

# Simulation-Based Optimization of Inventory Control Systems (Rn, Q) Multi Echelon - Multi Item

Seyed Mojtaba Kavooosi Davoodi<sup>1</sup>, Naeem Aghajani Delavar<sup>2</sup>, Zeinab Aghajani Delavar<sup>3</sup>,  
Mohammad Aghajani Delavar<sup>4</sup>, Esmail Najafi<sup>5</sup>

<sup>1,3,5</sup>Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

<sup>2</sup>Department of Industrial Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran

<sup>4</sup>Department of Civil Engineering, University of Tehran, Tehran, Iran

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## ABSTRACT

This research explores a multi echelon inventory control system consists of four main supplier, two central warehouse and four distributor pays the same place. Goods of the type of repair is inevitable investigated and policy a continue review (R, nQ). The purpose of this analysis is to determine the optimal order quantity (Q) and the optimal order point ® in policy (R, nQ) using optimization techniques based on simulation, to all local distributors, so the cost the total inventory is minimized. The total cost includes sum of the holding cost and shortage cost. The first phase of this research, physical and information flows between local distribution warehouse and manufacturing using Arena simulation software And the second phase by meta heuristics algorithms, the optimal values of decision variables, which include re-order point ® and the order quantity (Q) is determined.

**KEYWORDS:** Inventory Control Systems (R, nQ), multi echelon inventory control, simulation-based optimization

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## 1. INTRODUCTION

One of the most important aspects of inventory management that plays an important role in the production and distribution is multi echelon inventory management system. In many large factories correct distribution and control systems of final product is important. In practice, such systems usually have many difficulties because the distribution of inventory in a vast geographical area need to study many cases. Most notably the creation of central warehouses in different cities, select the appropriate retailers, select the appropriate policy for the distribution of inventory. Select the appropriate vehicle for goods transportation, transportation times and most other cases which can be important in timely reach of product to retailers and ultimately end consumers. In many cases, choosing the wrong product distribution policy may cause delay or loss orders. So, many manufacturing companies believe that choosing proper control policy and the distribution of products is as important as production of that product. Therefore investment in wide area of the field has been done. In practice when final products are stored in different warehouses and after that inventory is delivered to final consumers and retailers through warehouses, we are dealing with multi echelon inventory control systems which most large companies are faced with such systems. In recent decades, these systems have been considered for many applications in many industries and communication networks. Based on the assumptions and different parameters, different models have been proposed for them. In general, research and modeling of multi echelon inventory system began in the late 1950s and from that year onwards, researchers have been paying attention to it.

## 2. LITERATURE REVIEW

The literature reviewed in this paper will be divided into three general categories. At first part the control system (R, Q) is studied, in the second part the history of literature in the field of multi echelon inventory is reviewed. And ultimately in the third part, literature review in the field of using simulation-based optimization technique is checked.

### 2 -1 inventory control system (R, Q)

When (R, Q) policy is used, as soon as the inventory position (IP) 2 reaches to R (point of order) or below it, an order is issued. Order quantity is Q unit (Axsater, 2006). Since an order occurs exactly when the position of inventory is less or equal than R, we always have  $IP \leq R + Q$  so we should have in the stable condition

$R + 1 \leq IP \leq R + Q$ . In general, in the policy of (R, Q) when a request is entered into the system, the IP quantity decreases as much as demand. In Figure 1 (R,Q) policies are fully described.

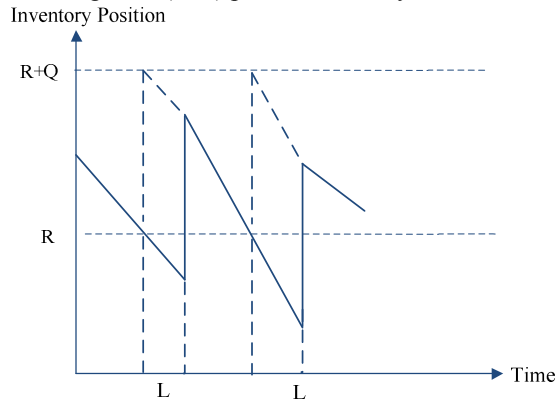


Figure 1: The Politics of (R, Q)

