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Some Basic Operations on Pythagorean Fuzzy Sets

[a]Khaista Rahman, [b]Saleem Abdullah, [c]Muhammad Sajjad Ali Khan, [d]Muhammad Ibrar, [e]Fawad Husain

[a, c]Department of Mathematics Hazara University, Mansehra, KPK, Pakistan
[b, d]Department of Mathematics Abdul Wali Khan University, Mardan, KPK, Pakistan
[e]Department of Mathematics Abbottabad University of Science and Technology Abbottabad, KPK, Pakistan

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ABSTRACT

In this paper we present a short but important summary of Pythagorean fuzzy sets. Especially we discuss several basic and important definitions of Pythagorean fuzzy sets, several operations on Pythagorean fuzzy sets, several algebraic laws of Pythagorean fuzzy sets. In the last we also present the concept of modal operators and normalization on Pythagorean fuzzy sets.
KEYWORDS: Pythagorean fuzzy sets, some algebraic laws, Normalization, Modal operators,

1. INTRODUCTION

The model of fuzzy set was familiarized by Zadeh [1]**.** In fuzzy set he only discussed the membership element, which is known as the degree of membership. He also developed many applications of the fuzzy set theory in many fields, such as engineering, management science and computer science etc. After the habituated of fuzzy set Atanassov [3] study fuzzy set theory and familiarized the idea of a new set called intuitionistic fuzzy set in which he considered both the membership and non-membership functions. Actually IPS is a generalized form of the fuzzy set. After the looks of intuitionistic fuzzy set theory many scholars studied in this field and familiarized several aggregation operators and their application also. After the appearances of intuitionistic fuzzy set model Yager [6] generalize intuitionistic fuzzy set and presented a new set called Pythagorean fuzzy set. In [7] Eiegwa et al argued some universal properties of intuitionistic fuzzy sets. There are many aggregation operators and its applications have been developed in [8, 9, 10] In this paper we introduce some basic definitions of Pythagorean fuzzy sets, several important operations on Pythagorean fuzzy sets, some basic algebraic laws of Pythagorean fuzzy sets, some model operators of Pythagorean fuzzy sets, and normalization of Pythagorean fuzzy sets.

The paper has been structured is as follows: In the next section, we give some basic results associated to the fuzzy sets. In section 3, some operators on fuzzy sets will be discussed. Finally, we win up our work by conclusion.

2. PRELIMINARIES

Definition 2.1 [1] *Let M be a fixed set, then a fuzzy sets Q in M can be define as:*

$$Q=\left\{\left(m,\lambda_Q\left(m\right)\right)\mid m\in M\right\}, \qquad (1)$$

where $\lambda_Q : M\rightarrow[0,1]$*, is called membership degree of* $m\in M$.

Definition 2.2 [2] *Let M be a fixed set, then an IFS in M can be define as:*

$$I=\{\left(m,\lambda_I(z),\eta_I(m)\right)\mid m\in M\}, \qquad (2)$$

where $\lambda_I\left(m\right)$ *and* $\eta_I\left(m\right)$ *are mappings from* M *to* $[0,1]$ *, with conditions* $0\le\lambda_I\left(m\right)\le1, 0\le\eta_I(m)\le1$ *and* $0\le\lambda_I(m)+\eta_I(m)\le1,$ *for all* $m\in M$ *. Let* $\pi_I(m)=1-\lambda_I(m)-\eta_I(m)$*, then it is called the intuitionistic fuzzy index of* $m\in M$ *to set* I*, representing the degree of indeterminacy* m *to* I*. Also* $0\le\pi_I(m)\le1$ *for every* $m\in M$.

