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Impacts of Magnetic Field on Fractionalized Viscoelastic Fluid

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ABSTRACT

This paper presents exact analytical solution for fractionalized second grade fluid with and without magnetohydrodynamics (MHD) effects due to accelerating plane. The solutions are obtained for velocity field and shear stress by employing Laplace transform with its inverse. General solutions are written in terms of generalized special function namely Fox-H functions $\boldsymbol{H}^{1,p}_{p,q+1}$ and satisfy initial and boundary conditions. Expressions of both velocity and shear stress have been particularized for some special cases for fractionalized and ordinary second grade and Newtonian fluids in presence and absence of magnetohydrodynamics (MHD). Finally the effects of pertinent rheological parameters namely viscosityν, time t, fractional parameter ξ, dynamics viscosity μ, material parameters λ, λ_1 and magnetic field$M$ have been analyzed and depicted for graphical illustrations.
KEY WORDS: Magnetic field, Discrete Laplace Transforms, Fox-H functions, Graphical analysis.

INTRODUCTION

The non-Newtonian fluids have been analyzed due to crucial consideration and attention of engineers, mathematicians, numerical analysts and scientist. Researchers have diverted their interest due to various applications of non-Newtonian fluid flows in industrial, engineering and technological advancement. Specifically, non-Newtonian fluid flows includes for instancecosmetic products, exotic lubricants, paints, certain oils, shampoos, applesauce, ketchup, clay coatings, suspension, colloidal and polymer solutions and several others. In order to disclose physical nature and structures of these fluids, there is no any single constitutive model or equation which can predict the rheology and characteristics of non-Newtonian fluids. In general, these fluids have been categorized into three classifications as (i) integral type, (ii) rate type and (iii) differential type. Among these three classified models, second grade is a model lies in subclass of differential type for which researchers can expect to investigate exact solutions because this model describes normal stress differences. Various research scholars of fluid mechanics have attempted subclass model of differential type. The literature also depicts interesting results which are referenced in [1-8].It is pointed out that, due to various causes and reasons the exact solutions are important because these solutions offer the exactness for examining the accurateness of various estimated and approximated solutions. We can also use these solutions to test or verify numerical schemes for studying very complex flow problems. By seeing these desires and needs the exact solutions of the equations describing the movements of viscoelastic fluids. On the other hand, many scientists and researchers are involved in finding such solutions in this field [9-18].The analysis of magnetohydrodynamics (MHD) is significant because magnetic field has very good interaction effects and influences on viscoelastic behavior of fluid flows. This happens in various industrial processes for instance, purification of crude oil, paper production, glass manufacturing, geophysics, MHD electrical power generation and magnetic materials processing. Exact solution on Micropolar fluid for three dimensional magnetohydrodynamics (MHD) stagnation-point flows is studied by Borrelli and et al. [19].Magnetohydrodynamics (MHD) aligned flow of a second grade fluid has been investigated for traveling wave solution in [20, 21]. Zaman and et al. has presented effects of MHD Axisymmetric Second-Grade Fluid for Hall current on flow over an Exponentially Stretching Sheet [22].Taza Gul and et al. has analyzed magnetohydrodynamics (MHD) third grade fluid on a vertical belt for thin film flow with no slip assumption on the boundary [23]. Sidra Abid and et al. has investigated second grade fluid between two vertical plates for magnetic hydrodynamic flow under oscillation of boundary conditions [24]. H. Rasheed and et al. has discussed a study of unsteady magnetohydrodynamics (MHD) third grade fluid for poiseuille and coquette flows [25]. We also include here similar study in few references [26-29] therein.

By the motivations of above investigations, our aim is to investigate exact analytical solution for fractionalized second grade fluid with and without magnetohydrodynamics (MHD) effects due to accelerating plane. The solutions are obtained for velocity field and shear stress by employing Laplace transform with its inverse. General solutions are

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written in terms of generalized special function namely Fox-H functions $\boldsymbol{H}^{1,p}_{p,q+1}$and satisfy initial and boundary conditions. Expressions of both velocity and shear stress have been particularized for some special cases for fractionalized and ordinary second grade and Newtonian fluids in presence and absence of magnetohydrodynamics (MHD). Finally, the comparison of different models in fractional and ordinary approach and the behavior of fluid are emphasized by implementing various rheological and material parameters by depicting graphical illustrations.

GOVERNING EQUATIONS

The governing equations to flow of an incompressible comprise momentum and continuity equations without body force are

$$\rho\frac{\partial \boldsymbol{V}}{\partial t}+\rho(\boldsymbol{V}.\boldsymbol{\nabla})\boldsymbol{V}=\boldsymbol{0},\quad \boldsymbol{\nabla}V=0, \tag{1}$$

$\boldsymbol{\nabla}$ represents del or nebla operator, ρ is density of fluid, $\boldsymbol{V}$ is the velocity of fluid, t is the time and $\boldsymbol{T}$ is the Cauchy stress tensor of second grade is related to the motion of fluid as [30, 31]

$$\boldsymbol{S}=\mu\boldsymbol{A_1}+\lambda_1\boldsymbol{A_2}+\lambda_2\boldsymbol{A}_1^2,\quad \boldsymbol{T}=-p\boldsymbol{I}+\boldsymbol{S}, \tag{2}$$

