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Regular and Intra-Regular Semihyper Groups in Terms of L-Fuzzy Soft Hyperideals

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ABSTRACT

In this paper, we introduce L-fuzzy soft quasi-hyperideal, L-fuzzy soft bi-hyperideal, L-fuzzy soft generalized bi-hyperideal, L-fuzzy soft interior hyperideal and discuss some important properties of these hyperideals. We use these hyperideals to characterize regular and intra-regular semihypergroups.
KEYWORDS AND PHRASES: L-fuzzy soft quasi-hyperideal, L-fuzzy soft bi-hyperideal, L-fuzzy soft generalized bi-hyperideal, L-fuzzy soft interior hyperideal and L-fuzzy soft Regular and Intra-regular semihypergroups.

1. INTRODUCTION

Most of the problems involving vagueness and uncertainties occur while modeling problems in medical sciences, engineering, social sciences, etc. To solve these problems many advanced mathematical tools were developed, like probability theory, fuzzy set theory, and rough set theory and interval mathematics. Each of these theories have their own intrinsic difficulties. In 1999, Molodtsov [21] introduced the concept of soft set theory as a completely innovative mathematical tool for modeling uncertainties in these fields. Therefore, it is said that the soft set theory is free from difficulties. In 2003, Maji et al. [19] defined the basic set theoretic operations of soft sets. In 2009, Ali et al. [3] introduced some new operations of soft sets and construct some counter examples. Many other authors also studied these structure further see [1, 4, 6, 11, 23, 25] and applied in their research areas. In 1967, Goguen [14] introduced L-fuzzy sets as a generalization of Zadeh's fuzzy sets [31]. In 2001, Maji et al. [18] initiated fuzzy soft sets and studied some operations. Later on, many mathematicians developed the work (see [2, 10, 12, 24, 28, 29]). In 2003, Li, Zheng and Hao [17] introduced L-fuzzy sof sets as an extension of fuzzy soft sets based on complete boolean lattice. In 2014, Ali, Shabir and Samina [5] applied L-fuzzy soft sets to semirings and investigated various important results. Shabir and Ghafoor [26] established L-fuzzy soft semigroups and several notions of the structure with examples. On the other hand, the concept of hyperstructure theory was proposed by Marty [20] in 1934, at the 8th congress of scandinavian mathematicians. The basic difference in a classical algebraic structure and an algebraic hyperstructure is that in a classical algebraic structure the composition of two elements is an element where as in algebraic hyperstructures the composition of two elements is a set. For this particular reason many mathematicians [7, 8, 9, 15, 22] attracted towards this direction and widely studied for their applications to various subjects of pure and applied mathematics. In [27] Shabir et al. initiated the study of L-fuzzy soft semihypergroups. In the present paper, we initiate the study of special type of the hyperideals of semihypergroups in the context of L-fuzzy soft sets, where L is a complete bounded distributive lattice. In Section 2, we give some basic definitions and results useful for further work. In Section 3, we introduce L-fuzzy soft quasi-hyperideals and prove some fundamental results. In Section 4, we define L-fuzzy soft bi-hyperideals and prove some related results. In Section 5, we define L-fuzzy soft generalized bi-hyperideals and illustrate some consequences. In Section 6, we introduce L-fuzzy soft interior hyperideals and construct some significant results. In Section 7, we characterize Regular and Intra-regular semihypergroups in terms of these L-fuzzy soft hyperideals. In Section 8, we give some concluding remarks.

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2. PRELIMINARIES

Definition 1 [7] *A non-empty set* S *together with a hyperoperation* $\circ$ *is called a hypergroupoid, where*

$$\circ : S \times S \rightarrow P^*(S)$$

and $P^*(S)$ *is the set of all non-empty subsets of* S. *We shall write* $\circ(a,b) = a \circ b$ *for any* $a,b \in S$ *. We denote a hypergroupoid by* $(S,\circ)$.

Definition 1 [7] *A hypergroupoid* $(S,\circ)$ *is called a semihypergroup if*

$$(a \circ b) \circ c = a \circ (b \circ c) \text{ for all } a,b,c \in S.$$

Let A,B *be the subsets of a semihypergroup* S *. Then the hyperproduct of* A *and* B *is defined by*

$$A \circ B = \cup_{x \in A,\, y \in B} (x \circ y).$$

We'll use $A \circ x$ *instead of* $A \circ \{x\}$ *and similarly* $x \circ A$ *instead of* $\{x\} \circ A$.

Definition 3 [7] *A non-empty subset* H *of a semihypergroup* S *is called a subsemihypergroup of* S *if* $H \circ H \subseteq H$ *, that is* $a \circ b \subseteq H$ *for all* $a,b \in H$.

Definition 4 [7] *Let* $(S,\circ)$ *be a semihypergroup and* $e \in S$ *. Then* e *is called the identity element of* S *, if* $e \in e \circ x = x \circ e$ *for all* $x \in S$.

Definition 5 [15] *A non-empty subset* I *of a semihypergroup* S *is called:*

1) a *left hyperideal* of S if for $a \in I$ and $b \in S$, we have $b \circ a \subseteq I$.
2) a *right hyperideal* of S if for $a \in I$ and $b \in S$, we have $a \circ b \subseteq I$.
3) a *hyperideal* of S, if it is both a left as well as a right hyperideal of S.

Definition 6 [9] *Let* x *be an element of a semihypergroup* S *. Then the smallest left (right) hyperideal of* S *generated by* x *is denoted by* $<x>_l \, (<x>_r)$ *, where* $<x>_l = (S \circ x) \cup \{x\} (<x>_r = (x \circ S) \cup \{x\})$ *and* $<x> = (S \circ x \circ S) \cup S \circ x \cup x \circ S \cup \{x\}$.

If S is a semihypergroup with identity element say e then $<x>_l = S \circ x (<x>_r = x \circ S)$ and $<x> = S \circ x \circ S$.

Definition 7 [8] *A non-empty subset* Q *of a semihypergroup* S *is called a Qausi-hyperideal of* S *if,* $Q \circ S \cap S \circ Q \subseteq Q$.

Definition 8 [8] *A subsemihypergroup* B *of a semihypergroup* S *is called a bi-hyperideal of* S *if,* $B \circ S \circ B \subseteq B$.

Definition 9 [8] *A non-empty subset* G *of a semihypergroup* S *is called a generalized bi-hyperideal of S if,* $G \circ S \circ G \subseteq G$.

Definition 10 [8] *A non-empty subset* I *of a semihypergroup* S *is called an interior hyperideal of* S *if,* $S \circ I \circ S \subseteq I$.

Definition 11 [8] *A semihypergroup* S *is called regular if, for all* $a \in S$ *there exists* $s \in S$ *such that* $a \in a \circ s \circ a$.

Definition 12 [8] *A semihypergroup* S *is called intra-regular, if for all* $a \in S$ *there exist* $x, y \in S$ *such that* $a \in x \circ a \circ a \circ y$.

Theorem 1 [8] *The following conditions for a semihypergroup* S *are equivalent:*

1) S is regular.
2) $A \circ B = A \cap B$, for every right hyperideal A and left hyperideal B of S .

Theorem 2 [8] *The following conditions for a semihypergroup* S *are equivalent:*

1) S is regular and intra-regular.
2) $B \circ B = B$ for each bi-hyperideal B of S .
3) $Q \circ Q = Q$ for each qausi-hyperideal Q of S .
4) $B_1 \cap B_2 = (B_1 \circ B_2) \cap (B_1 \circ B_2)$ for every bi-hyperideals B_1 and B_2 of S .
5) $R \cap L \subseteq (R \circ L) \cap (L \circ R)$ for every right hyperideal R and every left hyperideal L of S .
6) $<a>_r \cap <a>_l \subseteq (<a>_r \circ <a>_l) \cap (<a>_l \circ <a>_r)$ for all $a \in S$.

Definition 13 [17] *A partailly ordered set* $(L, \leq)$ *is called a lattice, if* $a \vee b \in L, a \wedge b \in L$ *for all* $a, b \in L$.

Definition 14 [17] *Let* $(L, \leq)$ *be a lattice. Then* L *is called*

1) a *complete lattice,* if $\vee N \in L, \wedge N \in L$ for every subset N of L .
2) a *bounded lattice,* if a top element $1_L \in L$ and a lower element $0_L \in L$.
3) a *distributive lattice,* if
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c), a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ for all $a, b, c \in L.$

Definition 15 [14] *Let* U *be a non-empty set and* L *be a complete bounded distributive lattice. Then an L-fuzzy set* A *in* U *is defined by a mapping* $A : U \to L$. *The set of all L-fuzzy sets in* U *is denoted by* L^U .