## 2.2 Multi echelon inventory control

In practice, when the products are distributed in a vast geographical area, the manufacturer specifies a number of storage locations which will be faced with a multi echelon inventory control. First researcher that wrote an article in this context was Sherbrooke in the 1986 (Sherbrooke, 1930). He proposed the first mathematical model called the metric 3 to determine the inventory echelon of repairable or coverable goods in two echelon inventory system to minimize the expected total overdue orders which satisfies the constraints of the budget. Metric model is an estimation technique and a key idea based on queuing theory. The successful application of metric was in automobile industry, computer electronic equipment and Rustenburg military matters (Cohen, 2000), (Rustenburg, 2001). In other researches in this area, solve optimization problems with constraints on time service was created that presented an innovative algorithm near to optimal answer for minimizing the extent of the cost of holding inventory system under time constraints. This approach differs from previous works because it clearly studied the response time limits (Caglar, 2006). They used a model similar to the metric model that use estimates for expected inventory echelon and overdue orders. Then applied an algorithm for solving problems (Caggiano, 2007). They presented an issue to achieve the required echelon which minimizes the total capital cost of the inventory system. Literature of traditional methods for solving problems of multi echelon inventory issues has three characteristics: 1. using the assumption of Poisson demand 2. Use single echelon estimates 3. Use a tree network for distribution services sectors.

## 2 -3 simulation-based optimization

Using optimization techniques based on simulation has been used in recent years. Instead of checking the system practically and incur high costs, by using this technique, in a short time with minimum cost, the results of the implementation of the policy on the computer are viewed and we decide to use the new system. The main approaches of optimization which are used for simulation-based optimization including random search (Andradottir, 2010), Barton response surface methodology (Barton, 2005), procedure based (Fumc, 2002), choosing and color (Kim, 2005), O' laffson innovative algorithms (O' laffson, 2005), which includes Tabu search, Genetic Algorithm, Scatter Search, and using simulation programs like Arena (Shing, 2013). They provided a simulation-based optimization algorithm for solving two echelon inventory with a number of limitations. This article aims to meet the needs of each warehouse and define the most optimal setting of inventory echelon for minimizing the inventory capital cost.

Figure 2 displays a good description of the relationship between simulation model and meta-heuristic algorithms.

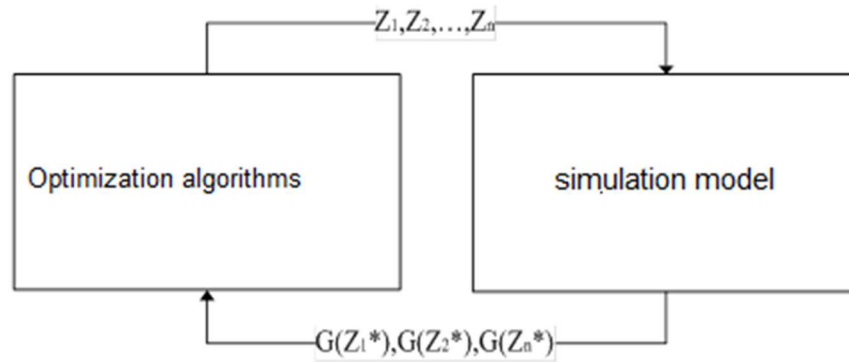


Figure 2: The relationship between simulation model and meta-heuristic algorithms

In Figure 3 types of simulation-based optimization problems have been classified (Pasandede, 2011)

