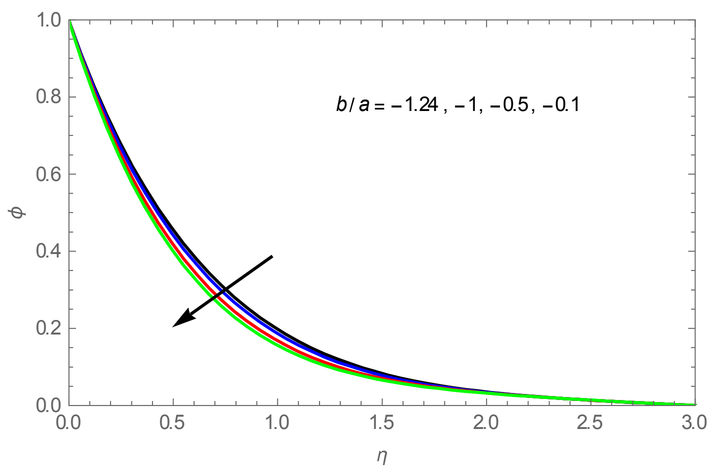

Fig.12: The plot for curves of ϕ under the effect of different values of $\frac{b}{a}$

$f_w = 2, M = 1, K = 0.1, \beta = 1, Rn = 0.1, \Pr = 1,$

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