Definition 16 [17] *Let* U *be an initial universe,* E *be the set of parameters and* L *be a complete bounded distributive lattice and* $A \subseteq E$. *Then an L-fuzzy soft set* f_A *over* U *is defined as* $f_A : E \to L^U$ *such that* $f_A(e) = \tilde{0}$ *for all* $e \notin A$, *where* $\tilde{0}$ *is the L-fuzzy set in* U *which maps every element of* U *on* L .

Definition 17 [17] *Some basic operations of L-fuzzy soft sets are given below:*

1) The *union* of two L-fuzzy soft sets f_A and g_B over U is denoted by $f_A \tilde{\cup} g_B = h_{A \cup B}$, where
$h_{A \cup B}(x) = f_A(x) \cup g_B(x)$ for all $x \in E$.
2) The *intersection* of two L-fuzzy soft sets f_A and g_B over U is denoted by $f_A \tilde{\cap} g_B = h_{A \cap B}$, where
$h_{A \cap B}(x) = f_A(x) \cap g_B(x)$ for all $x \in E$.
3) Let f_A and g_B be two L-fuzzy soft sets over U . Then f_A is said to be a *subset* of g_B , if
$f_A(x) \subseteq g_B(x)$ for all $x \in E$ and is denoted by $f_A \tilde{\subseteq} g_B$.
4) Two L-fuzzy soft sets f_A and g_B over U are said to be *equal* if $f_A(x) = g_B(x)$ for all $x \in E$ and is denoted by $f_A = g_B$.

Definition 18 [27] *By an L-fuzzy soft set of a semihypergroup* S *over* U, *we mean a map* $f_A : S \to L^U$ *such that* $f_A(s) = \tilde{0}$, *for all* $s \notin A$, *where* $\tilde{0}$ *is the L-fuzzy set in* U *which map every element of* U *on* L *and* $A \subseteq S$.

The *product* of two L-fuzzy soft sets f_A and g_B of a semihypergroup S over U is defined as

$$(f_A \circ g_B)(x) = \begin{cases} \bigcup_{x \in a \circ b} \{f_A(a) \cap g_B(b)\}, & \text{if } \exists\, a, b \in S \text{ such that } x \in a \circ b \\ 0, & \text{otherwise.} \end{cases}$$

for all $x \in S$.

Definition 19 [27] *Let* S *be a semihypergroup and* $A \subseteq S$. *Let* U *be an initial universe and* L *be a complete bounded distributive lattice. Then an L-fuzzy soft set* $C_A : S \to L^U$ *is defined as*

$$C_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A. \end{cases}$$

called the L-fuzzy soft characteristic function of A *over* U.

Proposition 1 [27] *Let* S *be a semihypergroup and* A, $B \subseteq S$ *be any non-empty subsets of* S. *Then*

1) $A \subseteq B$ if and only if $C_A \subseteq C_B$.
2) $C_A \tilde{\cap} C_B = C_{A \cap B}$ and $C_A \tilde{\cup} C_B = C_{A \cup B}$.
3) $C_A \circ C_B = C_{A \circ B}$.

Definition 20 [27] *Let* S *be a semihypergroup. Then an L-fuzzy soft set* f_A *of* S *over* U *is called an L-fuzzy soft subsemihypergroup of* S *if for all* $a \in x \circ y$, *we have* $\cap_{a \in x \circ y} \{f_A(a)\} \supseteq f_A(x) \cap f_A(y)$ *for all* $x, y \in S$.

Proposition 2 [27] *An L-fuzzy soft set* f_A *of a semihypergroup* S *over* U *is an L-fuzzy soft subsemihypergroup of* S *if and only if* $f_A \circ f_A \subseteq f_A$.

Corollary 1 [27] *Let* A *be a non-empty subset of a semihypergroup* S. *Then* A *is a subsemihypergroup of* S *if and only if the L-fuzzy soft characteristic function* C_A *of* A *is an L-fuzzy soft subsemihypergroup of* S *over* U.

Definition 21 [27] *Let* $\alpha \in L^U$ *and* f_A *be an L-fuzzy soft set of a semihypergroup* S *over* U. *Then* $\alpha -$ *cut of* f_A *is denoted and defined as* $f_A^{\alpha} = \{x \in S : f_A(x) \supseteq \alpha\}$.

Proposition 3 [27] *An L-fuzzy soft set* f_A *of a semihypergroup* S *over* U *is an L-fuzzy soft subsemihypergroup of* S *if and only if each non-empty* α *-cut of* f_A *is a subsemihypergroup of* S.

Definition 22 [27] *Let* S *be a semihypergroup and* f_A *be an L-fuzzy soft set of* S *over* U. *Then* f_A *is called*

1) an *L-fuzzy soft left hyperideal* of S over U if for each $x, y \in S$, we have $\cap_{a \in x \circ y} f_A(a) \supseteq f_A(y)$.
2) an *L-fuzzy soft right hyperideal* of S over U if for each $x, y \in S$, we have $\cap_{a \in x \circ y} f_A(a) \supseteq f_A(x)$.
3) an *L-fuzzy soft hyperideal* of S over U if it is both an L-fuzzy soft left hyperideal and an L-fuzzy soft right hyperideal of S over U.

Corollary 2 [27] *Let S be a semihypergroup S and $\emptyset \neq A \subseteq S$. Then A is a left (right) hyperideal of S if and only if the L-fuzzy soft characteristic function C_A of A is an L-fuzzy soft left (right) hyperideal of S over U.*

Proposition 4 [27] *If f_A and g_B are L-fuzzy soft right hyperideal and L-fuzzy soft left hyperideal of a semihypergroup S over U, then $f_A \circ g_B \tilde{\subseteq} f_A \tilde{\cap} g_B$.*

3. L-Fuzzy Soft Qausi-Hyperideal

In this Section, we introduce the L-fuzzy soft quasi-hyperideal of a semihypergroup S over U. We shall take L as a complete bounded distributive lattice throughout this paper.

Definition 23. *An L-fuzzy soft set f_A of a semihypergroup S over U is called an L-fuzzy soft qausi-hyperideal of S over U, if $\left(f_A \circ \tilde{1}\right) \tilde{\cap} \left(\tilde{1} \circ f_A\right) \tilde{\subseteq} f_A$.*

Proposition 5. *Every L-fuzzy soft qausi-hyperideal of a semihypergroup S over U is an L-fuzzy soft subsemihypergroup of S.*

Proof. Straightforward.

Proposition 6. *The intersection of any family of L-fuzzy soft qausi-hyperideals of a semihypergroup S over U is an L-fuzzy soft qausi-hyperideal of S over U.*

Proof. Strightforward.

Corollary 3. *Let f_A and g_B be L-fuzzy soft right hyperideal and L-fuzzy soft left hyperideal of S over U, respectively. Then $f_A \tilde{\cap} g_B$ is an L-fuzzy soft qausi-hyperideal of S over U.*

Proof. Let f_A and g_B be L-fuzzy soft right hyperideal and L-fuzzy soft left hyperideal of S over U, respectively · Then

$$\left[\left(f_A \tilde{\cap} g_B\right) \circ \tilde{1}\right] \tilde{\cap} \left[\tilde{1} \circ \left(f_A \tilde{\cap} g_B\right)\right] \tilde{\subseteq} \left(f_A \circ \tilde{1}\right) \tilde{\cap} \left(\tilde{1} \circ g_B\right)$$
$$\tilde{\subseteq} f_A \tilde{\cap} g_B.$$

Thus $f_A \tilde{\cap} g_B$ is an L-fuzzy soft qausi-hyperideal of S over U.

Lemma 1. *If f_A is an L-fuzzy soft set of a semihypergroup S over U, then $\left(f_A \tilde{\cup} \left(\tilde{1} \circ f_A\right)\right)$ and $\left(f_A \tilde{\cup} \left(f_A \circ \tilde{1}\right)\right)$ are L-fuzzy soft left and right hyperideals of S over U, respectively.*

Proof. Let f_A be an L-fuzzy soft set of S over U. Then

$$\begin{aligned}
\tilde{1} \circ \{f_A \tilde{\cap} (\tilde{1} \circ f_A)\} &= (\tilde{1} \circ f_A) \tilde{\cap} \{\tilde{1} \circ (\tilde{1} \circ f_A)\} \\
&= (\tilde{1} \circ f_A) \tilde{\cap} \{(\tilde{1} \circ \tilde{1}) \circ f_A\} \\
&\tilde{\subseteq} (\tilde{1} \circ f_A) \tilde{\cap} (\tilde{1} \circ f_A) \\
&= \tilde{1} \circ f_A \\
&\tilde{\subseteq} f_A \tilde{\cap} (\tilde{1} \circ f_A).
\end{aligned}$$

Thus $f_A \tilde{\cap} (\tilde{1} \circ f_A)$ is an L-fuzzy soft left hyperideal of S over U. Similarly, we can show that $f_A \tilde{\cap} (f_A \circ \tilde{1})$ is an L-fuzzy soft right hyperideal of S over U.