Definition 2.3 [6] *Let M be a fixed set, then a PFS in M can be defined as follows:*

***Corresponding Author:** Khaista Rahman, Department of Mathematics Hazara University, Mansehra, KPK, Pakistan.
Email: khaista355@yahoo.com

$$P=\{(m,\lambda_P(m),\eta_P(m))\mid m\in M\}, \qquad (3)$$

where $\lambda_P(m)$ *and* $\eta_P(m)$ *are mappings from* M *to* $[0,1]$ *, with conditions* $0\le\lambda_P(m)\le1,$ $0\le\eta_P(m)\le1$ *and also* $0\le\lambda_P^2(m)+\eta_P^2(m)\le1,$ *for all* $m\in M$ *, and they denote the degree of membership and degree of nonmembership of element* $m\in M$ *to set* P *, respectively. Let* $\pi_P(m)=\sqrt{1-\lambda_P^2(m)-\eta_P^2(m)}$*, then it is called the Pythagorean fuzzy index of element* $m\in M$ *to set* P*, representing the degree of indeterminacy of* m *to* P *. Also* $0\le\pi_P(m)\le1,$ *for every* $m\in M$

3. Some Operations on Pythagorean Fuzzy sets

In this section we discuss some operations and relations on Pythagorean fuzzy sets.

3.1 Some Relation on Pythagorean Fuzzy Sets.

Definition 3.1.1: *Let S and T be two Pythagorean fuzzy sets. Then S and T are called similar sets if the following conditions hold:*

$$\lambda_S(m)=\lambda_T(m) \text{ or } \eta_S(m)=\eta_T(m). \qquad (4)$$

Definition 3.1.2: *Let X and Y be two Pythagorean fuzzy sets. Then X and Y are called comparable sets if the following conditions hold:*

$$\lambda_S(m)=\lambda_T(m) \text{ and } \eta_S(m)=\eta_T(m). \qquad (5)$$

Definition 3.1.3: *Let S and T be two Pythagorean fuzzy sets. Then S and T are called equivalents sets if the following conditions hold:*

$$g\ :\ \lambda_S(m)\to\lambda_T(m) \text{ and } g\ :\ \eta_S(m)\to\eta_T(m),(6)$$

both are bijective functions.

Definition 3.1.4: *Let S and T be two Pythagorean fuzzy sets. Then S is called the subset of T and T is called the superset of S if the following conditions hold:*

$$\lambda_S(m)\le\lambda_T(m) \text{ and } \eta_S(m)\ge\eta_T(m). \qquad (7)$$

Definition 3.1.5: *Let S and T be two Pythagorean fuzzy sets. Then S is called the proper subset of T if the following properties hold:*

$$S\subseteq T, \text{ and also } \mathrm{S}\ne T. \qquad (8)$$

Definition 3.1.6: *Let S, T, and R be three Pythagorean fuzzy sets. Then*

(1) If S [illegible] S. Then S is said to be the reflexive relation,

(2) S [illegible] T and T [illegible] S. Then it is called symmetric relation

(3) S [illegible] T, T [illegible] R, and R [illegible] S. Then it is called transitive relation.

Definition 3.1.7: *If a relation is transitive, symmetric and reflexive, then it is known as equivalence.*

Theorem 3.1.8: *Let X and Y be two Pythagorean fuzzy sets, If X [illegible] Y and Y [illegible] X. Then X and Y are equivalence.*

Proof: Straightforward.

Theorem 3.1.9: *Let S and T be two Pythagorean fuzzy sets, If S [illegible] S, S [illegible] T and T [illegible] S. Then S and T are compatible.*

Proof: Straightforward.

