

Determine the optimal quantity of inventory control systems with dynamic demand for single product by the PSO algorithm

Naeem Aghajani Delavar¹, Seyed Mojtaba Kavooosi Davoodi², Zeinab Aghajani Delavar³,
Navid Aghajani Delavar⁴, Esmacil Najafi⁵

¹Department of Industrial Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran

^{2,3,5}Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

⁴Department of Management, Babol Branch, Islamic Azad University, Babol, Iran

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ABSTRACT

In two echelon inventory control system (R, Q), the most important factor is to determine the optimal amount of orders and re-order point. Choose the wrong policy to determine the order value can increase the storage costs and the cost is usually includes the sum of the holding cost and shortage cost. The other side of this policy may result in wrong deficiency and thus faced with lost sales or orders is overdue. In other words, should the level of inventory in the warehouse should be (R + 1, R + Q). Due to the high cost of warehouses, factory managers are looking for ways to decrease costs in warehouses. Given that the traditional methods of multiechelon control, in practice they are not accountable to the problem, we use new method to solve this problem. This article includes a case study of a central warehouse factory is tea cup. Considering the warehouse modeling inputs including retail demand and warehouse costs, including holding costs, shortage cost is done. The optimum amount of orders per period covered by the original PSO algorithm is obtained.

KEYWORD: two echelon inventory control, single product, order value, PSO algorithm

1. INTRODUCTION

In many large factories correct distribution and control system of final product is an important issue. In practice, such systems usually have many difficulties, because the distribution of a vast geographical area need to study many cases. Most important of which is creation of central warehouses in different cities, select the appropriate retailers, select the appropriate policy for the distribution, selected appropriate vehicle for transportation of goods, checking transportation times and many other items that can be important to send product on time to retailers and ultimately to the final consumer. In many cases choosing the wrong policy of product distribution can delay or loss of orders. So most of large manufacturing enterprises consider choose a correct control policy and distribution of products important as products themselves. Therefore, large investments have been made in this area. In practice, when the final product is stored in various warehouses and then stock reach to retailers and to the final consumer through warehouses dealing with multi echelon inventory control systems that most large companies are faced with this kind of system.

2. Background of the study

In practice, when the products are distributed in a vast geographical area, the manufacturer specifies a number of storage locations. In addition, to improve and increase efficiency in the delivery time show that multi echelon methods may reduce total cost in comparison with echelon techniques offered by Muckstadt and Thomas (Muckstadt, 1980). Multi echelon control system with confidence has been widely studied in the literature. The first person that has had most important papers in this field was Sherbrooke (Sherbrook, 1986). He offered the first mathematical model called Metric 1 (multi echelon technique for recoverable item control) to determine the level of repairable or coverable goods in two echelon inventory system to minimize the expected total overdue orders that satisfies budget constraints. Metric model is an estimation technique and key idea based on queuing theory. For example, Palm and Little have developed the metric method because of its success (Little, 1961), (Palm, 1938). Muckstadt (Muckstadt, 1973) has developed modmetric 2 model that was able to examine the structure of hierarchy services. Graves (4) replaced original model with an estimation of overdue distribution by binomial distribution. On that time Sherbrooke expanded Muckstadt' modmetric which used Graves estimation (Sherbrook, 1986). Successful application of metric was in Cohen et al (Cohen, 2000) automobile industry, computers and electronic equipment and military affairs of Rustenburg (Rustenburg, 2001).

In other researches in this field solving optimization problems with constraints on time service was created. Hopp et al (Hopp, 1999). provide a meta-heuristic algorithm for minimizing the total cost of factory preset inventory. This meta-heuristic algorithms, analyze the problems step by step. And for some simplify estimations, the steady state inventory and

Corresponding Author: Seyed Mojtaba Kavooosi Davoodi, Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran. Kavooosi_mojtaba@yahoo.com

overdue orders were used. Then they could treat storage issues using estimation and Lagrange Relaxation. Caglar and colleagues (Caglar, 2004) provided an innovative algorithm close to the optimum solution for minimizing the extent of the cost of holding inventory system under time constraints. This approach differs from previous works because it clearly survey response time limit. They used a model similar to the metric model that arise from estimations for expected inventory level and level of overdue orders. Then applied an innovative algorithm for solving problems. Caggiano and colleagues (Caggiano, 2007) presented an issue for achieve the required level that minimize total capital cost of the system. Literature of traditional methods for solving problems of multi echelon inventory has three characteristics: 1- The assumption of Poisson demand 2- Using of single-level estimates 3- Use a tree network for distribution services sectors.