Proposition 7. *A non-empty subset* Q *of a semihypergroup* S *is a qausi-hyperideal of* S *if and only if the L-fuzzy soft characteristic function* C_Q *of* Q *is an L-fuzzy soft qausi-hyperideal of* S *over* U.

Proof. Suppose that Q is a qausi-hyperideal of a semihypergroup S and C_Q is the L-fuzzy soft characteristic function of Q over U. Let a be an element of S.

If $a \in Q$, then $C_Q(a) = \tilde{1}$, so we have $\left[(C_Q \circ \tilde{1}) \tilde{\cap} (\tilde{1} \circ C_Q)\right](a) \leq \tilde{1} = C_Q(a)$.

If $a \notin Q$, then $C_Q(a) = \tilde{0}$. Let $\left[(C_Q \circ \tilde{1}) \tilde{\cap} (\tilde{1} \circ C_Q)\right](a) = \tilde{1}$.

Then $(C_Q \circ \tilde{1})(a) = \tilde{1}$ and $(\tilde{1} \circ C_Q)(a) = \tilde{1}$. This implies

$$\bigvee_{a \in x \circ y} \{C_Q(x) \wedge \tilde{1}(y)\} = \tilde{1} \text{ and } \bigvee_{a \in x \circ y} \{\tilde{1}(x) \wedge C_Q(y)\} = \tilde{1}.$$

This implies that there exist elements $b, c, d, e \in S$ with $a \in b \circ c$ and $a \in d \circ e$ such that $C_Q(b) = \tilde{1}$ and $C_Q(e) = \tilde{1}$. Hence $a \in b \circ c \subseteq Q \circ S$ and $a \in d \circ e \subseteq S \circ Q$, that is $a \in (Q \circ S) \cap (S \circ Q) \subseteq Q$, which contradicts our supposition that $a \notin Q$. Thus we have $(C_Q \circ \tilde{1}) \tilde{\cap} (\tilde{1} \circ C_Q) \tilde{\subseteq} C_Q$. Hence C_Q is an L-fuzzy soft qausi-hyperideal of the semihypergroup S over U.

Conversely, let C_Q be an L-fuzzy soft qausi-hyperideal of S over U. Let $a \in S$ be such that $a \in (Q \circ S) \cap (S \circ Q)$, that is $a \in Q \circ S$ and $a \in S \circ Q$. Then there exist elements $s, t \in S$ and $b, c \in Q$ such that $a \in b \circ s$ and $a \in t \circ c$. Thus we have

$$\begin{aligned}
(C_Q \circ \tilde{1})(a) &= \bigvee_{a \in x \circ y} \{C_Q(x) \wedge \tilde{1}(y)\} \\
&\geq C_Q(b) \wedge \tilde{1}(s) \\
&= \tilde{1} \wedge \tilde{1} = \tilde{1}.
\end{aligned}$$

Which shows that $(C_Q \circ \tilde{1})(a) \geq 1$. But $(C_Q \circ \tilde{1})(a) \leq 1$, this implies that $(C_Q \circ \tilde{1})(a) = 1$.

Similarly, we can show that $(\tilde{1} \circ C_Q)(a) = 1$.

Hence by hypothesis $C_Q(a) \geq \left[(C_Q \circ \tilde{1}) \tilde{\cap} (\tilde{1} \circ C_Q)\right](a) = 1 \wedge 1 = 1$. Which shows that $a \in Q$ and thus we have $(Q \circ S) \cap (S \circ Q) \subseteq Q$. Therefore Q is a qausi-hyperideal of S.

Theorem 3. *If an L-fuzzy soft set* f_A *of a semihypergroup* S *over* U *is an L-fuzzy soft qausi-hyperideal of* S *, then each non-empty* $\alpha-$ *cut* f_A^α *of* f_A *is a qausi-hyperideal of* S *.*

Proof. Suppose that f_A is an L-fuzzy soft qausi-hyperideal of S over U and $\alpha \in L^U$ be such that $f_A^\alpha \neq \emptyset$. Let $a \in (f_A^\alpha \circ S) \cap (S \circ f_A^\alpha)$. Then $a \in f_A^\alpha \circ S$ and $a \in S \circ f_A^\alpha$. Let $b, c \in f_A^\alpha$ and $s, t \in S$ be such that $a \in b \circ s$ and $a \in t \circ c$. Thus by hypothesis

$$
\begin{aligned}
f_A(a) &\geq \left[(f_A \circ \tilde{1}) \tilde{\cap} (\tilde{1} \circ f_A)\right](a) \\
&= (f_A \circ \tilde{1})(a) \wedge (\tilde{1} \circ f_A)(a) \\
&= \left[\bigvee_{a \in p \circ q} \{f_A(p) \wedge \tilde{1}(q)\}\right] \wedge \left[\bigvee_{a \in u \circ v} \{\tilde{1}(u) \wedge f_A(v)\}\right] \\
&\geq \{f_A(b) \wedge \tilde{1}(s)\} \wedge \{\tilde{1}(t) \wedge f_A(c)\} \\
&\geq f_A(b) \wedge f_A(c) \\
&\geq \alpha \wedge \alpha \qquad \text{because } b, c \in f_A^\alpha \\
&= \alpha
\end{aligned}
$$

This implies $a \in f_A^\alpha$ and so $(f_A^\alpha \circ S) \cap (S \circ f_A^\alpha) \subseteq f_A^\alpha$. Hence f_A^α is a qausi-hyperideal of S.

Proposition 8. *Every one-sided L-fuzzy soft hyperideal of a semihypergroup* S *over* U *is an L-fuzzy soft qausi-hyperideal.*

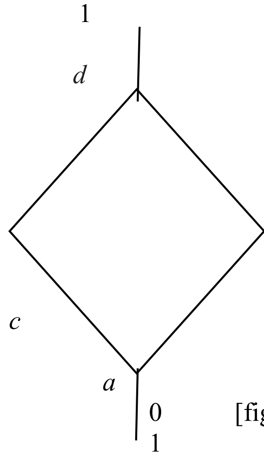
Proof. Straightforward.

Remark 1. *The converse of above Proposition is not true, in general.*

Example 1. *Consider the semihyperoup* $S = \{x, y, z, t\}$ *with hyperoperation* $\circ$ *defined in the following table*

$\circ$	x	y	z	t
x	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$
y	$\{x\}$	$\{x, y\}$	$\{x, z\}$	$\{x\}$
z	$\{x\}$	$\{x\}$	$\{x, y\}$	$\{x\}$
t	$\{x\}$	$\{x, t\}$	$\{x\}$	$\{x\}$

Let $U=\{p,q\}$ *and* $A=\{x,y\}\subseteq S$. *Let* $L=\{0,a,b,c,d,1\}$ *be a complete bounded distributive lattice shown in figure* 1 .



[figure 1]

Define $f_A : S\rightarrow L^U$ *as* $f_A(x)=\left\{\frac{p}{1},\frac{q}{a}\right\}, f_A(y)=\left\{\frac{p}{a},\frac{q}{d}\right\}, f_A(z)=f_A(t)=\bar{0}$. *Then* f_A *is an L-fuzzy soft quasi-hyperideal of* S *over* U .

Since

$\cap_{s\in y\circ y} f_A(s)=f_A(x)\cap f_A(z)=\left\{\frac{p}{1},\frac{q}{a}\right\}\cap\left\{\frac{p}{0},\frac{q}{0}\right\}=\left\{\frac{p}{0},\frac{q}{0}\right\}\not\supseteq\left\{\frac{p}{a},\frac{q}{d}\right\}=f_A(y)$.

Therefore f_A *is not an L-fuzzy soft right hyperideal of* S *over* U .

In addition $\cap_{s\in t\circ y} f_A(s)=f_A(x)\cap f_A(t)=\left\{\frac{p}{1},\frac{q}{a}\right\}\cap\bar{0}=\bar{0}\not\supseteq\left\{\frac{p}{a},\frac{q}{d}\right\}=f_A(y)$. Therefore f_A is not an L-fuzzy soft left hyperideal of S over U .