$\boldsymbol{S}$ is the extra-stress tensor, μ represents the dynamic viscosity, λ_1 and λ_2 material moduli, p is hydrostatic pressure, $\boldsymbol{I}$ is unit tensor identity,$-p\boldsymbol{I}$ **is** spherical stress and $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ first two Rivilin-Ericksen tensor defined by:

$$\boldsymbol{A_1}=(\boldsymbol{\nabla}V)^{\boldsymbol{T}}+(\boldsymbol{\nabla}V),\quad \boldsymbol{A_2}=\boldsymbol{A_1}(\boldsymbol{\nabla}V)^{\boldsymbol{T}}+(\boldsymbol{\nabla}V)\boldsymbol{A_1}+\frac{d\boldsymbol{A_1}}{dt}, \tag{3}$$

velocity field is assumed as

$$\boldsymbol{S}=\boldsymbol{S}(y,t),\qquad \boldsymbol{V}=\boldsymbol{V}(y,t)=u(y,t)\boldsymbol{i}, \tag{4}$$

For these flows the limitation of incompressibility is deliberately fulfilled. When $t=0$, the fluid is at rest then

$$\boldsymbol{S}=(y,0)=0,\qquad \boldsymbol{V}=(y,0)=0, \tag{5}$$

employing equation (5) in (1), yields governing equations

$$\frac{\partial u(y,t)}{\partial t}+M\left(\lambda_1\frac{\partial}{\partial t}+\mu\right)u(y,t)-\left(\lambda\frac{\partial}{\partial t}+\nu\right)\frac{\partial^2 u(y,t)}{\partial^2 t}=0, \tag{6}$$

$$\tau(y,t)-\left(\lambda_1\frac{\partial}{\partial t}+\mu\right)\frac{\partial u(y,t)}{\partial y}=0\,. \tag{7}$$

Equations (6) and (7) are transformed into fractionalized form as

$$\frac{\partial u(y,t)}{\partial t}-\frac{\partial^2 u(y,t)}{\partial^2 t}\left(\lambda\frac{\partial^\xi}{\partial t^\xi}+\nu\right)+M\left(\lambda_1\frac{\partial^\xi}{\partial t^\xi}+\mu\right)u(y,t)=0, \tag{8}$$

$$\tau(y,t)-\frac{\partial u(y,t)}{\partial y}\left(\lambda_1\frac{\partial^\xi}{\partial t^\xi}+\mu\right)=0\,. \tag{9}$$

Where, the fractional parameter is $0<\xi<1$ and the fractional differential operator $\frac{\partial^\xi}{\partial t^\xi}=D_t^\xi$ is described as

$$\frac{\partial^\xi}{\partial t^\xi}=D_t^\xi g(t)=\begin{cases}\dfrac{1}{\Gamma(1-k)}\displaystyle\int_0^t\frac{g'(s)}{(t-s)^k}ds, & 0<k<1\\ \dfrac{dg(t)}{dt}, & k=1\end{cases} \tag{10}$$

FORMULATION OF PROBLEM

Let us consider an incompressible magnetohydrodynamics (MHD) fractionalized second grade fluid owning the space on an infinitely plane which is positioned in the xz-plane and perpendicular to the y-axis. To begin with, the fluid is at rest and at the moment $t=0^+$ the plane start to accelerate in its plane. The fluid above the plane is progressively accelerated because of shear. Its governing equations (8-9) having (11-12) conditions are

$$u(y,0)=0,\qquad \tau(y,0)=0,\qquad y>0, \tag{11}$$

$$u(0,t)=UH(t)t^m\qquad t\geq 0. \tag{12}$$

Vivid natural conditions are

$$u(y,t),\qquad \frac{\partial u(y,t)}{\partial t}\to 0\qquad as\qquad y\to\infty\quad and\quad t>0, \tag{13}$$

can be also fulfilled.

ANALYTICAL SOLUTION OF VELOCITY FIELD

In order to find exact analytical solution for velocity field, we apply the Laplace transform formula to equation (8) and (11), we get

$$\left\{\frac{\partial^2}{\partial y^2}-\frac{M\left(\lambda_1 q^{\xi}+\mu\right)+q}{\left(\lambda q^{\xi}+\nu\right)}\right\}\bar{u}(y,q)=0, \tag{14}$$

subject to the boundary conditions

$$\bar{u}(0,q)=\frac{Um!}{q^{m+1}}, \tag{15}$$

Solving equation (14) and using equation (15), we find

$$\bar{u}(y,q)=\frac{Um!}{q^{m+1}}e^{-y\sqrt{\frac{M\left(\lambda_1 q^{\xi}+\mu\right)+q}{\left(\lambda q^{\xi}+\nu\right)}}}, \tag{16}$$

equation (16) is expressed in series form

$$\bar{u}(y,q)=\frac{Um!}{q^{m+1}}+Um!\sum_{i=1}^{\infty}\left(\frac{-y}{\sqrt{\nu}}\right)^{i}\sum_{j=0}^{\infty}\frac{(-\mu M)^{j}}{j!}\sum_{k=0}^{\infty}\frac{(-\lambda_1)^{k}}{k!\,\mu}\sum_{l=0}^{\infty}\frac{\left(-\frac{\lambda}{\nu}\right)^{l}\Gamma\left(1+\frac{i}{2}\right)\Gamma(1+j)\Gamma\left(l+\frac{i}{2}\right)}{l!\,\Gamma\left(1-j+\frac{i}{2}\right)\Gamma(1+j-k)\Gamma\left(\frac{i}{2}\right)q^{j-\xi k-\xi l-\frac{i}{2}+m+1}}, \tag{17}$$