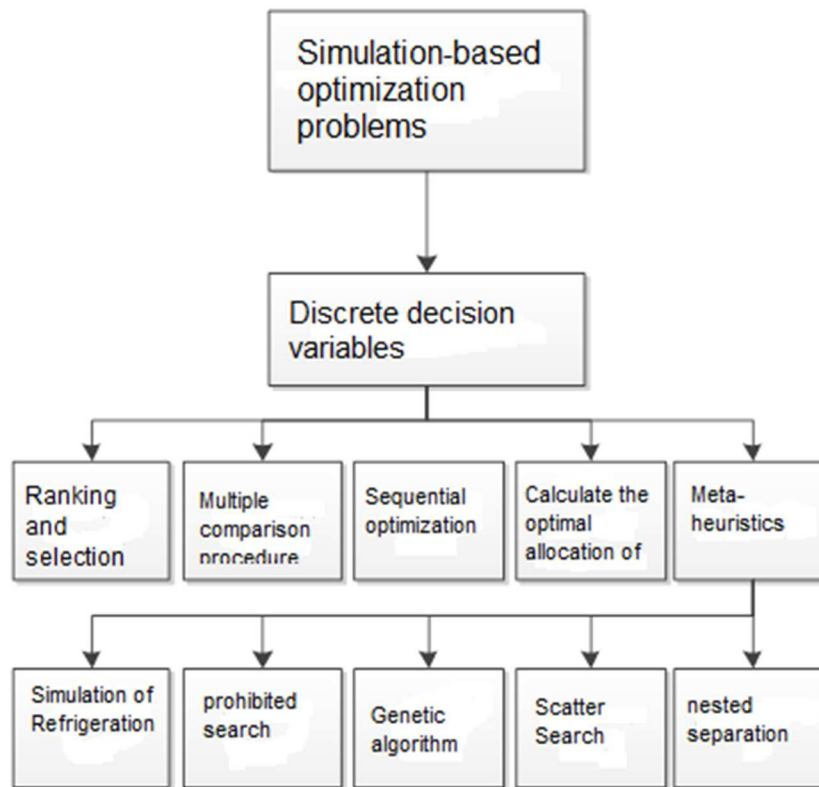


Figure 3: Classification of optimization problems based on simulation

### 3. RESEARCH METHODOLOGY

In this model first based on Pareto case of all company products in Padide Shimi Paydar Company Golrang industrial group, 20% of the goods that are equivalent to 80% of sales were selected. 20% of goods, including 10 production that include bleach, detergents, cylindrical dishwashing liquids, fabric softener, general dishwashing liquids, multipurpose bleach, Slave dishwashing liquid, soap, shampoo, hand washing liquid. These ten products produce by four manufacturer which include: X, Z, Y and W. Among all provincial distributors, the distributors of the four sites were chosen according to Pareto's theorem. This model consists of four local distributor. The local distributors including A, B, C and D. This model consists of two central warehouses named N and M. This research is based on the following assumptions:

- 1) Optimization is done at the strategic level.

- 2) Companies that have been considered in the inventory control system, has nothing to do with companies outside the system boundary, therefore no external supply system intended for them.
- 3) Also members who are at the same level, have no correlation in that level with other members.
- 4) Demand at the retail level to be considered definitive and may be characterized by a uniform distribution.
- 5) Production amount as manufacturers' output is considered either definite or possible with a uniform distribution.
- 6) Simulations have been carried out over a period of 90 days.
- 7) Each product is produced in a factory. In other words, a product is not produced in two factories.
- 8) Each producer can send additional products to one of central warehouses.
- 9) In each distributor there is demand for ten products.
- 10) For each of the distributors a periodic review system  $(R, nQ)$ , is considered. In other word each distributor independently has an order quantity  $(Q)$  and order point  $(R)$ . Browse cycle time of inventory is considered a day.
- 11) Delivery lead time from the manufacturer to the distributors and from the central warehouses to distributors is considered equal to zero.
- 12) In this issue all of the deficiencies is considered demand lag.