3. Case Study

Tea production factory, with brand Saed established in 2011 due to the increasing need of efficient and easy use of medicinal plants, after extensive research by experts in the field of medicinal plants in northwest area of Iran produce tea cup and herbal tea. Now the factory has a production capacity of one million pieces of tea cup per month. Due to the high cost of warehouses, factory managers are looking for ways to reduce costs in warehouses. Given that the traditional methods of multi echelon inventory control, in practice are not accountable to the problem, a new method to solve this issue were used. In this paper, we first define the problem then introduce parameters related to warehouse including maintenance cost and purchasing cost. Then will explain minimization function of warehouse. Finally, optimized results are presented by PSO algorithm.

4. RESEARCH METHODOLOGY

Due to the lack of background the factory are faced with the problem of the distribution of its products. Because central warehouse does not know how to order the factory that do not face with shortage. If the order quantity is too much it faced with increasing maintenance costs. Therefore, obtaining an optimal value of orders for each period according to the cost of maintenance, and purchase cost can cause saving in warehouse overall cost. Product is received by the central warehouse and delivered to the retailers. And finally products reach to final consumer by retailers. Based on multi echelon inventory control policy, permanent review of central warehouse check inventory levels every two days. If the inventory echelon is less than R , orders Q . In other words, the level of inventory in the warehouse should always be $(R + 1, R + Q)$. In the figure a part of the supply chain between warehouse and m retailer is plotted.

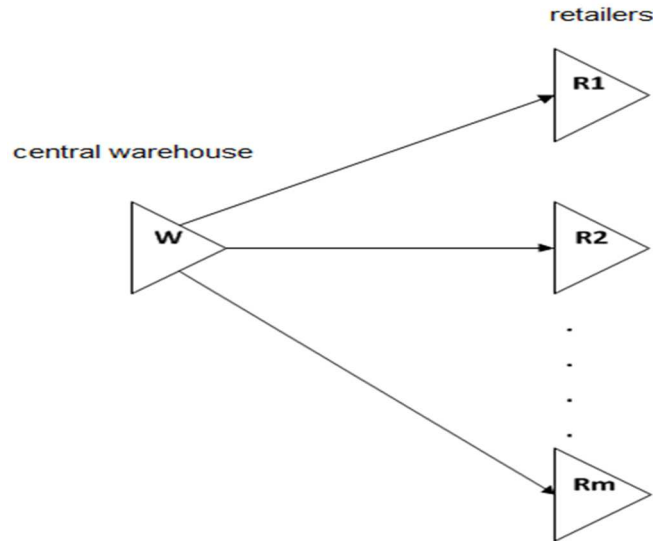


Figure 1: two echelon inventory control between central warehouse and retailers

Parameters of issue are shown below.

- H_t = total maintenance cost at time t in the central warehouse
- b_t = cost of goods at time t at a single central warehouse
- C_t = total purchase cost at time t in the central warehouse
- A_t = purchase cost of a single product at time t at the central warehouse
- I_t = Value of the packet order in warehouse at time t in the central warehouse
- Q_{min} = Minimum order quantity packets on the central warehouse
- Q_{max} = Maximum packet order in the central warehouse
- I_{t-1} = the previous inventory in the warehouse

D_t = the amount of retailer demand at time t
 I_t = inventory amount available at time t in the central warehouse
 I_{max} = maximum amount of inventory in the central warehouse

α = Rate of orders severity per period = $1 > \alpha > 0$

To solve the problem, the following assumptions are considered.

1. The retail demand is constant.
2. The cost of maintenance, purchase cost and demand are considered as input.
3. The production rate is infinite.
4. Lead time in all models considered to be zero.

5. Modeling issue

In modeling of this issue we seek to minimize the purchase cost and maintenance costs in central warehouse. Due to the fact that purchase cost and maintenance cost are directly related to the amount of the order, so by determining the optimal order quantity, total cost of purchase and maintenance costs will be optimized. To obtain purchase cost, multiplied the order from the central warehouse to upstream manufacturer by the purchase cost of a single product, that purchase cost in the first half of the year is 750 Rials in the second half of the year is 800 Rials. Transportation costs are constant throughout the period and considered 50 Rials per cup.

$$I_t = I_{t-1} + Q_t - D_t$$

$$Q_t \geq 0 \quad \forall t$$

$$D_t \geq 0 \quad \forall t$$

at $\sum Q_t = (C_t)$ purchase cost

It $\sum b_t = (H_t)$ Maintenance costs

The top model in general, now we expressed this model in more detail. As is clear from the above formula the level of inventory in the central warehouse equal to prior inventory in warehouse, which normally is zero at time zero, ie I_0 which central warehouse order added to it and the amount of retailers demand deducted from it.

$$\min \sum \text{at } \sum Q_t + \sum I_t$$

$$I_t = I_{t-1} + Q_t - D_t$$

$$Q_t \geq 0 \quad \forall t$$

$$D_t \geq 0 \quad \forall t$$

By obtaining optimum Q_t in above model, It will be optimized and by obtaining optimal I_t , the minimization model which is a combination of Q_t and I_t also reaches optimality. The information includes the amount of retailers demand per month, the cost of purchasing goods, maintenance costs over the course and initial inventory from the central warehouse gathered and expressed in tables.