Inverting equation (17) by Laplace transform and expressing as Fox H-function [32], we find expression for velocity field:

$$\begin{aligned}u(y,t)&=UH(t)t^{m}+UH(t)m!\sum_{i=1}^{\infty}\left(\frac{-y}{\sqrt{\nu}}\right)^{i}\sum_{j=0}^{\infty}\frac{(-\mu M)^{j}}{j!}\sum_{k=0}^{\infty}\frac{1}{k!}\frac{(-\lambda_1)^{k}}{\mu}t^{j-\xi k-\frac{i}{2}+m}\\&\times\boldsymbol{H}_{3,5}^{1,3}\left[\frac{\lambda}{\nu t^{\xi}}\middle|\begin{matrix}\left(-\frac{i}{2},0\right),(-j,0),\left(1-\frac{i}{2},1\right)\\(0,1),\left(j-\frac{i}{2},0\right),(k-j,0),\left(\frac{i}{2},0\right),\left(\frac{i}{2}+\xi k-j-m,-\xi\right)\end{matrix}\right],\end{aligned} \tag{18}$$

Where, the Fox-H function is described as

$$\sum_{k}^{\infty}\frac{(-Q)^{k}\prod_{j=1}^{p}\Gamma\left(c_j+C_j k\right)}{k!\,\prod_{j=1}^{q}\Gamma\left(d_j+D_j k\right)}=\boldsymbol{H}_{p,q+1}^{1,p}\left[Q\middle|\begin{matrix}(1-c_1,C_1),(1-c_2,C_2),\dots,\left(1-c_p,C_p\right)\\(0,1),(1-d_1,D_1),(1-d_2,D_2),\dots,\left(1-d_q,D_q\right)\end{matrix}\right], \tag{19}$$

ANALYTICAL SOLUTION OF SHEAR STRESS

Applying discrete Laplace transform to equation (9) and (11), we get suitable expression

$$\bar{\tau}(y,q)=\left(\lambda_1 q^{\xi}+\mu\right)\frac{\partial\bar{u}(y,s)}{\partial y}, \tag{20}$$

employing equation (16) into equation (20), we have

$$\bar{\tau}(y,q)=-\frac{Um!\left(\lambda_1 q^{\xi}+\mu\right)}{q^{m+1}}\sqrt{\frac{M\left(\lambda_1 q^{\xi}+\mu\right)+q}{\left(\lambda q^{\xi}+\nu\right)}}e^{-\sqrt{\frac{M\left(\lambda_1 q^{\xi}+\mu\right)+q}{\left(\lambda q^{\xi}+\nu\right)}}y}, \tag{21}$$

rewriting equation (21) in terms of series form for suitable expression of $\bar{\tau}(y,q)$,

$$\bar{\tau}(y,q) = -\frac{Um!}{2\sqrt{\nu\mu}}\sum_{i=0}^{\infty}\left(\frac{-y\mu\sqrt{M}}{\nu}\right)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{-1}{\mu}\right)^{j}\sum_{k=0}^{\infty}\frac{(-\lambda_1)^k}{k!}\sum_{l=0}^{\infty}\frac{\left(-\frac{\lambda}{\nu}\right)^{l}\Gamma\left(\frac{3+i}{2}\right)\Gamma\left(1+\frac{3+i}{2}\right)\Gamma\left(l+\frac{3+i}{2}\right)}{l!\,\Gamma\left(\frac{3+i}{2}\right)\Gamma\left(\frac{3+i}{2}-j\right)\Gamma\left(\frac{3+i}{2}-k+1\right)} \times \frac{1}{q^{j-\xi k-\xi l-\frac{i+1}{2}+m+1}}, \tag{22}$$

Inverting equation (22) by Laplace transform and expressing as Fox H-function [32], we find expression for shear stress:

$$\tau(y,t) = -\frac{UH(t)m!}{2\sqrt{\nu\mu}}\sum_{i=0}^{\infty}\left(\frac{-y\mu\sqrt{M}}{\nu}\right)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{-1}{\mu}\right)^{j}\sum_{k=0}^{\infty}\frac{(-\lambda_1)^k}{k!}t^{j-\xi k-\frac{i+1}{2}+m} \times \boldsymbol{H}_{3,5}^{1,3}\left[\frac{\lambda}{\nu t^{\xi}}\middle|\begin{matrix}\left(1-\frac{3+i}{2},0\right),\left(-\frac{3+i}{2},0\right),\left(1-\frac{3+i}{2},1\right)\\(0,1),\left(1-\frac{3+i}{2},0\right),\left(1+j-\frac{3+i}{2},0\right),\left(k-\frac{3+i}{2},0\right),\left(j-\xi k-\frac{i+1}{2}+m+1,-\xi\right)\end{matrix}\right]. \tag{23}$$

5. LIMITING CASES

5.1. ORDINARY MAGNETIZED SECOND GRADE FLUID WHEN $\xi = 1$ AND $M \neq 0$

Letting $\xi = 1$ and $M \neq 0$ in equations (18) and (23) the solutions are recovered for ordinary second grade fluid with magnetic field

$$u(y,t) = UH(t)t^m + UH(t)m!\sum_{i=1}^{\infty}\left(\frac{-y}{\sqrt{\nu}}\right)^{i}\sum_{j=0}^{\infty}\frac{(-\mu M)^j}{j!}\sum_{k=0}^{\infty}\frac{1}{k!}\frac{(-\lambda_1)^k}{\mu}t^{j-k-\frac{i}{2}+m} \times \boldsymbol{H}_{3,5}^{1,3}\left[\frac{\lambda}{\nu t}\middle|\begin{matrix}\left(-\frac{i}{2},0\right),(-j,0),\left(1-\frac{i}{2},1\right)\\(0,1),\left(j-\frac{i}{2},0\right),(k-j,0),\left(\frac{i}{2},0\right),\left(\frac{i}{2}+k-j-m,-1\right)\end{matrix}\right], \tag{24}$$