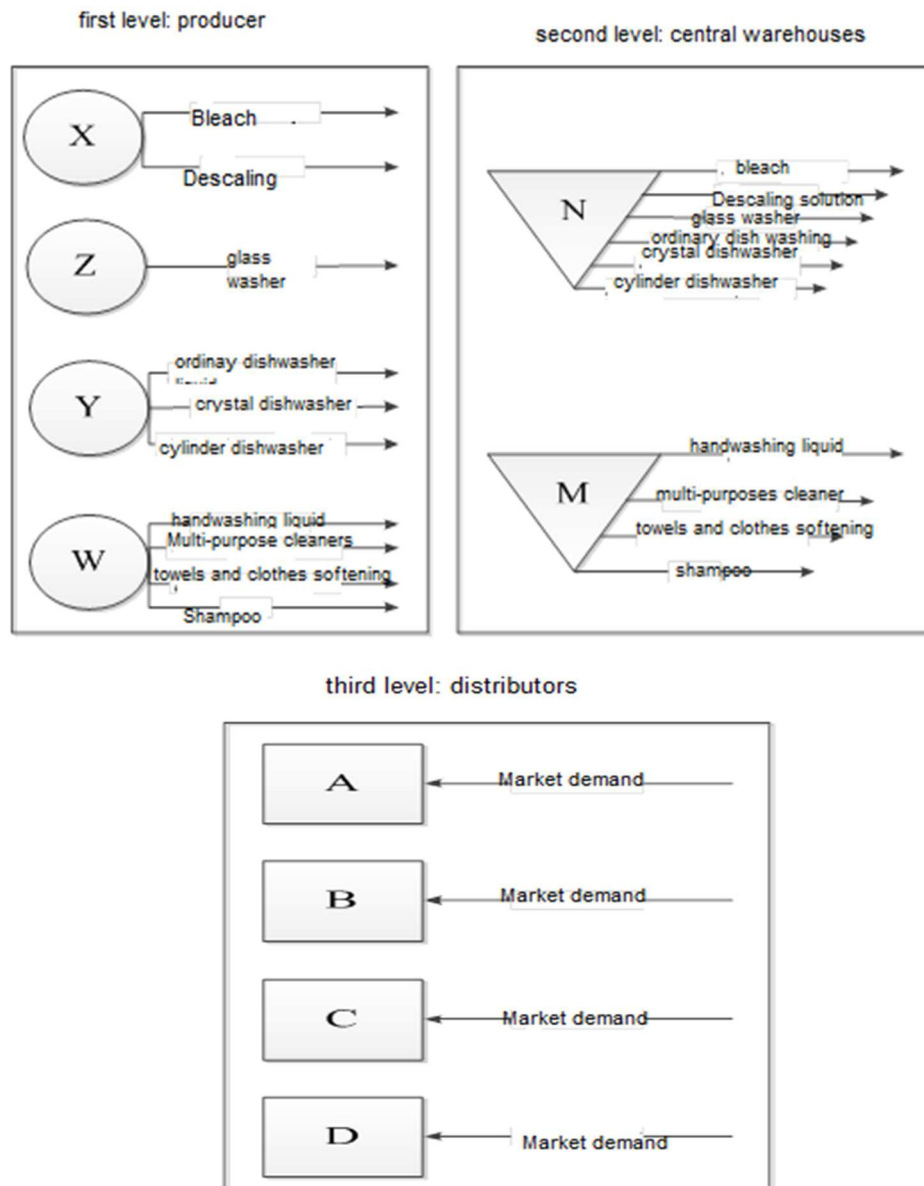


Figure (4) the flow of materials and information on the inventory control system

**3-1 modeling and data analysis**

In the present study, due to the nature of the data, assumptions and variables, in order to model multi-level inventory control system according to performance indicators, resource and constraints, Arena Simulation Software is used. Also EXCEL software is used for data retrieval of Arena software. Optquest tool is used for optimizing decision variables in Arena. In this tool, in order to optimize the parameters two multi-innovative algorithms Tabu Search and Scatter Search are used.

**3 -2 Re-order point and order quantities**

In Table 1 and Table 2 the basic information for the construction of a simulation model is shown. Table 1 relates to reorder point (R) and Table 2 shows the order quantity (Q) in each local distributors.

**Table 1: Order quantity (Q) (in boxes)**

product	A	B	C	D
Bleach	2600	1000	700	500
Detergent	1300	500	400	260
Cylinder dish washing liquid	960	360	280	180
Clothes softening	1560	600	440	300
Dishwashing liquids	2200	800	600	400
Versatile bleach	960	360	280	200
hand washing liquid	3200	1120	880	600
Soap	1300	500	400	260
Shampoo	1300	500	400	260
Crystal dishwashing liquid	960	360	280	200

**Table 2: amount of re order (R) (per box)**

products	A	B	C	D
Bleach	1300	500	350	250
Detergent	650	250	200	130
Cylinder dish washing liquid	480	180	140	100
Clothes softening	780	300	220	150
Dishwashing liquids	1100	400	300	200
Versatile bleach	480	180	140	100
hand washing liquid	1600	560	440	300
Soap	650	250	200	130
Shampoo	650	250	200	130
Crystal dishwashing liquid	480	180	140	100

**3.3 Simulation of inventory control system**

Due to the large volume of data such as the daily production per product, the quantity demanded per day, the numbers of the order (R) and order quantity (Q), all data is collected in the form of an Excel file. Since the Arena software has connectivity to Excel files, all information such as the amount of demand and production values is read in Excel files through Arena software.