4. L-Fuzzy Soft Bi-Hyperideal

In this section, we define the L-fuzzy soft bi-hyperideals and prove some related results.

Definition 21. *An L-fuzzy soft subsemihypergroup* f_A *of a semihypergroup* S *over* U *is called an L-fuzzy soft bi-hyperideal of* S *over* U *if* $\cap_{s\in a\circ b\circ c}\{f_A(s)\}\supseteq f_A(a)\cap f_A(c)$ *for all* $a,b,c\in S$.

Proposition 9. *Let* f_A *be an L-fuzzy soft set of a semihypergroup* S *over* U . *Then* f_A *is an L-fuzzy soft bi-hyperideal of* S *over* U *if and only if*

(1) $f_A\circ f_A\subseteq f_A$.

(2) $f_A\circ\bar{1}\circ f_A\subseteq f_A$.

Proof. Suppose that f_A is an L-fuzzy soft bi-hyperideal of S over U . Then f_A is an L-fuzzy soft subsemihypergroup of S over U and by Proposition 2, $f_A\circ f_A\subseteq f_A$. Let $x\in S$. If there do not exist $y,z\in S$ such that $x\in y\circ z$ then $\left(f_A\circ\bar{1}\circ f_A\right)(x)=\bar{0}\subseteq f_A(x)$ and so, $f_A\circ\bar{1}\circ f_A\subseteq f_A$. Otherwise,

$$\begin{aligned}(f_A * \tilde{1} * f_A)(x) &= \bigcup_{x\in y\circ z}\left\{f_A(y)\cap\left(\tilde{1} * f_A\right)(z)\right\}\\ &= \bigcup_{x\in y\circ z}\left[f_A(y)\cap\left\{\bigcup_{z\in p\circ q}\left\{\tilde{1}(p)\cap f_A(q)\right\}\right\}\right]\\ &= \bigcup_{x\in y\circ z}\bigcup_{z\in p\circ q}\left\{f_A(y)\cap \tilde{1}(p)\cap f_A(q)\right\}\\ &= \bigcup_{x\in y\circ z}\bigcup_{z\in p\circ q}\left\{f_A(y)\cap f_A(q)\right\}.\end{aligned}$$

Since $x\in y\circ z$ and $z\in p\circ q$ this implies that $x\in y\circ z\subseteq y\circ(p\circ q)$. Since f_A is an L-fuzzy soft bi-hyperideal of S over U, so $\cap_{x\in y\circ p\circ q} f_A(x)\supseteq f_A(y)\cap f_A(q)$. This implies that $f_A(x)\supseteq f_A(y)\cap f_A(q)$. Thus

$$\begin{aligned}\cup_{x\in y\circ z}\cup_{z\in p\circ q}\{f_A(y)\cap f_A(q)\} &\subseteq \cup_{x\in y\circ z}\cup_{z\in p\circ q} f_A(x)\\ &= f_A(x).\end{aligned}$$

Therefore $f_A * \tilde{1} * f_A \subseteq f_A$.

Conversely, assume that $f_A * f_A \subseteq f_A$ and $f_A * \tilde{1} * f_A \subseteq f_A$. Since $f_A * f_A \subseteq f_A$, so by Proposition 2, f_A is an L-fuzzy soft subsemihypergroup of S over U. Now for $a,b,x\in S$, let $p\in a\circ x\circ b$. Then there exists $q\in a\circ x$ such that $p\in q\circ b$. Thus by hypothesis, we have

$$\begin{aligned}f_A(p) &\supseteq \left(f_A * \tilde{1} * f_A\right)(p)\\ &= \bigcup_{p\in u\circ v}\left\{\left(f_A * \tilde{1}\right)(u)\cap f_A(v)\right\}\\ &\supseteq \left(f_A * \tilde{1}\right)(q)\cap f_A(b)\\ &= \bigcup_{q\in s\circ t}\left\{f_A(s)\cap \tilde{1}(t)\right\}\cap f_A(b)\\ &\supseteq f_A(a)\cap \tilde{1}(x)\cap f_A(b)\\ &= f_A(a)\cap f_A(b)\end{aligned}$$

Thus $\cap_{p\in a\circ x\circ b} f_A(p)\supseteq f_A(a)\cap f_A(b)$.

Hence f_A is an L-fuzzy soft bi-hyperideal of S over U.

Proposition 10. *A non-empty subset* B *of a semihypergroup* S *is a bi-hyperideal of* S *if and only if the L-fuzzy soft characteristic function* C_B *of* B *is an L-fuzzy soft bi-hyperideal of* S *over* U.

Proof. Suppose that B is a bi-hyperideal of S. Then by Corollary 1, C_B is an L-fuzzy soft subsemihypergroup of S over U. Let $x,y,z\in S$.

If $x,z\in B$, then $C_B(x)=C_B(z)=\tilde{1}$. So for every $s\in x\circ y\circ z\subseteq B\circ S\circ B\subseteq B$, we have

$$C_B(s)=\tilde{1}=C_B(x)\cap C_B(z)$$

This implies $\bigcap_{s\in x\circ y\circ z} C_B(s)=\tilde{1}=C_B(x)\cap C_B(z)$

If $x\notin B$ or $z\notin B$, then $C_B(x)=\tilde{0}$ or $C_B(z)=\tilde{0}$. Thus we have

$$C_B(s) \supseteq \tilde{0} = C_B(x) \cap C_B(z)$$

This implies $\cap_{s \in x \circ y \circ z} C_B(s) \supseteq C_B(x) \cap C_B(z)$

Which shows that C_B is an L-fuzzy soft bi-hyperideal of S over U.

Conversely, assume that C_B is an L-fuzzy soft bi-hyperideal of S over U. Then by Corollary 1, B is a subsemihypergroup of S. Let $s \in B \circ S \circ B$. Then there exist $x, z \in B$ and $y \in S$ such that $s \in x \circ y \circ z$. Since

$$\cap_{s \in x \circ y \circ z} C_B(s) \supseteq C_B(x) \cap C_B(z) = \tilde{1} \cap \tilde{1} = \tilde{1}.$$

Hence for each $s \in x \circ y \circ z$, we have $C_B(s) = \tilde{1}$ and so $s \in B$. Thus $B \circ S \circ B \subseteq B$. Therefore, B is a bi-hyperideal of S.

Theorem 4. *An L-fuzzy soft set f_A of a semihypergroup S over U is an L-fuzzy soft bi-hyperideal of S over U if and only if each non-empty $\alpha-$ cut f_A^α of f_A is a bi-hyperideal of S.*

Proof. Suppose that f_A is an L-fuzzy soft bi-hyperideal of S over U. Then f_A is an L-fuzzy soft subsemihypergroup of S. By Proposition 3, f_A^α is a subsemihypergroup of S. Let $a, b \in f_A^\alpha$. Then $f_A(a) \supseteq \alpha$ and $f_A(b) \supseteq \alpha$. Let $s \in S$. Then by hypothesis

$$\cap_{x \in a \circ s \circ b} f_A(x) \supseteq f_A(a) \cap f_A(b)$$
$$\supseteq \alpha.$$

This implies $f_A(x) \supseteq \alpha$ for every $x \in a \circ s \circ b$ and so $x \in f_A^\alpha$. This shows that $f_A^\alpha \circ S \circ f_A^\alpha \subseteq f_A^\alpha$. Hence f_A^α is a bi-hyperideal of S.

Conversely, suppose that each non-empty subset f_A^α of f_A is a bi-hyperideal of S. Then f_A^α is a subsemihypergroup of S. By Proposition 3, f_A is an L-fuzzy soft subsemihypergroup of S over U. Now we show $\cap_{x \in a \circ s \circ b} f_A(x) \supseteq f_A(a) \cap f_A(b)$.

If $f_A(a) \cap f_A(b) = \tilde{0}$, then there is nothing to prove.

If $f_A(a) \cap f_A(b) \neq \tilde{0}$. Let $f_A(a) \cap f_A(b) = \alpha \in L^U$. Then

$$\alpha = f_A(a) \cap f_A(b) \subseteq f_A(a)$$
and $$\alpha = f_A(a) \cap f_A(b) \subseteq f_A(b)$$

This implies $a, b \in f_A^\alpha$. Since each $f_A^\alpha \neq \emptyset$ is a bi-hyperideal of S, so for all $s \in S$, $a \circ s \circ b \subseteq f_A^\alpha$. Let $x \in a \circ s \circ b$. Then $f_A(x) \supseteq \alpha$ for every $x \in a \circ s \circ b$.