Definition 3.1.10: *Let S and T be two Pythagorean fuzzy sets. Then their basic properties can be defined as:*

$$S \bigcup T = \left\{ \left\langle m, \max\left(\lambda_S(m), \lambda_T(m)\right), \min\left(\eta_S(m), \eta_T(m)\right) \right\rangle \mid m \in M \right\}. \tag{9}$$

$$S \bigcap T = \left\{ \left\langle m, \min\left(\lambda_S(m), \lambda_T(m)\right), \max\left(\eta_S(m), \eta_T(m)\right) \right\rangle \mid m \in M \right\}. \tag{10}$$

$$S + T = \left\{ \left\langle \begin{array}{c} m, \sqrt{\left(\lambda_S(m)\right)^2 + \left(\lambda_T(m)\right)^2 - \left(\lambda_S(m)\right)^2 \left(\lambda_T(m)\right)^2}, \\ \eta_S(m)\eta_T(m) \end{array} \right\rangle \mid m \in M \right\}. \tag{11}$$

$$S \times T = \left\{ m \left\langle \begin{array}{c} m, \sqrt{\left(\eta_S(m)\right)^2 + \left(\eta_T(m)\right)^2 - \left(\eta_S(m)\right)^2 \left(\eta_T(m)\right)^2}, \\ \lambda_S(m)\lambda_T(m) \end{array} \right\rangle \mid m \in M \right\}. \tag{12}$$

$$S^c = \{\left\langle m, \eta_S(m), \lambda_S(m) \right\rangle \mid m \in M\}. \tag{13}$$

3.2 Some Basic Algebraic Operations on Pythagorean Fuzzy Sets

Definition 3.2.1: Let X, Y, and R be three Pythagorean fuzzy sets, and then the followings laws hold:

$$(S^c)^c = S \tag{14}$$

$$S \bigcup S = S \tag{15}$$

$$S \bigcap S = S \tag{16}$$

$$S \bigcup T = T \bigcup S \tag{17}$$

$$S \bigcap T = T \bigcap S \tag{18}$$

$$\left(S \bigcup T\right)^c = S^c \bigcap T^c \tag{19}$$

$$\left(S \bigcap T\right)^c = S^c \bigcup T^c \tag{20}$$

$$S \bigcup \left(T \bigcap S\right) = S \tag{21}$$

$$S \bigcap \left(T \bigcup S\right) = S \tag{22}$$

$$\left(S \bigcup T\right) \bigcup R = S \bigcup \left(T \bigcup R\right) \tag{23}$$

$$\left(S \bigcap T\right) \bigcap R = S \bigcap \left(T \bigcap R\right) \tag{24}$$

$$S \bigcup \left(T \bigcap R\right) = \left(S \bigcup T\right) \bigcap \left(S \bigcup R\right) \tag{25}$$

$$S \bigcap \left(T \bigcup R\right) = \left(S \bigcap T\right) \bigcup \left(S \bigcap R\right) \tag{26}$$

$$S+T=T+S \qquad (27)$$

$$S\times T=T\times S \qquad (28)$$

$$(S+T)+R=T+(S+R) \qquad (29)$$

$$(S\times T)\times R=T\times(S\times R) \qquad (30)$$

$$(S+T)^c=S^c\times T^c \qquad (31)$$

$$(S\times T)^c=S^c+T^c \qquad (32)$$

$$S+(T\cup R)=(S+T)\cup(S+R) \qquad (33)$$

$$S+(T\cap R)=(S+T)\cap(S+R) \qquad (34)$$

$$S\times(T\cup R)=(S\times T)\cup(S\times R) \qquad (35)$$

$$S\times(T\cap R)=(S\times T)\cap(S\times R) \qquad (36)$$

3.3: Some Modal Operators on Pythagorean Fuzzy

We are going now to define two modal operators on Pythagorean fuzzy set, which convert every Pythagorean fuzzy set into PS.