$$\tau(y,t) = -\frac{UH(t)m!}{2\sqrt{\nu\mu}}\sum_{i=0}^{\infty}\left(\frac{-y\mu\sqrt{M}}{\nu}\right)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{-1}{\mu}\right)^{j}\sum_{k=0}^{\infty}\frac{(-\lambda_1)^k}{k!}t^{j-k-\frac{i+1}{2}+m} \times \boldsymbol{H}_{3,5}^{1,3}\left[\frac{\lambda}{\nu t}\middle|\begin{matrix}\left(1-\frac{i+3}{2},0\right),\left(-\frac{i+3}{2},0\right),\left(1-\frac{i+3}{2},1\right)\\(0,1),\left(1-\frac{i+3}{2},0\right),\left(1+j-\frac{i+3}{2},0\right),\left(k-\frac{i+3}{2},0\right),\left(j-k-\frac{i+1}{2}+m+1,-1\right)\end{matrix}\right]. \tag{25}$$

5.2. FRACTIONALIZED AND NON-MAGNETIZED SECOND GRADE FLUID WHEN $\xi \neq 1$ AND $M = 0$

Making $M = 0$ and $\xi \neq 1$ in equations (18) and (23) the solutions are recovered for fractionalized second grade fluid without magnetic field

$$u(y,t) = UH(t)t^m + UH(t)m!\sum_{i=1}^{\infty}\left(\frac{-y}{\sqrt{\nu}}\right)^{i}\sum_{k=0}^{\infty}\frac{1}{k!}\frac{(-\lambda_1)^k}{\mu}t^{j-\xi k-\frac{i}{2}+m} \times \boldsymbol{H}_{3,5}^{1,3}\left[\frac{\lambda}{\nu t^{\xi}}\middle|\begin{matrix}\left(-\frac{i}{2},0\right),(-j,0),\left(1-\frac{i}{2},1\right)\\(0,1),\left(j-\frac{i}{2},0\right),(k-j,0),\left(\frac{i}{2},0\right),\left(\frac{i}{2}+\xi k-j-m,-\xi\right)\end{matrix}\right], \tag{26}$$

$$\tau(y,t) = -\frac{UH(t)m!}{2\sqrt{\nu\mu}}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{-1}{\mu}\right)^{j}\sum_{k=0}^{\infty}\frac{(-\lambda_1)^k}{k!}t^{j-\xi k-\frac{i+1}{2}+m} \times \boldsymbol{H}_{3,5}^{1,3}\left[\frac{\lambda}{\nu t^{\xi}}\middle|\begin{matrix}\left(1-\frac{i+3}{2},0\right),\left(-\frac{i+3}{2},0\right),\left(1-\frac{i+3}{2},1\right)\\(0,1),\left(1-\frac{i+3}{2},0\right),\left(1+j-\frac{i+3}{2},0\right),\left(k-\frac{i+3}{2},0\right),\left(j-\xi k-\frac{i+1}{2}+m+1,-\xi\right)\end{matrix}\right]. \tag{27}$$

5.3. MAGNETIZED NEWTONIAN FLUID WHEN $M \neq 0$ AND $\lambda = \lambda_1 = 0$

Making $M \neq 0$ and $\lambda = \lambda_1 = 0$ in equations (18) and (23) the solutions are recovered for magnetized Newtonian fluid with magnetic field

$$u(y,t) = UH(t)t^m + UH(t)m! \sum_{i=1}^{\infty} \left(\frac{-y}{\sqrt{\nu}}\right)^i \boldsymbol{H}_{1,3}^{1,1} \left[\frac{\mu M}{t} \middle| \begin{matrix} \left(-\frac{i}{2}, 0\right) \\ (0,1), \left(-\frac{i}{2}, 1\right), \left(\frac{i}{2} - m, 1\right) \end{matrix} \right] t^{-\frac{i}{2}+m}, \tag{28}$$

$$\tau(y,t) = -\frac{UH(t)\mu m!}{\sqrt{\nu}} \sum_{i=0}^{\infty} \left(\frac{-y}{\sqrt{\nu}}\right)^i \boldsymbol{H}_{1,3}^{1,1} \left[\frac{\mu M}{t} \middle| \begin{matrix} \left(-\frac{i+1}{2}, 0\right) \\ (0,1), \left(-\frac{i+1}{2}, -1\right), \left(\frac{i+1}{2} - m, 1\right) \end{matrix} \right] t^{-\frac{i+1}{2}+m}. \tag{29}$$

5.4. NEWTONIAN FLUID WHEN $M = 0$ AND $\lambda = \lambda_1 = 0$

In similar pattern, making $M = 0$ and $\lambda = \lambda_1 = 0$ in equations (18) and (23) the solutions are recovered for Newtonian fluid without magnetic field can be retrieved.

CONCLUSION

In this investigation, the unsteady flow of magnetized fractional second grade fluid is analyzed over an accelerating plate in which assumption between presence and absence of magnetohydrodynamics (MHD) is emphasized. The exact solutions for velocity field and shear stress are perused by employing discrete Laplace transform with its inverse. The general solutions are established in series form and written as Fox-H function $\boldsymbol{H}_{p,q+1}^{1,p}$satisfying initial and boundary conditions. These general solutions have been particularized for four models namely (i) ordinary magnetized second grade fluid (ii) fractionalized and non-magnetized second grade fluid (iii) magnetized Newtonian fluid and (iv) Newtonian fluid. On the other hand, the impacts of rheological parameters, fractional parameter and magnetic parameter have been illustrated for motion of fluid by depicting several graphs. The role of magnetic parameter as expected has brought interesting result with respect to fractional parameter and rheological parameters. However, major finding as listed below:

(i) The expressions for the velocity field equation (18) and shear stress equation (23) have been presented in terms of Fox-H function $\boldsymbol{H}_{p,q+1}^{1,p}$and four models have been reduced using $\xi = 1$ and $M \neq 0$, $\xi \neq 1$ and $M = 0$, $M \neq 0$ and $\lambda = \lambda_1 = 0$, $M = 0$ and $\lambda = \lambda_1 = 0$ respectivelyfrom general solutions.