Thus $\cap_{x \in a \circ s \circ b} f_A(x) \supseteq f_A(a) \cap f_A(b)$ and hence f_A is an L-fuzzy soft bi-hyperideal of S over U.

Proposition 11. *Every L-fuzzy soft qausi-hyperideal of a semihypergroup S over U is an L-fuzzy soft bi-hyperideal of S over U.*

Proof. Straightforward.

Corollary 4. *Every one-sided L-fuzzy soft hyperideal of a semihypergroup* S *over* U *is an L-fuzzy soft bi-hyperideal of* S *over* U *.*
Proof. Striaghtforward.

Remark 2. *The converse of above corollary is not true in general, as shown in the following Example.*
Example 2. *Let* $S=\{x,y,z,t\}$ *be a semihypergroup with hyperoperation* $\circ$ *defined in the following table*

$\circ$	x	y	z	t
x	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$
y	$\{x\}$	$\{x,y\}$	$\{x,t\}$	$\{x\}$
z	$\{x\}$	$\{x\}$	$\{x,y\}$	$\{x\}$
t	$\{x\}$	$\{x,t\}$	$\{x\}$	$\{x\}$

Let $U=\{p,q\}$ *be the initial universe and* $L=\{0,a,b,c,d,1\}$ *be a complete bounded distributive lattice of Example. Let* $B=\{x,y\}\subseteq S$ *. Define an L-fuzzy soft set* $f_B : S \to L^U$ *as* $f_B(x)=\left\{\frac{p}{1},\frac{q}{a}\right\}, f_B(y)=\left\{\frac{p}{b},\frac{q}{d}\right\}, f_B(z)=f_B(t)=\tilde{0}$ *. Then* f_B *is an L-fuzzy soft bi-hyperideal of* S *over* U *.*
As $\cap_{s\in y\circ z} f_B(s)=f_B(x)\cap f_B(t)=\left\{\frac{p}{1},\frac{q}{a}\right\}\cap\tilde{0}=\tilde{0}\nsupseteq\left\{\frac{p}{b},\frac{q}{d}\right\}=f_B(y)$ *. Therefore* f_B *is not an L-fuzzy soft right hyperideal of* S *over* U *.*
Further $\cap_{s\in t\circ y} f_B(s)=f_B(x)\cap f_B(t)=\left\{\frac{p}{1},\frac{q}{a}\right\}\cap\tilde{0}=\tilde{0}\nsupseteq\left\{\frac{p}{b},\frac{q}{d}\right\}=f_B(y)$. Therefore f_B is not an L-fuzzy soft left hyperideal of S over U .

Proposition 12. *The product of two L-fuzzy soft bi-hyperideals of* S *over* U *is an L-fuzzy soft bi-hyperideal of* S *over* U *.*
Proof. Straightforward.

5. L-Fuzzy Soft Generalized bi-hyperideal

In this Section, we define the L-fuzzy soft generalized bi-hyperideal. We shall also prove some fundamental results of L-fuzzy soft generalized bi-hyperideals.

Definition 25. *An L-fuzzy soft set* f_G *of a semihypergroup* S *over* U *is called an L-fuzzy soft generalized bi-hyperideal of* S *over* U *if* $\cap_{a\in x\circ y\circ z}\{f_G(a)\}\supseteq f_G(x)\cap f_G(z)$ *for all* $x,y,z\in S$ *.*

Proposition 13. *Let* f_G *be an L-fuzzy soft set of a semihypergroup* S *over* U *. Then* f_G *is an L-fuzzy soft generalized bi-hyerideal of* S *over* U *if and only if* $f_G \circ \tilde{1} \circ f_G \subseteq f_G$ *.*
Proof. The proof is similar to the proof of Proposition 9.

Proposition 14. *A non-empty subset G of a semihypergroup S is a generalized bi-hyperideal of S if and only if the L-fuzzy soft characteristic function C_G of G is an L-fuzzy soft generalized bi-hyperideal of S over U.*

Proof. The proof is similar to the proof of Proposition 10.

Theorem 5. *An L-fuzzy soft set f_G of a semihypergroup S over U is an L-fuzzy soft generalized bi-hyperideal of S over U if and only if each non-empty $\alpha-$ cut f_G^{α} of f_G is a generalized bi-hyperideal of S.*

Proof. The proof is similar to the proof of Theorem 4.

Remark 3. *Every L-fuzzy soft bi-hyperideal of a semihypergroup S over U is an L-fuzzy soft generalized bi-hyperideal of S over U. But the converse is not true in general, because every L-fuzzy soft set of a semihypergroup S over U is not an L-fuzzy soft subsemihypergroup.*

Example 3. *Consider the semihypergroup $S=\{x,y,z,t\}$ with hyperoperation $\circ$ defined by*

$\circ$	x	y	z	t
x	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$
y	$\{x\}$	$\{x\}$	$\{x,t\}$	$\{x\}$
z	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$
t	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$

Let $U=\{l,m\}$ be the initial universe and $L=\{0,a,b,c,d,1\}$ be a complete bounded distributive lattice of Example 1. Let $G=\{x,y,z\}\subseteq S$. Define an L-fuzzy soft set $f_G : S\rightarrow L^U$ as $f_G(x)=\left\{\frac{l}{1},\frac{m}{a}\right\}, f_G(y)=\left\{\frac{l}{a},\frac{m}{b}\right\}, f_G(z)=\left\{\frac{l}{c},\frac{m}{d}\right\}, f_G(t)=\tilde{0}$. Then f_G is an L-fuzzy soft generalized bi-hyperideal of S over U. For f_G to be an L-fuzzy soft bi-hyperideal, we calculate $f_G \circ f_G$.

For $t\in S$, we have

$$\begin{aligned}
(f_G \circ f_G)(t) &= \bigvee_{t\in u\circ v}\{f_G(u)\wedge f_G(v)\}\\
&= \{f_G(y)\wedge f_G(z)\}\\
&= \left\{\frac{l}{a},\frac{m}{b}\right\}\wedge\left\{\frac{l}{c},\frac{m}{d}\right\}\\
&= \left\{\frac{l}{c},\frac{m}{d}\right\}\neq\tilde{0}=f_G(t)
\end{aligned}$$

So, $f_G \circ f_G \not\tilde{\subseteq} f_G$ and hence by Proposition 9, f_G is not an L-fuzzy soft bi-hyperideal of S over U.

Proposition 15. *The product of two L-fuzzy soft generalized bi-hyperideals of S over U is an L-fuzzy soft generalized bi-hyperideal.*

Proof. The proof follows from Proposition 12.

6. L-Fuzzy Soft Interior hyperideal

In this Section, we define L-fuzzy soft interior hyperideal and prove some important results. We also give some counter examples.

Definition 26. *An L-fuzzy soft set* f_I *of a semihypergroup* S *over* U *is called an L-fuzzy soft interior hyperideal of* S *over* U *if* $\cap_{s\in a\circ b\circ c}\{f_I(s)\}\supseteq f_I(b)$ *for all* $a,b,c\in S$.

Proposition 16. *Let* f_I *be an L-fuzzy soft set of a semihypergroup* S *over* U *. Then* f_I *is an L-fuzzy soft interior hyperideal of* S *over* U *if and only if* $\tilde{1}\circ f_I\circ\tilde{1}\subseteq f_I$.

Proof. The proof is similar to the proof of Proposition 9.

Proposition 17. *A non-empty subset* I *of a semihypergroup* S *is an interior hyperideal of* S *if and only if the L-fuzzy soft characteristic function* C_I *of* I *is an L-fuzzy soft interior hyperideal of* S *over* U .

Proof. The proof is similar to the proof of Proposition 10.

Theorem 6. *An L-fuzzy soft set* f_I *of a semihypergroup* S *over* U *is an L-fuzzy soft interior hyperideal of* S *over* U *if and only if each non-empty* $\alpha-$ *cut* f_I^{α} *of* f_I *is an interior hyperideal of* S .

Proof. The proof is similar to the proof of Theorem 4.

Lemma 2. *Every L-fuzzy soft hyperideal* f_I *of a semihypergroup* S *over* U *is an L-fuzzy soft interior hyperideal of* S *over* U .

Proof. Straightforward.