Table 1: amount of demand, purchase cost and storage cost in central warehouse

Month	Total demand from retailers (number)	purchase cost (Rials)	Maintenance costs
April	50200	37650000	1040000
May	49800	37350000	1016000
June	40000	30000000	872000
July	30000	22500000	680000
August	26000	19500000	560000
September	27500	20625000	608000
October	30500	24400000	692000
November	36200	28960000	800000
December	46000	36800000	980000
January	53000	42400000	1040000
February	59000	47200000	1136000
March	54500	43600000	1088000

The amount of orders per month obeys the following formula. Factory and central warehouse, according to their agreement consider a maximum demand for the central warehouse. The maximum order is equal to I_{max} is considered about 700000. On the other hand for α it is determined at any time, according to the seasons and the demand each month. If α equals 1, Q_t is equal to the maximum amount of order. Or in other words the rate of demand intensity reach to its peak.

Usually in the cold season of the year that the per capita consumption of tea is high this rate is close to zero, and if the rate is zero, Q_t is equal to minimum order quantities or in other words, the intensity of the demand rate reach to the lowest level. Usually in warm seasons this rate is close to zero.

$$Q_t = \min ([Q_{min} + (Q_{max} - Q_{min}) \times \alpha], Q_{max})$$

Based on central warehouse policy when the inventory is close to zero, the order will be sent to the factory. Since in assumptions the lead time amount is considered equal to zero, inventory echelon immediately increase in order size. This factor causes that in months which maintenance and purchase costs is higher they be ordered and warehouse incur additional cost. For example, if the central warehouse in August increase its orders, and orders for some courses the purchase cost will be much lighter.

6. PSO algorithm

PSO algorithm first time was introduced By Kennedy and Eberhart (Kennedy, 1995) in an article entitled a new optimizer using particle swarm theory. This algorithm is categorized based on a population-based algorithms that became modeled according to social behavior of some birds, so PSO algorithm is a continuous algorithm. This algorithm is used to solve the model. For writing this algorithm MATLAB 2012 program has been used. Inputs, for solving this problem is the amount of demand per month maintenance cost per month, purchase costs per month, the maximum order quantity, minimum order quantity and intensity of demand in any month which are random numbers between zero and one. All required entries were calculated in Table 1. The input for the algorithm also minimizing function which includes maintenance cost, purchase costs and orders per month. The formula is shown in Equation 2. Therefore output of the algorithm include the total cost of maintenance and purchase costs, inventory, and optimal order amount for each period. It should be noted that for objective function, the total amount of maintenance cost and ordering costs, an amount of fines is considered for lack of orders per month. If these penalties are not considered algorithm tend to minimum cost and finds lowest cost in not ordering. Generally, total cost is negative and as a result inventory amount is negative and no order is registered. The ultimate objective function is as follows.

$$Z = ((at \times Q_t) + (It \times bt) + \alpha \times V_{MIN}) \times (1 + \beta \times V_{MAX})$$

$$V_{MIN} = \text{Mean} (\text{Max} (0, -I))$$

$$V_{MAX} = \text{Mean} (\text{Max} (I_{max} - 1, 0))$$

In the above expression for the beta and alpha special coefficient is considered that this coefficient is dependent on the size of the problem. For this issue alpha coefficient is considered 1000 and beta coefficient is 100/000/000. Acceptable answers are $V_{MIN} = 0$ and for $V_{MAX} = 0$. If two above parameters have values other than zero the objective function will be worse. While the algorithm moves toward better solutions. We performed algorithm with an initial population of two hundred, and five hundred repetitions. Ultimately the optimal answers are obtained at the output of the algorithm as follows.