Definition 3.3.1: *Let X be a Pythagorean fuzzy set in Z, where Z be a fixed set. Then the following conditions hold:*

$$\Box S=\left\{\left\langle m,\lambda_S(m)\mid m\in M\right\rangle\right\}=\left\{\left\langle m,\lambda_S(m),1-\lambda_S(m)\mid m\in M\right\rangle\right\} \qquad (37)$$

$$\Diamond S=\left\{\left\langle m,1-\eta_S(m)\mid m\in M\right\rangle\right\}=\left\{\left\langle m,1-\eta_S(m),\eta_S(m)\mid m\in M\right\rangle\right\} \qquad (38)$$

Theorem 3.3.2: *Let X be a Pythagorean fuzzy set in Z, where Z be a fixed set. Then the following conditions hold:*

(1) $\Box\Box S=S$

(2) $\Box\Diamond S=\Diamond S$

(3) $\Diamond\Box S=S$

(4) $\Diamond\Diamond S=S$

Proof: Here we prove only (1) and (2), (3), (4) can be proved by the using of (1) and (37), (38).

$$S=\{\langle m,\lambda_S(m),\eta_S(m)\rangle\mid m\in M\}$$

$$\begin{aligned}\Box S&=\{\langle m,\lambda_S(m)\rangle\mid m\in M\}\\&=\{\langle m,\lambda_S(m),1-\lambda_S(m)\rangle\mid m\in M\}\\&=\{\langle m,\lambda_S(m),\eta_S(m)\rangle\mid m\in M\}\end{aligned}$$

$$\begin{aligned}\Box\Box S&=\{\langle m,\lambda_S(m)\rangle\mid m\in M\}\\&=\{\langle m,\lambda_S(m),1-\lambda_S(m)\rangle\mid m\in M\}\\&=\{\langle m,\lambda_S(m),\eta_S(m)\rangle\mid m\in M\}\\&=S.\end{aligned}$$

Theorem 3.3.3: *Let S and T be two Pythagorean fuzzy sets in M, where M be a fixed set. Then the following are hold:*

(1) $\Box(S\cap T)=S\cap T$

(2) $\Diamond\left(S\bigcap T\right)=\Diamond S\bigcap\Diamond T$

(3) $\Box\left(S\bigcup T\right)=S\bigcup T$

(4) $\Diamond\left(S\bigcup T\right)=\Diamond S\bigcup\Diamond T$

(5) $\Box\left(S+T\right)=S+\Box T$

(6) $\Box\left(S\times T\right)=S\times\Box T$

(7) $\Diamond\left(S+T\right)=\Diamond S+\Diamond T$

(8) $\Diamond\left(S\times T\right)=\Diamond S\times\Diamond T$

Proof: We can prove only (1) and (2) and the remaining are straightforward.

(1) As we know that

$$S\bigcap T=\left\{\left\langle m,\min\left(\lambda_S(m),\lambda_T(m)\right),\max\left(\eta_S(m),\eta_T(m)\right)\right\rangle \mid m\in M\right\}$$

$$\Box\left(S\bigcap T\right)=\left\{\left\langle m,\min\left(\lambda_S(m),\lambda_T(m)\right)\right\rangle \mid m\in M\right\}$$

$$=\left\{\left\langle m,\lambda_S(m)\right\rangle \mid m\in Z\right\}\cap\left\{\left\langle m,\lambda_T(m)\right\rangle \mid m\in M\right\}$$

$$X\bigcap Y.$$

(2) As we know that

$$S\bigcap T=\left\{\left\langle m,\min\left(\lambda_S(m),\lambda_T(m)\right),\max\left(\eta_S(m),\eta_T(m)\right)\right\rangle \mid m\in M\right\}$$

$$S\bigcap T=\left\{\left\langle m,\min\left(1-\eta_S(m),1-\eta_T(m)\right),\max\left(1-\lambda_T(m),1-\lambda_T(m)\right)\right\rangle \mid m\in M\right\}$$

$$\Diamond\left(S\bigcap T\right)=\left\{\left\langle m,\min\left(1-\eta_S(m),1-\eta_T(m)\right)\right\rangle \mid m\in M\right\}$$

$$=\left\{\left\langle m,\min\left(1-\eta_S(m)\right)\right\rangle \mid m\in Z\right\}\cap\left\{\left\langle m,\min\left(1-\eta_T(m)\right)\right\rangle \mid m\in M\right\}$$

$$=\Diamond S\bigcap\Diamond T.$$

Theorem 3.3.4: *Let S and T be two Pythagorean fuzzy sets in M, where M be a fixed set. Then the following conditions hold:*