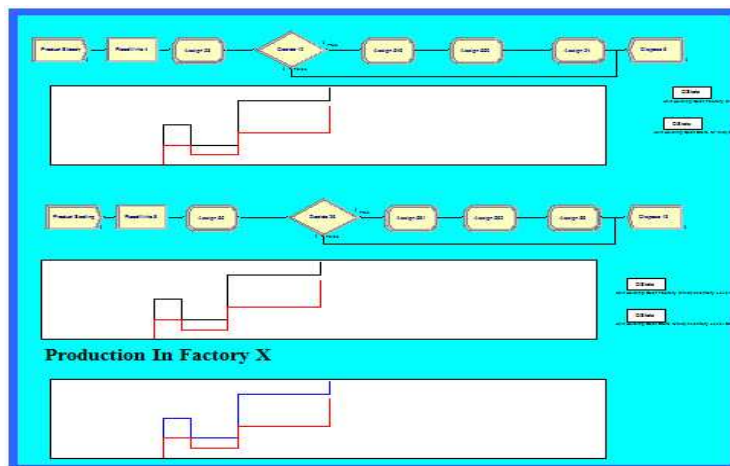


Figure (5): part of the simulation model in manufacturers' level

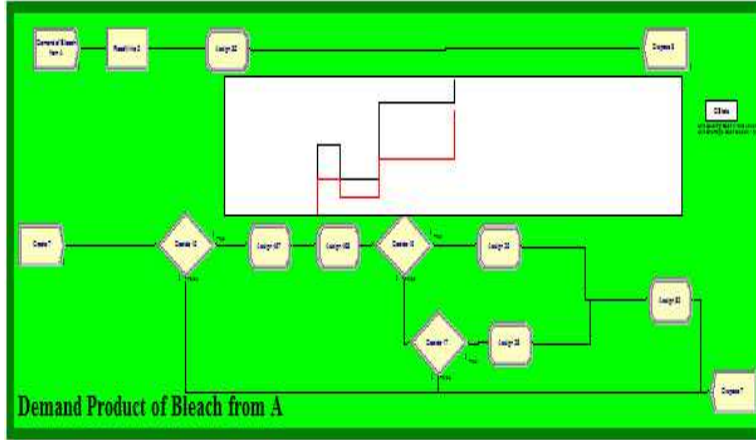


Figure (6): part of the simulation model in distributors' level

In (R, nQ) policies when the position of inventory reach to re-order R or be less than it, the order is issued equal to the size of a stack Q. (If the inventory position is too low, you may need more than one Q stacking order to reach the position above R. In this case ordering policy is called (R, nQ). Amount of N in the policy is calculated according to equation 1.

$$n = \left\lceil \frac{R-IL}{Q} \right\rceil \tag{1}$$

**3-4 optimization decision variables**

In order to optimize the decision variables, including the re-order point (R) and order quantity (Q), we use OptQuest tool, a powerful optimization tool in Arena. This optimization tool uses two meta-heuristic algorithm, Tabu search and scatter search for the optimization variables. Each R, Q represents the initial points to start the optimization by meta-heuristic algorithms. Table 3 and Table 4 respectively show the optimal R and Q and demand and production of both is considered definitive.

**Table (3): Optimal order quantity (Q) (in boxes)**

product	A	B	C	D
Bleach	2611	997	766	500
Detergent	1300	498	401	260
Cylinder dish washing liquid	978	360	279	180
Clothes softening	1560	600	440	314
Dishwashing liquids	2200	799	648	398
Versatile bleach	941	358	280	200
hand washing liquid	3200	1120	908	600
Soap	1300	530	399	260
shampoo	1300	508	430	260
Crystal dishwashing liquid	960	360	280	200

**Table (4): The optimal amount of re order point (R) (per box)**

product	A	B	C	D
Bleach	1302	500	346	250
Detergent	650	251	200	132
Cylinder dish washing liquid	480	198	138	100
Clothes softening	785	300	216	164
Dishwashing liquids	1142	401	297	200
Versatile bleach	480	179	138	100
hand washing liquid	1600	560	440	300
Soap	650	250	200	131
shampoo	650	249	194	130
Crystal dishwashing liquid	480	180	140	100

Pasandideh, Akhavan and Tokhmechi presented a model for multi echelon inventory control that includes a supplier, a central warehouse and some local distributors. In that article the optimal amount of (R, Q) was calculated by genetic algorithm. Due to the proximity with the research model, this model can be used to better understand the optimization problem (Pasandideh, 2011).

$$\min: TC = m \sum_{i=1}^k \bar{I}_{ri}(R_{ri}, Q_{ri}) + \sum_{i=1}^k C_i Q_{wi} \bar{I}_{wi}(R_{wi}, Q_{wi})$$

s. t.