Remark 4. *The converse of above lemma is not true in general, as shown in the following Example.*

Example 4. *Let* $S=\{x,y,z,t\}$ *be the semihypergroup of Example 3. Let* $U=\{p,q\}$ *be the initial universe and* $L=\{0,a,b,c,d,1\}$ *be a complete bounded distributive lattice of Example 1. Let* $I=\{x,y\}\subseteq S$ *. Define an L-fuzzy soft set* $f_I : S\to L^U$ *as* $f_I(x)=\left\{\frac{p}{1},\frac{q}{a}\right\}, f_I(y)=\left\{\frac{p}{1},\frac{q}{d}\right\}, f_I(z)=f_I(t)=\tilde{0}$ *. Then* f_I *is an L-fuzzy soft interior hyperideal of* S *over* U .

Now as $\cap_{s\in y\circ z} f_I(s)=f_I(x)\cap f_I(z)=\left\{\frac{p}{1},\frac{q}{a}\right\}\cap\tilde{0}=\tilde{0}\nsupseteq\left\{\frac{p}{1},\frac{q}{d}\right\}=f_I(y)$.

Therefore f_I *is not an L-fuzzy soft right hyperideal of* S *over* U *. Subsequently* f_I *is not an L-fuzzy soft hyperideal of* S *over* U .

Proposition 18. *Let* S *be a semihypergroup with identity* e *. Then* f_I *is an L-fuzzy soft hyperideal of* S *over* U *if and only if* f_I *is an L-fuzzy soft interior hyperideal of* S *over* U .

Proof. Straightforward.

7. Regular and Intra-regular Semihypergroups

In this Section, we characterize the regular and intra-regular semihypergroups by using the properties of their L-fuzzy soft hyperideals.

Theorem 7. *A semihypergroup S is regular if and only if $f_A \circ g_B = f_A \tilde{\cap} g_B$ for every L-fuzzy soft right hyperideal f_A and L-fuzzy soft left hyperideal g_B of S over U.*

Proof. Suppose that S is a regular semihypergoup. Let f_A and g_B be L-fuzzy soft right and L-fuzzy soft left hyperideals of S over U, respectively. Since S is regular, so for every $a \in S$ there exists $x \in S$ such that $a \in a \circ x \circ a$. Let $p \in x \circ a$ be such that $a \in a \circ p$. Then

$$
\begin{aligned}
(f_A \circ g_B)(a) &= \bigvee_{a \in b \circ c} \{ f_A(b) \wedge g_B(c) \} \\
&\geq f_A(a) \wedge g_B(p) \\
&\geq f_A(a) \wedge g_B(a) \quad \text{because } g_B(p) \geq \bigwedge_{p \in x \circ a} g_B(p) \geq g_B(a) \\
&= \left(f_A \tilde{\cap} g_B\right)(a)
\end{aligned}
$$

This implies $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B$ and by Proposition 4, $f_A \circ g_B \tilde{\subseteq} f_A \tilde{\cap} g_B$.

Hence $f_A \circ g_B = f_A \tilde{\cap} g_B$.

Conversely, suppose that $f_A \circ g_B = f_A \tilde{\cap} g_B$ for every L-fuzzy soft right hyperidal f_A and L-fuzzy soft left hyperideal g_B of S over U. Let A and B be right and left hyperideals of S, respectively. Then by Corollary 2, the L-fuzzy soft characteristic functions C_A and C_B of A and B are L-fuzzy soft right hyperideal and L-fuzzy soft left hyperideal of S over U, respectively. By hypothesis $C_A \circ C_B = C_A \tilde{\cap} C_B$ and by Proposition 1, we have

$$C_{A \circ B} = C_A \circ C_B = C_A \tilde{\cap} C_B = C_{A \cap B}.$$

This implies $A \circ B = A \cap B$. Hence by Theorem 1, S is a regular semihypergroup.

Theorem 8. *The following assertions are equivalent for a semihypergroup S.*

(1) S is regular.

(2) $f_A \circ \tilde{1} \circ f_A = f_A$ for every L-fuzzy soft generalized bi-hyperideal f_A of S over U.

(3) $f_A \circ \tilde{1} \circ f_A = f_A$ for every L-fuzzy soft bi-hyperideal f_A of S over U.

(4) $f_A \circ \tilde{1} \circ f_A = f_A$ for every L-fuzzy soft qausi-hyperideal f_A of S over U.

Proof. $(1) \Rightarrow (2)$ Let S be a regular semihypergroup and f_A be an L-fuzzy soft generalized bi-hyperideal of S over U. Since S is regular, so for every $a \in S$ there exists $s \in S$ such that $a \in a \circ s \circ a$. Let $x \in a \circ s$ be such that $a \in x \circ a$. Then

$$\begin{aligned}(f_A \circ \tilde{1} \circ f_A)(a) &= \bigvee_{a \in b \circ c} \left\{ (f_A \circ \tilde{1})(b) \wedge f_A(c) \right\} \\ &\geq (f_A \circ \tilde{1})(a) \wedge f_A(a) \\ &= \bigvee_{x \in p \circ q} \left\{ f_A(p) \wedge \tilde{1}(q) \right\} \wedge f_A(a) \\ &\geq f_A(a) \wedge \tilde{1}(x) \wedge f_A(a) \\ &= f_A(a) \wedge f_A(a) \\ &= f_A(a)\end{aligned}$$

Thus we have $f_A \circ \tilde{1} \circ f_A \,\widetilde{\supseteq}\, f_A$. Since f_A is an L-fuzzy soft generalized bi-hyperideal of S over U , so $f_A \circ \tilde{1} \circ f_A \,\widetilde{\subseteq}\, f_A$. Hence $f_A \circ \tilde{1} \circ f_A = f_A$.

$(2) \Rightarrow (3)$ and $(3) \Rightarrow (4)$ are straightforward.

$(4) \Rightarrow (1)$ Let f_A and g_B be L-fuzzy soft right and L-fuzzy soft left hyperideals of S over U , respectively. Then by Corollary 3, $f_A \widetilde{\cap} g_B$ is an L-fuzzy soft qausi-hyperideal of S over U . Hence by hypothesis

$$\begin{aligned}f_A \widetilde{\cap} g_B &= (f_A \widetilde{\cap} g_B) \circ \tilde{1} \circ (f_A \widetilde{\cap} g_B) \\ &\widetilde{\subseteq} f_A \circ \tilde{1} \circ g_B \\ &\widetilde{\subseteq} f_A \circ g_B.\end{aligned}$$

But $f_A \circ g_B \,\widetilde{\subseteq}\, f_A \widetilde{\cap} g_B$ is always true, for every L-fuzzy soft right hyperideal f_A and L-fuzzy soft left hyperideal g_B of S over U . Hence $f_A \circ g_B = f_A \widetilde{\cap} g_B$. Therefore by Theorem 7, S is a regular semihypergroup.

Theorem 9. *The following assertions are equivalent for a semihypergroup* S :

1) S is regular.
2) $f_A \widetilde{\cap} g_B \,\widetilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft left hyperideal g_B of S over U .
3) $f_A \widetilde{\cap} g_B \,\widetilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft left hyperideal g_B of S over U .
4) $f_A \widetilde{\cap} g_B \,\widetilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft left hyperideal g_B of S over U .

Proof. $(1) \Rightarrow (2)$ Let f_A and g_B be any L-fuzzy soft generalized bi-hyperideal and L-fuzzy soft left hyperideal of S over U , respectively. Since S is regular, so for every $a \in S$ there exists $x \in S$ such that $a \in a \circ x \circ a$. Let $p \in x \circ a$ be such that $a \in a \circ p$. Then

$$(f_A \tilde{\circ} g_B)(a) = \bigvee_{a\in b\circ c}\{f_A(b) \wedge g_B(c)\}$$
$$\geq f_A(a) \wedge g_B(p)$$

Also $g_B(p) \geq \bigvee_{p\in x\circ a} g_B(a)$

$\geq g_B(a)$ because g_B is L-fuzzy soft left hyperideal.

Thus we have $(f_A \tilde{\circ} g_B)(a) \geq f_A(a) \wedge g_B(a) = (f_A \tilde{\cap} g_B)(a)$ and hence $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\cap} g_B$.

$(2)\Rightarrow(3)\Rightarrow(4)$ are straight forward.

$(4)\Rightarrow(1)$ Let f_A and g_B be any L-fuzzy soft right hyperideal and L-fuzzy soft left hyperideal of S over U, respectively. Then f_A is an L-fuzzy soft qausi-hyperideal of S over U. Then by hypothesis $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \tilde{\circ} g_B$. But $f_A \tilde{\circ} g_B \tilde{\subseteq} f_A \tilde{\cap} g_B$ always holds, for every L-fuzzy soft right hyperideal f_A and L-fuzzy soft left hyperideal g_B of S over U. Thus $f_A \tilde{\cap} g_B = f_A \tilde{\circ} g_B$. Therefore by Theorem 7, S is a regular semihypergroup.