(1) $S\subseteq T$ if and only if $\Box S\subseteq\Box T$

(2) $S\subseteq\Diamond T$ if and only if $\Diamond S\subseteq\Diamond T$

Proof: (1) As we know that

$$\Box T=\{\left\langle m,\lambda_T(m)\right\rangle \mid m\in M\}$$

$$=\{\left\langle m,\lambda_T(m),1-\lambda_T(m)\right\rangle \mid m\in M\}$$

$$=\{\left\langle m,\lambda_T(m),\eta_T(m)\right\rangle \mid m\in M\}$$

$$=T.$$

Similarly $\Box S=S$. Thus $\Box S\subseteq\Box T$. Conversely, suppose $\Box S\subseteq\Box T.$ Since $\Box S=S.$ Thus $S\subseteq\Box T.$

(2) Since

$$\Diamond T=\{\left\langle m,1-\eta_T(m)\right\rangle \mid m\in Z\}$$

$$=\{\left\langle m,1-\eta_T(m),\eta_T(m)\right\rangle \mid m\in Z\}$$

$$=\{\left\langle m,\lambda_T(m),\eta_T(m)\right\rangle \mid m\in Z\}$$

$$T.$$

Similarly $\lozenge S = S$. Thus $\lozenge S \subseteq \lozenge T$. Conversely, suppose $\lozenge S \subseteq \lozenge T$. Since $\lozenge S = S$. Thus $S \subseteq \lozenge T$.

3.4 Normalization of Pythagorean Fuzzy Sets

Definition 3.4.1: *Let M be a complete set, then the normalization of Pythagorean fuzzy set S can be represented by NORM(S) and define as followings:*

$$NORM\left(S\right) = \{\left\langle m, \lambda_{NORM(S)}(m), \eta_{NORM(S)}(m) \right\rangle \mid m \in M\}, \quad (39)$$

where

$$\lambda_{NORM(S)}(m) = \frac{\lambda_{(S)}(m)}{\sup\left(\lambda_{(S)}(m)\right)}, \quad (40)$$

and

$$\eta_{NORM(S)}(m) = \frac{\eta_{(S)}(m) - \inf\left(\eta_{(S)}(m)\right)}{1 - \inf\left(\eta_{(S)}(m)\right)}, \text{ for } M = \{m\}. \quad (41)$$

Including $\pi_{NORM(S)}\left(m\right)$. *Thus equation (39) can be written as:*

$$NORM\left(S\right) = \{\left\langle m, \lambda_{NORM(S)}(m), \eta_{NORM(S)}(m), \pi_{NORM(S)}\left(m\right) \right\rangle \mid m \in M\}, \quad (42)$$

where

$$\pi_{NORM(S)}\left(m\right) = \sqrt{1 - \left(\lambda_{NORM(S)}(m)\right)^2 - \left(\eta_{NORM(S)}(m)\right)^2}. \quad (43)$$

Theorem 3.4.2: *Let M be a universal set, and S be the Pythagorean fuzzy set in M. Then the following conditions hold:*

(1) $\pi_X\left(m\right) = 0$. Then $\pi_{NORM(S)}\left(m\right) = 0$

(2) $NORM\left(\square S\right) = \square\left(NORM\left(S\right)\right)$

(3) $NORM\left(\lozenge S\right) = \lozenge\left(NORM\left(S\right)\right)$

Proof: Straightforward.