$$\frac{1}{k} \sum_{i=1}^k \frac{D_{wi}}{Q_{wi}} \leq N_w, \tag{4}$$

$$\frac{1}{k} \sum_{i=1}^k \frac{D_{ri}}{Q_{ri}} \leq N_r, \tag{4}$$

$$\sum_{i=1}^k \bar{B}_{wi}(R_{wi}, Q_{wi}) \leq B_w, \tag{5}$$

$$\sum_{i=1}^k \bar{B}_{ri}(R_{ri}, Q_{ri}) \leq B_r, \tag{6}$$

$$\sum_{i=1}^k C_i Q_{wi} Q_{ri} \leq X_w, \tag{7}$$

$$m \sum_{i=1}^k C_i Q_{ri} \leq X_r, \tag{8}$$

$$\sum_{i=1}^k g_i Q_{wi} \leq G_w \tag{9}$$

$$R_{wi} \geq -Q_{wi} \quad i = 1, \dots, k \tag{10}$$

$$R_{ri} \geq -Q_{ri} \quad i = 1, \dots, k \tag{11}$$

$$Q_{wi} \geq 1 \quad i = 1, \dots, k \tag{12}$$

$$Q_{ri} \geq 1 \quad i = 1, \dots, k \tag{13}$$

$$Q_{ri}, R_{ri}, Q_{wi}, R_{wi}: \text{Integer} \tag{14}$$

Equation (2) expresses the objective function of the problem. The first part of the objective function is cost of local distributors and the second part is the cost of inventory in warehouse. Equation (3) and (4) are in order to ensure that the average number of warehouse orders and distributors is not greater than warehouse target order and distributors. Equation (5) and (6) in order to ensure that the average number of backlog of warehouse and each distributor, is not further than warehouse and distributors' backlog. Equation (7) is to ensure that the warehouse purchase cost is not more than its whole capital. Equation (8) is to ensure that the distributors purchase cost is not more than its whole capital. Equation (9) ensures that the goods storage space in warehouse is not further than whole warehouse space. Equation (10) and (11) are to ensure that the backlog of warehouse and each distributor will be fulfilled after arriving orders. Equation (12) and (13) are to ensure that the size of orders in the warehouse and each distributor is greater than zero. Finally, equation (14) indicates that the value of the order and point of order in each local distributor and central warehouse is an integer. After optimization by OptQuest tool, optimization results are presented in a graph of the change in the objective function and the results of the re-ordering of optimal locations and quantities of orders. Note that for R, Q of each product automatically is considered an upper and lower limit by the software. In fact, each R, Q represents a starting point for optimization by meta- innovative algorithms. After optimization of the model and the results we conclude that OptQuest cannot guarantee that find optimum global point in any case, furthermore, it is a challenging problem; Because not only there are too many states to consider possibilities, but also the target (output simulation model that should be optimized) cannot be measured with certainty.

### 1. NUMERICAL RESULTS

In this section the results of performance indicators relating to the implementation of the model are provided. Considering them multi-echelon, multi- item inventory control models will be evaluated. All costs related to the implementation of the simulation results is presented in Table 5. Figure 7 compares the costs, including the cost of maintenance and deficit cost. All outputs including costs in manufacturing level, central warehouses and distributors is obtained by Arena simulation software. In mentioned study, only deficit and maintenance costs are calculated. Because these two costs depends on the number of items and may vary in each moment of time. Cost recording and ordering will not be considered. Figure 6 shows the values of the objective function in the simulation is 755 repetition. As it is specified in figure the algorithm starts from a point. And according to the decision variables and objective function of the problem including R, Q reach to optimum point. The algorithm stops in condition that there is no improvement in the objective function in hundred consecutive iterations. The objective function is the sum of the costs of maintenance and deficit costs in each level.