(ii) Figure 1 is plotted, as time increases shear stress is increasing and the velocity field has oscillating behavior on fluid motion.

(iii) Figures 2 and3 depict the influence of material parameters $\lambda, \lambda_1$for which shear stress is stretching in reciprocally when the free stream exceeds boundary.

(iv) Figures 4 and 5shows the effects of viscosity and fractional parameter in which smaller values of both parameters produces thickness of fluid flows over the boundary.

(v) In figure 6, as the value of m increases the velocity is rapidly increasing and shear stress is slowly increasing, this is due to the fact of nonlinear behavior of fluid motion. And figure 7 represents the effect of increasing magnetic field reduces and slow down the motion of fluid.

(vi) In figure 8 is drawn for the comparison of ordinary four models for $t = 2$ seconds, where velocity field for Newtonian model without magnetic field slow down the motion of fluid and corresponding shear stress Newtonian model without magnetic field is fastest as expected.

(vii) In similar pattern, figure 9 is drawn for the comparison of fractionalized four models for $t = 6$ seconds, where, velocity field for Newtonian model without magnetic field slow down the motion of fluid and corresponding shear stress Newtonian model without magnetic field is fastest along with scattering behavior of fluid flows. This is due to the fact that fractionalized models describe complete history for the fractional parameter between $0 < \xi < 1$ and on the whole domain.

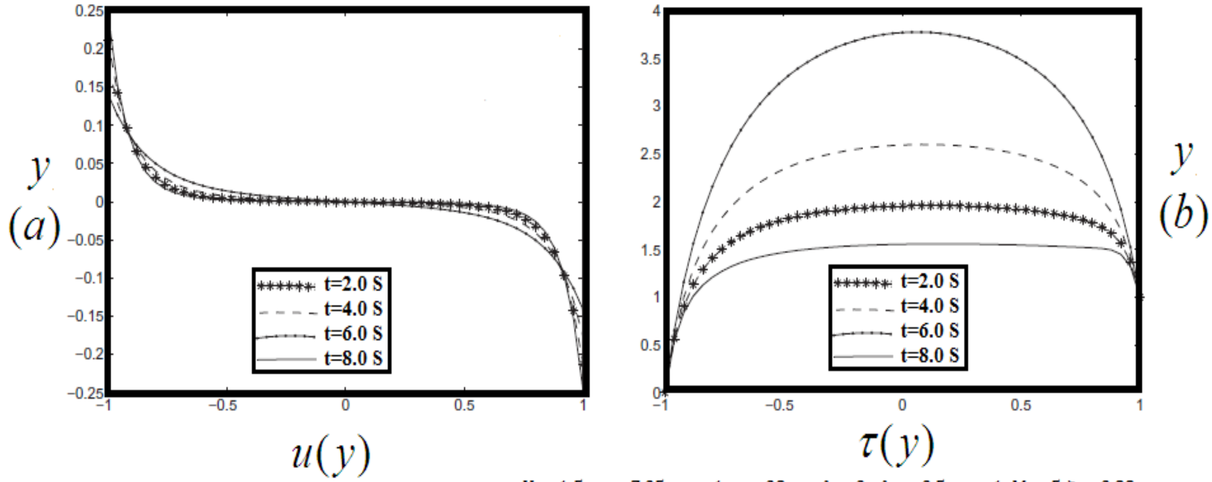

Figure 1: Profile of velocity field and shear stress $U = 1.5,\ \nu = 7.25,\ y = 1, \mu = 28,\ s,\ \lambda = 3,\ \lambda_1 = 3.5, m = 1, M = 5, \xi = 3.88$.

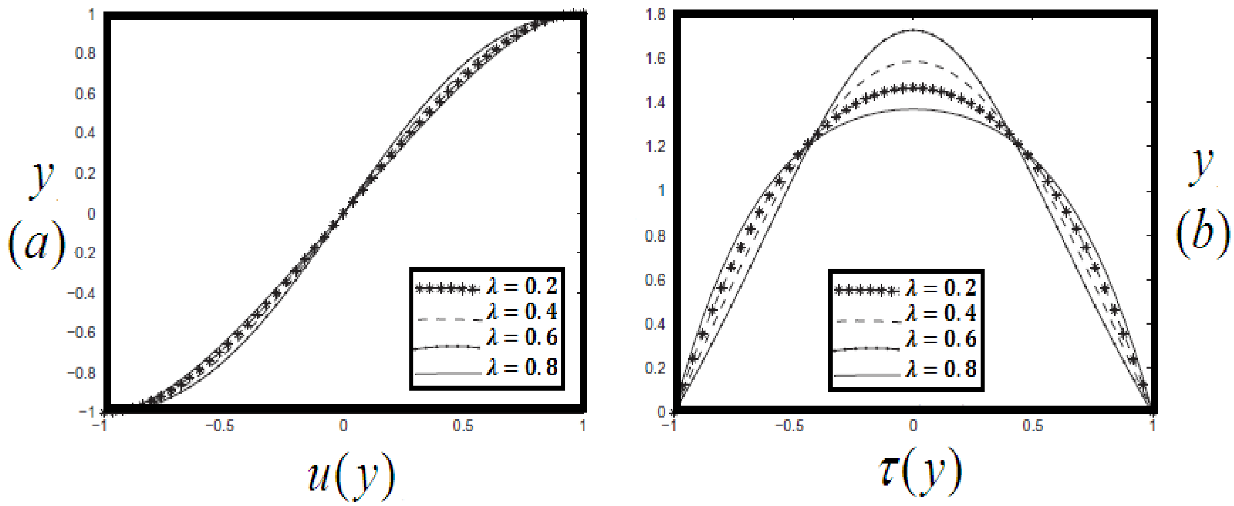

Figure 2: Profile of velocity field and shear stress $U = 1,\ \nu = 2.25,\ y = 1, \mu = 28,\ \ t = 1\,s,\ \lambda_1 = 2, m = 2, M = 2, \xi = 3.88$.