Example 3.4.3: *Let* $M = \{m_1, m_2, m_3\}$, *and S be a Pythagorean fuzzy set such that* $S = \{\langle 0.5, 0.6\rangle, \langle 0.7, 0.4\rangle, \langle 0.6.0.5\rangle\}$. *Then sup* $\left(\lambda_{(S)}(m)\right) = 0.7$ *and in f* $\left(\eta_{(S)}(m)\right) = 0.4$. *Thus* $\lambda_{NORM(S)}(m_1) = 0.714, \lambda_{NORM(S)}(m_2) = 1.0, \lambda_{NORM(S)}(m_3) = 0.857,$ *and* $\eta_{NORM(S)}(m_1) = 0.333, \eta_{NORM(S)}(m_2) = 0.0, \eta_{NORM(S)}(m_3) = 0.166.$ *Clearly*

$$0 \le \left(\lambda_{NORM(S)}(m_1)\right)^2 + \left(\eta_{NORM(S)}(m_1)\right)^2 \le 1$$

$$0 \le \left(\lambda_{NORM(S)}(m_2)\right)^2 + \left(\eta_{NORM(S)}(m_2)\right)^2 \le 1$$

$$0 \le \left(\lambda_{NORM(S)}(m_3)\right)^2 + \left(\eta_{NORM(S)}(m_3)\right)^2 \le 1$$

Thus $NORM\left(S\right)$ *is a Pythagorean fuzzy set.*

We are going now to prove the Theorem 3.3.2 . *Since*

$$\left(\lambda_{NORM(S)}(m_2)\right)^2 + \left(\eta_{NORM(S)}(m_2)\right)^2 = 1.$$

And

$$\pi_{NORM(S)}(m_2) = \sqrt{1-\left(\lambda_{NORM(S)}(m_2)\right)^2 - \left(\eta_{NORM(S)}(m_2)\right)^2}$$
$$= \sqrt{1-1}$$
$$= 0$$

If we put $m_2 = m$ *, we get* $\pi_{NORM(S)}(m) = 0.$

Now we prove that $NORM(\Box S) = \Box(NORM(S))$ *. For this we have* $\Box S = S.$ *Then we have*

$$NORM(\Box S) = \{\langle 0.714, 0.333\rangle, \langle 1.0, 0.0\rangle, \langle 0.857, 0.166\rangle\}.$$

If

$$NORM(\Box S) = \{\langle 0.714, 0.333\rangle, \langle 1.0, 0.0\rangle, \langle 0.857, 0.166\rangle\}.$$

Then

$$\Box(NORM(S)) = \{\langle 0.714, 0.333\rangle, \langle 1.0, 0.0\rangle, \langle 0.857, 0.166\rangle\}.$$

Thus

$$NORM(\Box S) = \Box(NORM(S)).$$

Next we are going to show that $NORM(\Diamond S) = \Diamond(NORM(S))$ *. As we know that* $\Diamond S = S.$ *Then we have*

$$NORM(\Diamond S) = \{\langle 0.714, 0.333\rangle, \langle 1.0, 0.0\rangle, \langle 0.857, 0.166\rangle\}.$$

If

$$NORM(\Diamond S) = \{\langle 0.714, 0.333\rangle, \langle 1.0, 0.0\rangle, \langle 0.857, 0.166\rangle\}.$$

Then

$$\Diamond(NORM(S)) = \{\langle 0.714, 0.333\rangle, \langle 1.0, 0.0\rangle, \langle 0.857, 0.166\rangle\}.$$

Thus $NORM(\Diamond S) = \Diamond(NORM(S)).$

Theorem 3.4.4: *Let M be a universal set, and S be the Pythagorean fuzzy set in M. Then the following conditions hold:*

(1) $NORM(M) = NORM(\Diamond M)$

(2) $\Box(NORM(S)) = \Diamond(NORM(S))$

Proof : As we know that

$$S = \{\langle m, \lambda_S(m), \eta_S(m)\rangle \mid m \in M\}$$

As we also know that the operators convert the Pythagorean fuzzy set to fuzzy set, since we have $\lambda_S(m) = 1 - \eta_S(m)$ and also $\eta_S(m) = 1 - \lambda_S(m)$ for every $m \in M$. Thus

$$\Box S = \{\langle m, \lambda_S(m), 1-\lambda_S(m)\rangle \mid m \in M\}$$
$$= \{\langle m, \lambda_S(m), \eta_S(m)\rangle \mid m \in M\}$$
$$S.$$