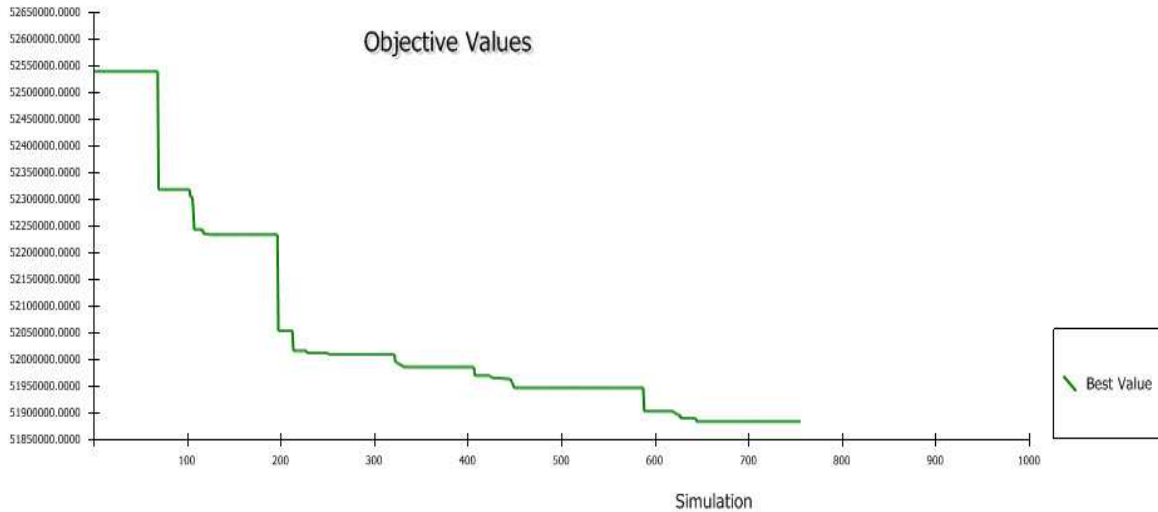


Figure (7): varying the values of the objective function in 755 simulation repetition

Registration and purchase ordering costs are calculated as a constant number in the objective function which we ignore them. Maintenance costs include all cost of maintenance in factories, central warehouse and distributors. In order to calculate the cost of maintaining in each factory, central warehouse and distributors, high-level of inventory diagram multiplied in goods cost of each unit. Finally, all cost of maintenance in each level added and deficit costs only are calculated compensable. In order to calculate deficit costs in any of the factories, central warehouses and distributors low-level of inventory diagram multiplied in deficit costs of each goods' unit. And finally, all deficit costs in each level are added together.

**Table 5 compares the costs in optimize and current state**

Sum of costs	current state	Optimal state
The total cost of maintaining in distributor A	6568346	6565086
The total cost of maintaining in distributor B	1685897	1680462
The total cost of maintaining in distributor C	1149698	1140278
The total cost of maintaining in distributor D	642751	638755
The total cost of maintaining in central warehouse	39578760	40321200
The total cost of maintaining in factories	1989000	1929000
The total deficit costs in total products	0	0
Total costs	51884100	52540040

Stability of production and demand values and the policy of (R, nQ) has led a situation in which significant difference was not seen between the current state and the optimal state. Figure 7 clearly shows that the highest cost is related to maintenance in central warehouse and after that the cost of maintaining in distributor A.

**Table 6 improvement in costs**

State	Total cost	Repeat in simulation	Percent of improvement
Current	52540040	repetition 755	percent 1.2
Optimal	51884100		

Because the manufacturers send all products to their central warehouses, causes that cost of maintaining at this level has highest value. Also, due to high demand in the distributor A at this level cost of maintaining have gone up.