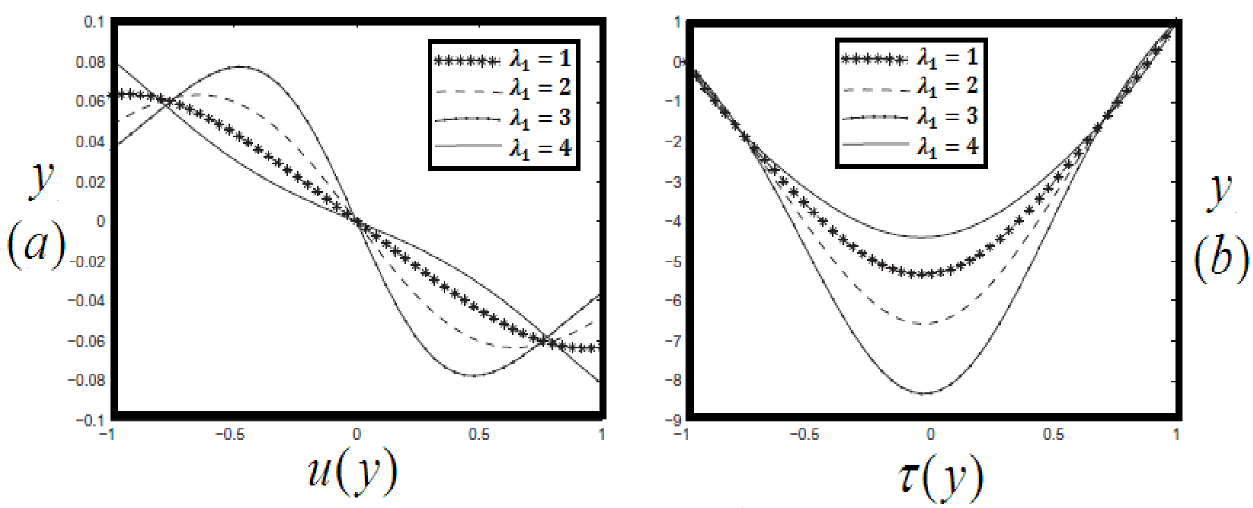

Figure 3: Profile of velocity field and shear stress $U = 5,\ \nu = 4.50,\ y = 1, \mu = 15,\ \ t = 2\,s,\ \lambda = 2, m = 2, M = 2, \xi = 3.88$.

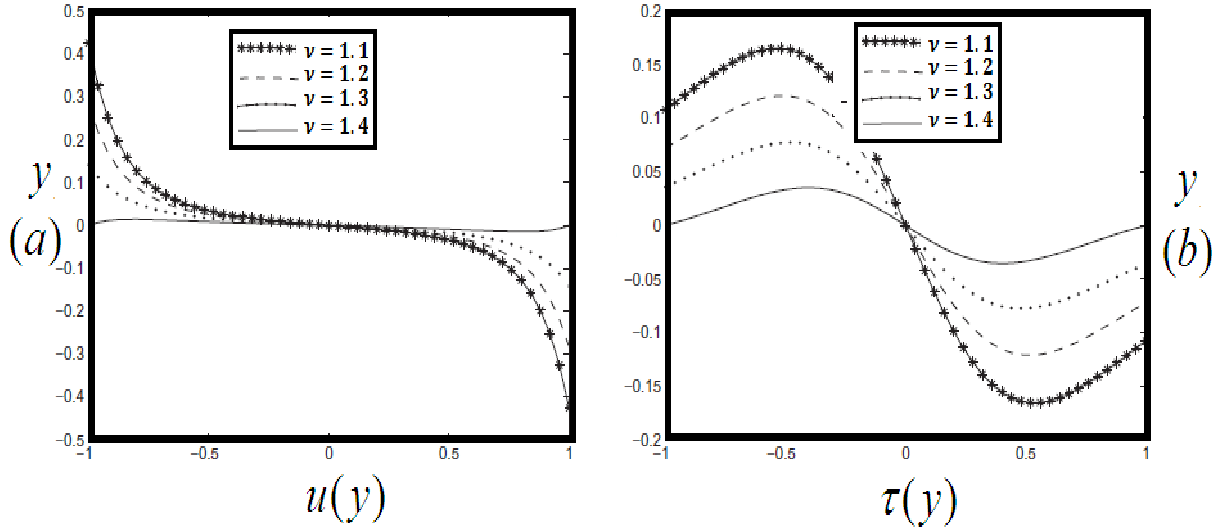

Figure 4: Profile of velocity field and shear stress $U = 0.2,\ \lambda_1 = 0.25,\ y = 1, \mu = 2.5,\ \ t = 10\,s,\ \lambda = 0.8, m = 1, M = 0.22, \xi = 125$.