Table 2: The order quantity and inventory echelon at central warehouse in optimal state

Month	Order quantity (Q)	Inventory levels (I)
April	60264	10064
May	39739	3
June	40010	13
July	30470	483
August	58026	32509
September	64144	69153
October	31213	69866
November	36334	70000
December	35814	69814
January	48832	65646
February	27089	37735
March	20765	0

As you see in table 2 the largest amount of orders were recorded in August and September and the lowest orders were in February and March. The reason is low purchase price and maintenance cost of the product, in the months of August and September. In April due to the lack of orders in previous months and low order cost in first half of the year in order to compensate inventories many orders have been recorded. Also, due to the increased cost of production, average of inventory levels in the second half of the year is more than the first half of the year. This indicates that the increase in the cost of purchasing the inventory echelon in warehouse increase.

7. The results of the algorithm

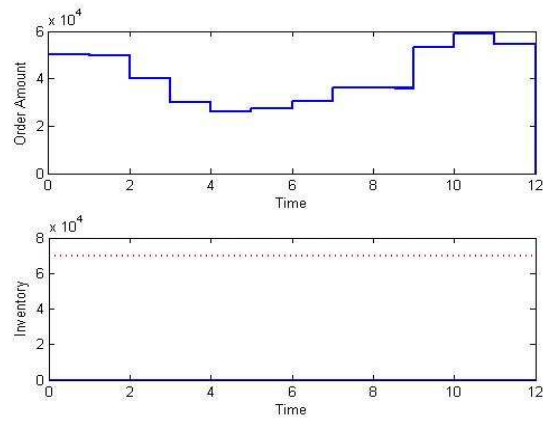


Figure 2: The ordering amount and inventory echelon in current state

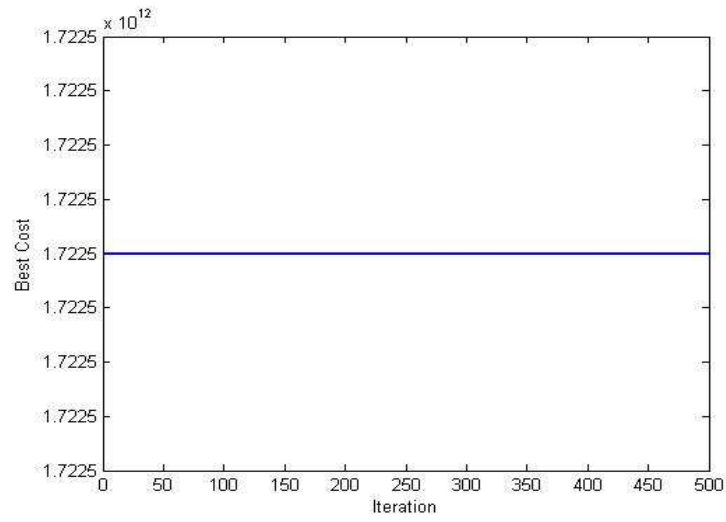


Figure 3: The cost amount in the current situation

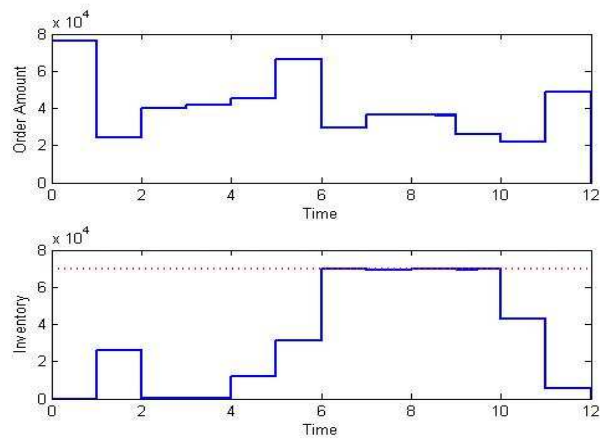


Figure 4: The order quantity and inventory echelon in optimal situation

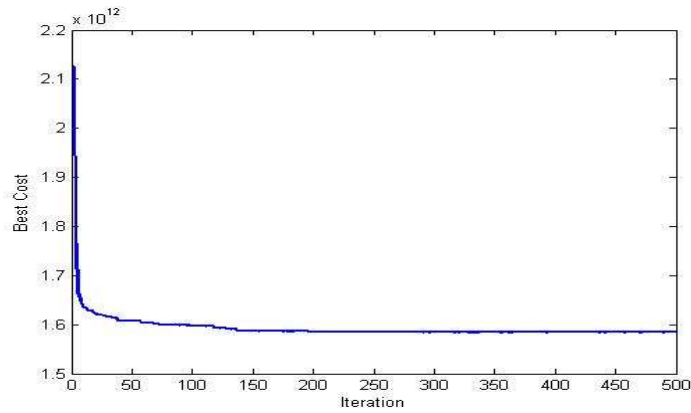


Figure 5: The cost amount in optimal situation

Figure 2 and Figure 4 shows the amount of orders and inventory echelon at all times, which is the output of MATLAB software. The red line in the above figures, shows the highest inventory echelon. In comparison Figure 3 and Figure 5, we note that the optimal cost approximately is 1.58×10^{12} and the best cost in this case indicates the number $10^{12} \times 1.7225$ which covers a constant process at all times. Amount of optimal cost reduction compared to the current state is approximately 9%. This amount of reduction in higher numbers will significantly increase. Figure 2 and Figure 3 shows graphs of the best cost, inventory echelon and order quantities in current state, and Figure 4 and Figure 5 shows the graph of the best cost, inventory echelon and order quantity in optimal state. Comparing Figure 2 and Figure 4, we note that in optimal state order quantity in the warm months is more than order in the cold months. It is due to price increase in the cold months. In Figure 2, the order quantity is equal to demand in each period, and as a result in each period the inventory echelon has become zero. No relationship exists between order quantity in warehouse and purchasing cost in warm and cold months of the year. Figure 5 shows the changes in the cost in 500 repetition. As can be seen in the first iterations the amount of cost greatly reduced, but after hundred repetition reduce rate in cost amount decreases and almost after 220th repetition significant changes in total cost hasn't been seen. Figure 3 shows the changes in the cost of 500 repetitions. As can be seen in all period the amount of the costs are fix.

8. Conclusion