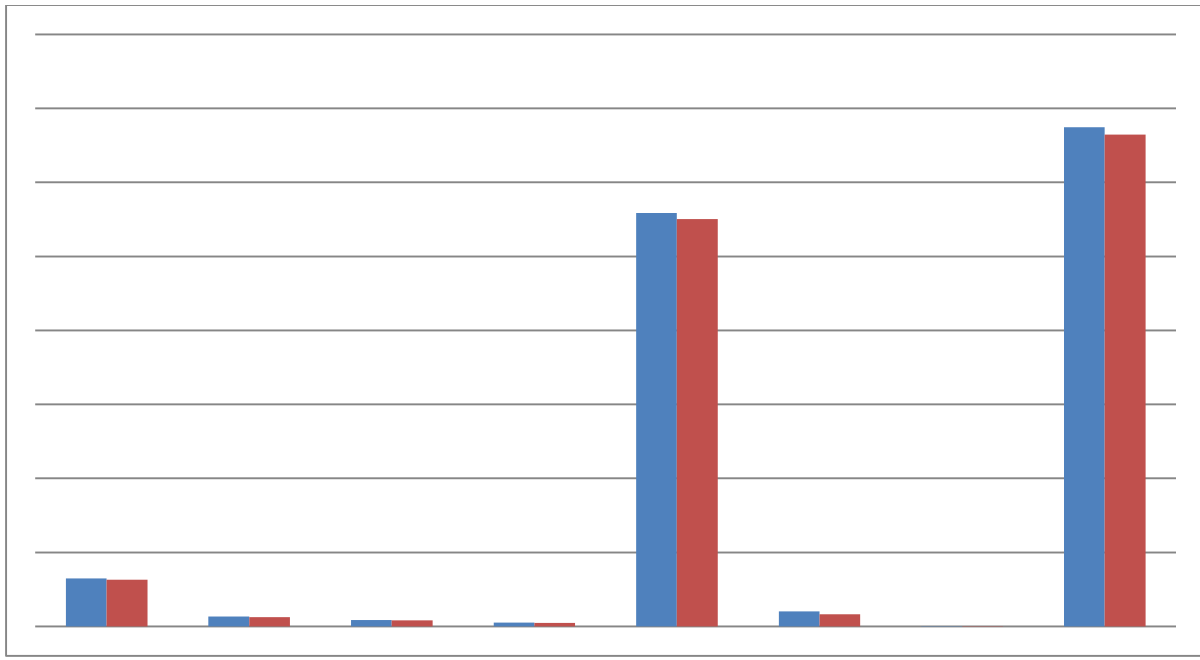


Figure 8: Comparison of costs in optimization and the current state

## 2. CONCLUSIONS AND RECOMMENDATIONS

With regard to the implementation of the model and compare the costs of inventory control system (R, Q) we determined that using (R, nQ) policies, inventory distribution and control systems has more assurance store, because inventory position is always in interval  $R + 1 \leq IP \leq R + nQ$  that includes wider range in comparison to inventory control system. Predicted transport costs in inventory control (R, nQ) is easily possible because every time you order, amount of order is equal to nQ, but in other systems amount of order is different in each time and also transportation costs vary. Using optimization techniques based on simulation and determine optimal values of the decision variables, causing a reduction in the total cost of the inventory control system. In Table 5, costs reduction is displayed in current and optimal state.

Suggestions for future research are as follows:

- The use of simulation-based optimization approach to optimize the production level.
- The use of simulation-based optimization approach to optimize inventory control of other systems such as inventory control system (s, S).
- Comparison of costs in inventory control systems (R, Q) and (s, S) with simulation-based optimization approach.
- The percentage of increase and decrease in costs in possible mode of inventory control systems (R, Q) and (s, S).
- Comparison of shortcomings in inventory control system (R, Q) and (R, nQ).

## REFERENCES

- Andradóttir, S., & Kim, S.-H. (2010). Full sequential procedures for comparing constrained systems via simulation. *Naval Research Logistics* 57, 403–421.
- Barton, R. (2005). Response surface methodology. Amsterdam: Elsevier: In: Henderson SG, Nelson BL, editors. *Handbooks in operations research and management science: simulation*.
- Caglar, D., Li, C.-L., & Simchi-Levi, D. (2006). Two-echelon spare parts inventory system subject to a service constraint. *IIE Transactions* 36, 655-666.
- Caggiano, K.E., Jackson, P.L., & J.A., Rapp. (2007). Optimizing service parts inventory in a multi-echelon, multi-item supply chain with time-based customer service level agreements. *Operations Research* 55, 303-318.
- Cohen, M.A., Cull, C., & Willen, D. (2000). Saturn's supply chain innovation: high value in after sales service. *MIT Sloan Management Review* 41, 93-101.

- Fu MC. (2002). Optimization for simulation: theory vs. practice. *Informs Journal on Computing* 14(3), 192–215.
- Kim S-H., & Nelson BL. (2005). Selecting the best system. Amsterdam: Elsevier: In: HendersonSG, NelsonBL editors. *Handbooks in operations research and management science simulation*.
- Rustenburg, W.D., Van Houtum,G.J., & Zijm,W.H.M.,. (2001). Spare parts management at complex technology-based organizations: an agenda for research. . *International Journal of Production Economics* 71, 177-193
- Seyed Hamid Reza Pasandideh, Seyed Taghi Akhavan N, & Nafiseh Tokhmehchi. (2011). A parameter-tuned genetic algorithm to optimize two-echelon continuous review inventory systems. *Expert Systems with Applications* 38, 11708–11714.
- Sherbrooke, C.C. (1930). METRIC: a multi-echelon technique for recoverable item control. *Operations Research* 16, 122-141.
- Shing Chih Tsai., & Ya-Xin Zheng. (2013). A simulation optimization approach for a two- echelon inventory system with service level constraints. *Department of Industrial and Information Management* 229, 364-374.
- Sven Axsater. (2006). *INVENTORY CONTROL*. New York: Springer Science.
- O’ laffson S. (2005). *Metaheuristics*. Amsterdam: Elsevier: In: Henderson SG, Nelson BL, editors. *Handbooks in operations research and management science simulation*.