Theorem 10. *The following assertions are equivalent for a semihypergroup* S:

1) S is regular.
2) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \tilde{\circ} g_B$ for every L-fuzzy soft right hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
3) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \tilde{\circ} g_B$ for every L-fuzzy soft right hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
4) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \tilde{\circ} g_B$ for every L-fuzzy soft right hyperideal f_A and every L-fuzzy soft qausi- hyperideal g_B of S over U.

Proof. The proof is similar to the proof of Theorem 9.

Theorem 11. *The following assertions are equivalent for a semihypergroup* S:

1) S is intra-regular.
2) $L \cap R \subseteq L \circ R$ for every right hyperideal R and every left hyperideal L of S.
3) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \tilde{\circ} g_B$ for every L-fuzzy soft right hyperideal g_B and every L-fuzzy soft left hyperideal f_A of S over U.

Proof. $(1)\Rightarrow(3)$ Let f_A and g_B be any L-fuzzy soft left hyperideal and L-fuzzy soft right hyperideal of S over U, respectively and $a \in S$. Since S is intra-regular, so there exist $x, y \in S$ such that $a \in x \circ a \circ a \circ y$. Let $p \in x \circ a$ and $q \in a \circ y$ be such that $a \in p \circ q$. Then we have

$$(f_A \tilde{\circ} g_B)(a) = \bigvee_{a\in s\circ t}\{f_A(s) \wedge g_B(t)\}$$
$$\geq f_A(p) \wedge g_B(q)$$

Since f_A and g_B are L-fuzzy soft left and L-fuzzy soft right hyperideals of S over U, respectively. Therefore

$$f_A(p) \supseteq \cap_{p \in x \circ a} f_A(p) \supseteq f_A(a)$$
$$\text{and } g_B(q) \supseteq \cap_{q \in a \circ y} g_B(q) \supseteq g_B(a).$$

Thus

$$(f_A \tilde{\wedge} g_B)(a) \subseteq f_A(p) \cap g_B(q)$$
$$\subseteq f_A(a) \cap g_B(a)$$
$$= \left(f_A \tilde{\circ} g_B\right)(a)$$

Hence $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B$.

$(3) \Rightarrow (2)$ Let L and R be any left and right hyperideals of S , respectively. Then by Corollary 2, the L-fuzzy soft characteristic function C_L of L is an L-fuzzy soft left hyperideal of S over U and the L-fuzzy soft characteristic function C_R of R is an L-fuzzy soft right hyperideal of S over U . By hypothesis, we have $C_L \tilde{\circ} C_R \tilde{\supseteq} C_L \tilde{\wedge} C_R$ and by Proposition 1, we have

$$C_{L \circ R} = C_L \tilde{\circ} C_R \tilde{\supseteq} C_L \tilde{\wedge} C_R = C_{L \cap R}.$$

This implies $L \cap R \subseteq L \circ R$.

$(2) \Rightarrow (1)$ Let $a \in S$. Let $< a >_l$ and $< a >_r$ be left and right hyperideals of S generated by a , respectively. Then

$$\begin{aligned} a &\in < a >_l \cap < a >_r \\ &\subseteq < a >_l \circ < a >_r \\ &= (\{a\} \cup S \circ a) \circ (\{a\} \cup a \circ S) \\ &= (a \circ a) \cup (a \circ a \circ S) \cup (S \circ a \circ a) \cup (S \circ a \circ a \circ S). \end{aligned}$$

If $a \in a \circ a$, then $a \in a \circ a \subseteq a \circ a \circ a \circ a \subseteq S \circ a \circ a \circ S$.

If $a \in a \circ a \circ S$, then $a \in a \circ a \circ S \subseteq a \circ (a \circ a \circ S) \circ S \subseteq a \circ (a \circ a) \circ (S \circ S) \subseteq S \circ a \circ a \circ S$.

If $a \in S \circ a \circ a$, then $a \in S \circ a \circ a \subseteq S \circ (S \circ a \circ a) \circ a \subseteq (S \circ S) \circ (a \circ a) \circ a \subseteq S \circ a \circ a \circ S$.

Thus in each case $a \in S \circ a \circ a \circ S$. This means that for each $a \in S$ there exist $x, y \in S$ such that $a \in x \circ a \circ a \circ y$. Hence S is an intra-regular semihypergroup.

Theorem 12. *The following assertions are equivalent for a semihypergroup* S :

1) S is regular and intra-regular.
2) Every L-fuzzy soft qausi-hyperideal of S over U is idempotent.
3) Every L-fuzzy soft bi-hyperideal of S over U is idempotent.
4) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B$ for every L-fuzzy soft qausi-hyperideal f_A and g_B of S over U .
5) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U .
6) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U .

7) $f_A \tilde{\wedge} g_B \,\tilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft qausi-hyperideal g_B of S over U.
8) $f_A \tilde{\wedge} g_B \,\tilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft bi-hyperideal f_A and g_B of S over U.
9) $f_A \tilde{\wedge} g_B \,\tilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
10) $f_A \tilde{\wedge} g_B \,\tilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft qausi-hyperideal g_B of S over U.
11) $f_A \tilde{\wedge} g_B \,\tilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
12) $f_A \tilde{\wedge} g_B \,\tilde{\subseteq}\, f_A \circ g_B$ for every L-fuzzy soft generalized bi-hyperideal f_A and g_B of S over U.

Proof. $(1) \Rightarrow (12)$ Let f_A and g_B be any L-fuzzy soft generalized bi-hyperideals of S over U and $a \in S$. Then by our hypothesis there exist $x, y, z \in S$ such that $a \in a \circ x \circ a$ and $a \in y \circ a \circ a \circ z$. Thus

$$\begin{aligned} a &\in a \circ x \circ a \\ &\subseteq a \circ x \circ (a \circ x \circ a) \\ &\subseteq (a \circ x) \circ a \circ (x \circ a) \\ &\subseteq (a \circ x) \circ (y \circ a \circ a \circ z) \circ (x \circ a) \\ &= (a \circ x \circ y \circ a) \circ (a \circ z \circ x \circ a). \end{aligned}$$

Then for some $p \in a \circ x \circ y \circ a$ and $q \in a \circ z \circ x \circ a$, we have $a \in p \circ q$. Thus we have

$$\begin{aligned} (f_A \circ g_B)(a) &= \bigcup_{a \in s \circ t} \{ f_A(s) \cap g_B(t) \} \\ &\supseteq f_A(p) \cap g_B(q) \end{aligned}$$

Since f_A and g_B are L-fuzzy soft generalized bi-hyperideals, so

$$\begin{aligned} \cap_{p \in a \circ (x \circ y) \circ a} f_A(p) &\supseteq f_A(a) \cap f_A(a) \\ &= f_A(a) \\ \text{and} \cap_{q \in a \circ (z \circ x) \circ a} g_B(q) &\supseteq g_B(a) \cap g_B(a) \\ &= g_B(a). \end{aligned}$$

Thus

$$\begin{aligned} (f_A \circ g_B)(a) &\supseteq f_A(p) \cap g_B(q) \\ &\supseteq f_A(a) \cap g_B(a) \\ &= \left(f_A \tilde{\wedge} g_B \right)(a) \end{aligned}$$

Hence $f_A \tilde{\wedge} g_B \,\tilde{\subseteq}\, f_A \circ g_B$.

It is clear that $(12) \Rightarrow (11) \Rightarrow (10) \Rightarrow (4) \Rightarrow (2)$, $(12) \Rightarrow (9) \Rightarrow (8) \Rightarrow (7) \Rightarrow (4)$, $(12) \Rightarrow (6) \Rightarrow (5) \Rightarrow (4)$ and $(8) \Rightarrow (3) \Rightarrow (2)$.

$(2)\Rightarrow(1)$ Let Q be a qausi-hyperideal of S. Then by Proposition 7, the L-fuzzy soft characteristic function C_Q of Q is an L-fuzzy soft qausi-hyperideal of S over U. By hypothesis

$$C_{Q\circ Q} = C_Q \circ C_Q = C_Q.$$

Thus $Q \circ Q = Q$. Hence by Theorem 2, S is regular and intra-regular semihypergroup.