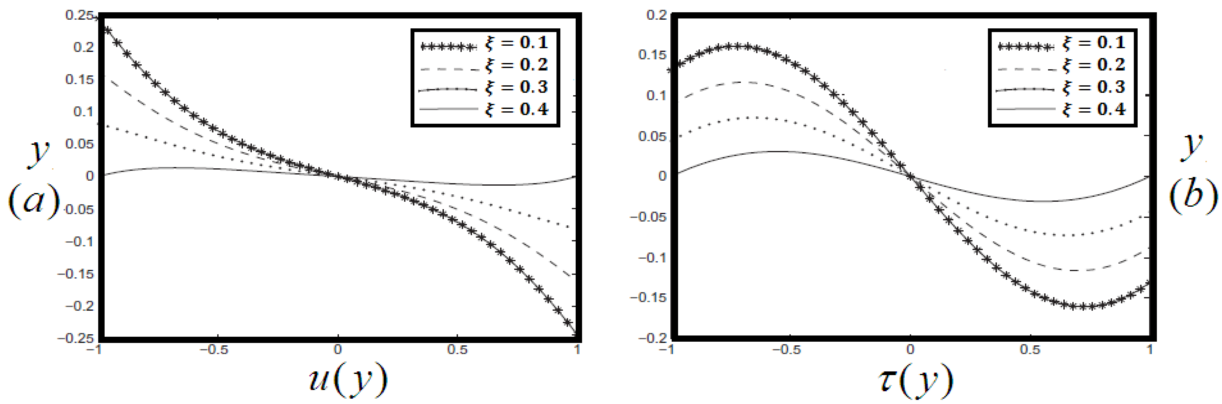

Figure 5: Profile of velocity field and shear stress $U = 5,\ \lambda_1 = 2,\ y = 1, \mu = 25,\ \ t = 2\,s,\ \lambda = 1.5, m = 1, M = 5, \nu = 4.5$.

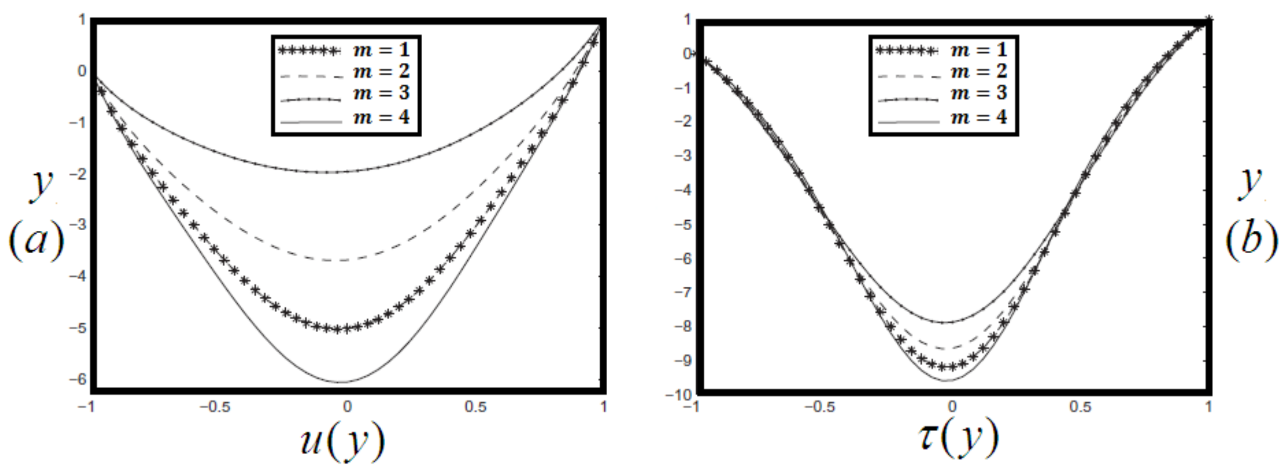

Figure 6: Profile of velocity field and shear stress $U = 2,\ \lambda_1 = 3.02,\ y = 1, \mu = 52,\ \ t = 2\,s,\ \lambda = 1.5, \xi = 0.5, M = 5, \nu = 4.5$.

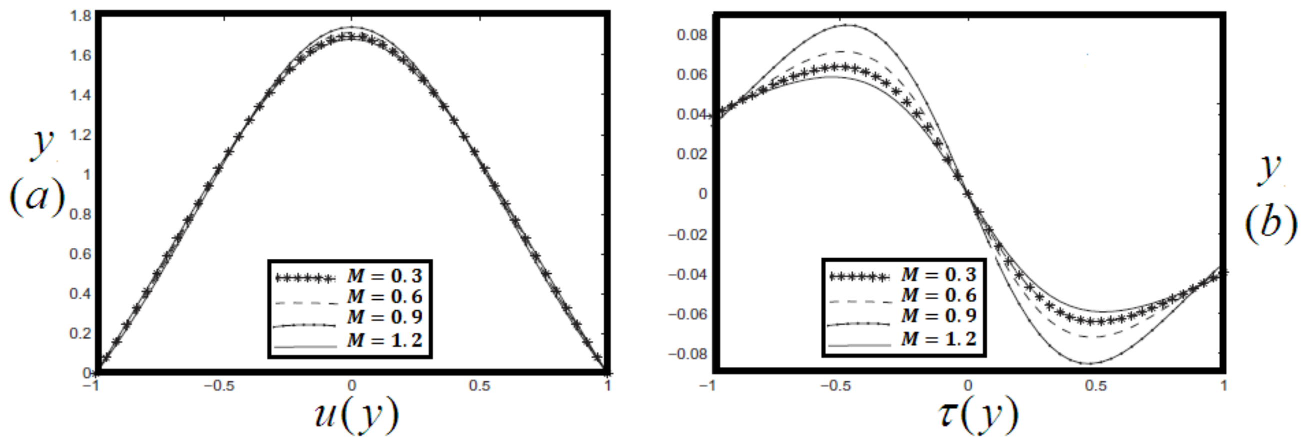

Figure 7: Profile of velocity field and shear stress $U = 2,\ \lambda_1 = 3.02,\ y = 1, \mu = 52,\ \ t = 2\,s,\ \lambda = 1.5, \xi = 0.5, \xi = 07, \nu = 4.5$.