Again we have

$$\Diamond S = \{\langle m, 1-\eta_S(m), \eta_S(m)\rangle \mid m \in M\}$$
$$= \{\langle m, \lambda_S(m), \eta_S(m)\rangle \mid m \in M\}$$
$$= S.$$

Then we automatically say that $\Box S = \Diamond S = S.$ As $\pi_S(m) = 0.$ Thus

$$NORM\left(S\right)=NORM\left(\Diamond S\right)$$

Next we prove that

$$\Box\left(NORM\left(S\right)\right)=\Diamond\left(NORM\left(S\right)\right)$$

Since

$$NORM\left(S\right)=\{\left\langle m,\lambda_{NORM(S)}(m),\eta_{NORM(S)}(m),\pi_{NORM(S)}\left(m\right)\right\rangle\mid m\in M\}$$

As $\pi_S\left(m\right)=0.$ So $\pi_{NORM(S)}\left(m\right)=0.$ Therefore the above equation can be written as

$$NORM\left(S\right)=\{\left\langle m,\lambda_{NORM(S)}(m),\eta_{NORM(S)}(m)\right\rangle\mid m\in M\}.$$

As

$$\left(\lambda_{NORM(S)}(m)\right)^2+\left(\eta_{NORM(S)}(m)\right)^2=1.$$

Thus

$$\begin{aligned}\Box\left(NORM\left(S\right)\right)&=\{\left\langle m,\lambda_{NORM(S)}(m),1-\lambda_{NORM(S)}(m)\right\rangle\mid m\in M\}\\&=\{\left\langle m,\lambda_{NORM(S)}(m),\eta_{NORM(S)}(m)\right\rangle\mid m\in M\}\\&=NORM\left(S\right).\end{aligned}$$

Again

$$\begin{aligned}\Diamond\left(NORM\left(S\right)\right)&=\{\left\langle m,1-\eta_{NORM(S)}(m),\eta_{NORM(S)}(m)\right\rangle\mid m\in M\}\\&=\{\left\langle m,\lambda_{NORM(S)}(m),\eta_{NORM(S)}(m)\right\rangle\mid m\in M\}\\&=NORM\left(S\right).\end{aligned}$$

Thus

$$S\cap T=\left\{\left\langle m,\min\left(\lambda_S\left(m\right),\lambda_T\left(m\right)\right),\max\left(\eta_S\left(m\right),\eta_T\left(m\right)\right)\right\rangle\mid m\in M\right\}$$

$$S\cap T=\left\{\left\langle m,\min\left(1-\eta_S\left(m\right),1-\eta_T\left(m\right)\right),\max\left(1-\lambda_S\left(m\right),1-\lambda_T\left(m\right)\right)\right\rangle\mid m\in M\right\}$$

$$\begin{aligned}\Diamond\left(S\cap T\right)&=\left\{\left\langle m,\min\left(1-\eta_S\left(m\right),1-\eta_T\left(m\right)\right)\right\rangle\mid m\in M\right\}\\&=\left\{\left\langle m,\min\left(1-\eta_S\left(m\right)\right)\right\rangle\mid m\in Z\right\}\cap\left\{\left\langle m,\min\left(1-\eta_T\left(m\right)\right)\right\rangle\mid m\in M\right\}\\&=\Diamond S\cap\Diamond T.\end{aligned}$$

CONCLUSION

In this work, we presented successfully some important summary of Pythagorean fuzzy sets. We discussed some several basic operators and important definitions on Pythagorean fuzzy sets. In addition we presented the concept of model operators and normalization Pythagorean fuzzy sets.

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