Theorem 13. *Let S be a semihypergroup. Then the following statements are equivalent:*

1) S is both regular and intra-regular.
2) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft right hyperideal f_A and every L-fuzzy soft left hyperideal g_B of S over U.
3) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft right hyperideal f_A and every L-fuzzy soft qausi-hyperideal g_B of S over U.
4) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft right hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
5) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft right hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
6) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft left hyperideal f_A and every L-fuzzy soft qausi-hyperideal g_B of S over U.
7) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft left hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
8) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft left hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
9) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft qausi-hyperideal f_A and g_B of S over U.
10) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
11) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
12) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft bi-hyperideal f_A and g_B of S over U.
13) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
14) $f_A \tilde{\cap} g_B \tilde{\subseteq} (f_A \circ g_B) \tilde{\cap} (g_B \circ f_A)$ for every L-fuzzy soft generalized bi-hyperideal f_A and g_B of S over U.

Proof. $(1)\Rightarrow(14)$ Let f_A and g_B be L-fuzzy soft generalized bi-hyperideals of S over U. Then by Theorem 12, $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B$. Also we have

$$f_A \tilde{\circ} g_B \tilde{=} g_B \tilde{\circ} f_A$$
$$\tilde{\supseteq} g_B \tilde{\wedge} f_A .$$

Therefore $f_A \tilde{\circ} g_B \tilde{\supseteq} (f_A \tilde{\wedge} g_B) \tilde{\circ} (g_B \tilde{\wedge} f_A)$.

It is clear that $(14) \Rightarrow (13) \Rightarrow (12) \Rightarrow (9) \Rightarrow (6) \Rightarrow (2)$, $(14) \Rightarrow (11) \Rightarrow (10) \Rightarrow (9)$, $(14) \Rightarrow (5) \Rightarrow (4) \Rightarrow (3) \Rightarrow (2)$.

$(2) \Rightarrow (1)$ Let f_A and g_B be any L-fuzzy soft right hyperideal and L-fuzzy soft left hyperideal of S over U, respectively. Then we have

$$f_A \tilde{\circ} g_B \tilde{\supseteq} (f_A \tilde{\wedge} g_B) \tilde{\circ} (g_B \tilde{\wedge} f_A)$$
$$\tilde{\supseteq} (g_B \tilde{\wedge} f_A)$$

So by Theorem 11, S is intra-regular. Since

$$f_A \tilde{\circ} g_B \tilde{\supseteq} (f_A \tilde{\wedge} g_B) \tilde{\circ} (g_B \tilde{\wedge} f_A)$$
$$\tilde{\supseteq} (f_A \tilde{\wedge} g_B)$$

Also $f_A \tilde{\wedge} g_B \tilde{\supseteq} f_A \tilde{\circ} g_B$, always holds for every L-fuzzy soft right hyperideal f_A and L-fuzzy soft left hyperideal g_B . Therefore we have $f_A \tilde{\circ} g_B \tilde{=} f_A \tilde{\wedge} g_B$. Thus by Theorem 7, S is regular.

Theorem 14. *Let S be a semihypergroup. Then the following statements are equivalent:*

1) S is regular and intra-regular.
2) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft left hyperideal g_B of S over U .
3) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft right hyperideal g_B of S over U .
4) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft qausi-hyperideals f_A and g_B of S over U .
5) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U .
6) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U .
7) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft left hyperideal g_B of S over U .
8) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft right hyperideal g_B of S over U .
9) $f_A \tilde{\circ} g_B \tilde{\supseteq} f_A \tilde{\wedge} g_B \tilde{\wedge} f_A$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft qausi-hyperideal

g_B of S over U.

10) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B \circ f_A$ for every L-fuzzy soft bi-hyperideals f_A and g_B of S over U.
11) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B \circ f_A$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
12) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B \circ f_A$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft left hyperideal g_B of S over U.
13) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B \circ f_A$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft right hyperideal g_B of S over U.
14) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B \circ f_A$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft qausi-hyperideal g_B of S over U.
15) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B \circ f_A$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
16) $f_A \tilde{\cap} g_B \tilde{\subseteq} f_A \circ g_B \circ f_A$ for every L-fuzzy soft generalized bi-hyperideals f_A and g_B of S over U.

Proof $(1) \Rightarrow (16)$ Let f_A and g_B be any L-fuzzy soft generalized bi-hyperideals of S over U and $a \in S$. Since S is regular as well as intra-regular, so there exist $x, y, z \in S$ such that $a \in a \circ x \circ a$ and $a \in y \circ a \circ a \circ z$. Thus

$$\begin{aligned} a &\in a \circ x \circ a \\ &\subseteq (a \circ x \circ a) \circ x \circ (a \circ x \circ a) \\ &\subseteq a \circ x \circ (y \circ a \circ a \circ z) \circ x \circ (y \circ a \circ a \circ z) \circ x \circ a \\ &= (a \circ x \circ y \circ a) \circ (a \circ z \circ x \circ y \circ a) \circ (a \circ z \circ x \circ a). \end{aligned}$$

Then for some $p, q, r \in S$, we have $p \in a \circ x \circ y \circ a$, $q \in a \circ z \circ x \circ y \circ a$ and $r \in a \circ z \circ x \circ a$ such that $a \in p \circ q \circ r$. Let $l \in p \circ q$ be such that $a \in l \circ r$. Thus we have

$$\begin{aligned} (f_A \circ g_B \circ f_A)(a) &= \bigvee_{a \in s \circ t} \{(f_A \circ g_B)(s) \wedge f_A(t)\} \\ &\geq (f_A \circ g_B)(l) \wedge f_A(r) \\ &= \bigvee_{l \in b \circ c} \{f_A(b) \wedge g_B(c)\} \wedge f_A(r) \\ &\geq f_A(p) \wedge g_B(q) \wedge f_A(r) \end{aligned}$$

Since f_A and g_B are L-fuzzy soft generalized bi-hyperideals of S over U, so we have

$$f_A(p) \geq \bigwedge_{p \in a \circ (x \circ y) \circ a} f_A(p) \geq f_A(a) \wedge f_A(a) = f_A(a),$$

$$g_B(q) \geq \bigwedge_{q \in a \circ (z \circ x \circ y) \circ a} g_B(q) \geq g_B(a) \wedge g_B(a) = g_B(a)$$

and $$f_A(r) \geq \bigwedge_{r \in a \circ (z \circ x) \circ a} f_A(r) \geq f_A(a) \wedge f_A(a) = f_A(a)$$

This shows that

$$\begin{aligned}(f_A \circ g_B \circ f_A)(a) &\geq f_A(a) \wedge g_B(a) \wedge f_A(a)\\ &= f_A(a) \wedge f_A(a) \wedge g_B(a)\\ &= f_A(a) \wedge g_B(a)\\ &= \left(f_A \tilde{\wedge} g_B\right)(a)\end{aligned}$$

Hence $f_A \tilde{\wedge} g_B \subseteq f_A \circ g_B \circ f_A$.
It is clear that $(11) \Rightarrow (6) \Rightarrow (5) \Rightarrow (4) \Rightarrow (3)$, $(16) \Rightarrow (15) \Rightarrow (14) \Rightarrow (13) \Rightarrow (8) \Rightarrow (3)$, $(14) \Rightarrow (12) \Rightarrow (7) \Rightarrow (2)$ and $(16) \Rightarrow (11) \Rightarrow (10) \Rightarrow (9) \Rightarrow (1)$.

$(3) \Rightarrow (1)$ Let f_A be an L-fuzzy soft qausi-hyperideal of S over U . Since $\breve{1}$ is an L-fuzzy soft right hyperideal of S over U , thus we have $f_A = f_A \tilde{\wedge} \breve{1} \subseteq f_A \circ \breve{1} \circ f_A$.
Hence by Theorem 8, S is regular. Let g_B be any L-fuzzy soft right hyperideal of S over U . Since f_A is an L-fuzzy soft qausi-hyperideal of S over U , so we have

$$\begin{aligned}f_A \tilde{\wedge} g_B &\subseteq f_A \circ g_B \circ f_A\\ &\subseteq f_A \circ \left(g_B \circ \breve{1}\right)\\ &\subseteq f_A \circ g_B.\end{aligned}$$

Hence by Theorem 11, S is intra-regular.
Similarly, we can prove $(2) \Rightarrow (1)$.

8. CONCLUSION

In this paper, we have introduced L-fuzzy soft quasi-hyperideal, L-fuzzy soft bi-hyperideal, L-fuzzy soft generalized bi-hyperideal and L-fuzzy soft interior hyperideal of a semihypergroup S over U . We have investigated some important algebraic properties of these hyperideals. Also we have characterized Regular and Intra-regular semihypergroups in terms of these L-fuzzy soft hyperideals, where L is a complete bounded distributive lattice. In future we will study prime and semiprime L-fuzzy soft bi-hyperideals of a semihypergroup S over U .

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