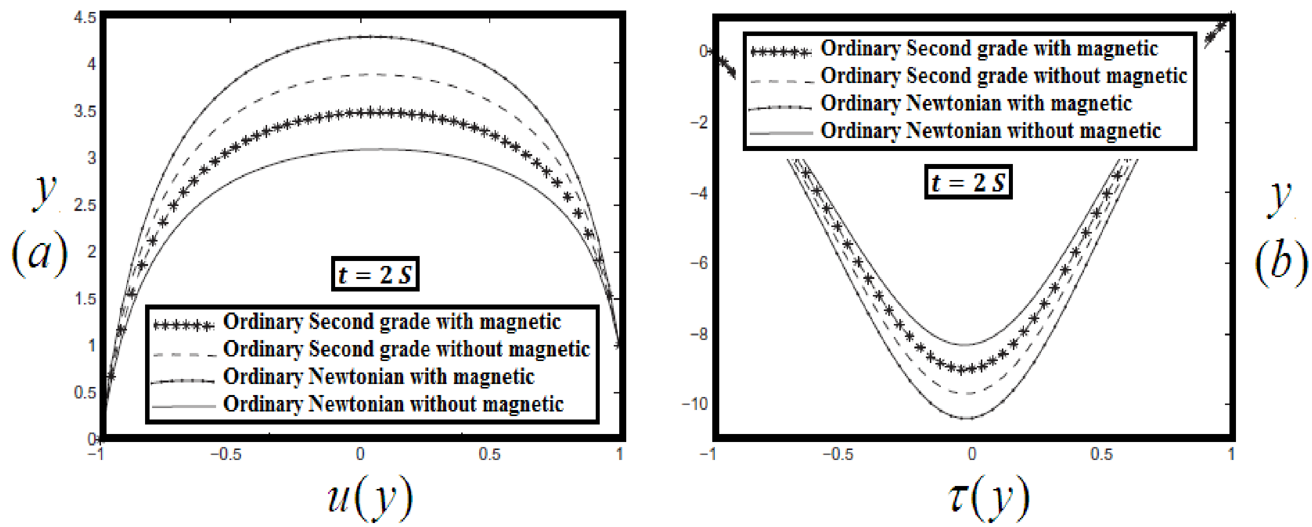

Figure 8: Comparison of velocity field and shear stress $U = 2,\ \lambda_1 = 0.2,\ y = 1, \mu = 2, M = 2.5, \lambda = 1., \xi = 0.5, \xi = 0.2, \nu = 4.5$.

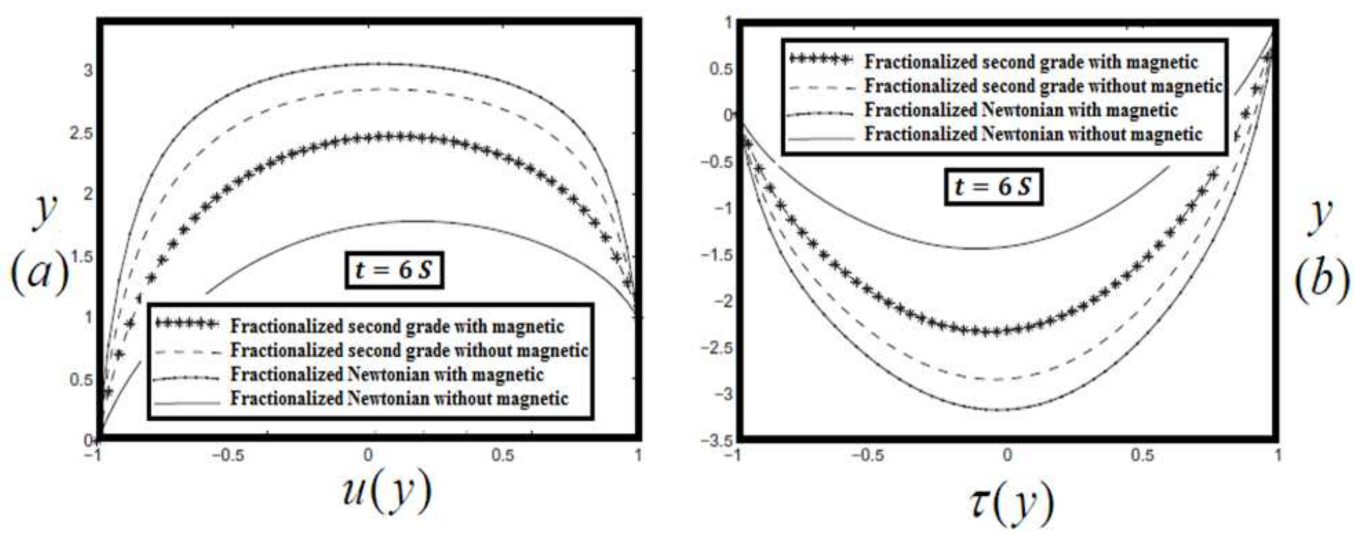

Figure 9: Comparison of velocity field and shear stress $U = 2,\ \lambda_1 = 0.2,\ y = 1, \mu = 2, M = 2.5, \lambda = 1., \xi = 0.5, \xi = 0.2, \nu = 4.5$.

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