As we saw in the previous section, traditional methods for optimizing inventory control system does not work in practice. Therefore, there is need for new ways for optimizing use of inventory control systems. One of these methods is the use of meta-heuristic algorithms. These algorithms to achieve optimal answer use estimation methods. Every time using algorithm we achieved different answers which are close to the optimal answer, but these methods are usually used because of high speed in reaching optimal answer. In this study examined the optimal order of two echelon inventory control systems. Calculation of this amount, by exact methods, is very difficult and complex and it is even impossible in some multi-products. In future researches, it is recommended that instead of using meta-heuristic algorithms, simulation and emulation program, to solve such problems to be used. Considering that this research has been studied as a single product, in future researches the research could be on multi-Products. As in this study, the optimal order quantity is calculated by meta-heuristic algorithms, it is suggested that for future research optimum re-order point (R) be investigated.

REFERENCES

- Caggiano, K.E. Jackson, P.L., Muckstadt, J.A., Rapp. (2007). Optimizing service parts inventory in a multi-echelon, multi-item supply chain with time-based customer service level agreements. *Operations Research* 55, 303–318.
- Caglar, D., Li,C.-L., Simchi-Levi, D., (2004). Two-echelon spare parts inventory system subject to a service constraint. *IIE Transactions* 36, 655–666.
- Cohen, M.A.,Cull,C.,Lee,H.L.,Willen, D., (2000). Saturn's supply chain innovation: high value in after sales service. *MIT Sloan Management Review* 41, 93–101.
- Graves, S.C., (1985). A multi-echelon inventory model for a repairable item with one-for-one replenishment. *Management Science* 31, 1247–1256.
- Hopp, W.J.,Zhang, R.Q.,Spearman, M.L., (1999). An easily implementable hierarchical heuristic for a two-echelon spare parts distribution system. *IIE Transactions* 31, 977–988.

- Kennedy J, Eberhart R, (1995). A new optimizer using particle swarm theory. *Micro Machine and Human Science*, 95, 39 - 43.
- Little, J.D.C., (1961). A proof of the queueing formula: $L = kW$. *Operations Research* 9, 383–387.
- Muckstadt, J.A., (1973). A model for a multi-item, multi-echelon, multi-indenture. *Management Science* 20, 472–481.
- Muckstadt, J.A., Thomas, L.J., (1980). Are multi-echelon inventory methods worth implementing in systems with low-demand-rate items? *Management Science* 26, 483–494.
- Palm, C., (1938). Analysis of the Erlang traffic formula for busy signal arrangements. *Ericsson Technics* 5, 39–58.
- Rustenburg, W.D., Van Houtum, G.J., Zijm, W.H.M., (2001). Spare parts management at complex technology-based organizations: an agenda for research. *International Journal of Production Economics* 71, 177–193.
- Sherbrooke, C.C., (1986). METRIC: a multi-echelon technique for recoverable item control. *Operations Research* 16, 122–141.
- Sherbrooke, C.C., (1986). VARI-METRIC: improved approximations for multi-indenture, multi-echelon availability models. *Operations Research* 34, 311–319.