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Unsteady Transient Couette and Poiseuille Flow Under The effect Of Magneto-hydrodynamics and Temperature

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ABSTRACT

Current article deal with the study of non-Newtonian Couette and Poiseuille flow between two periodically and parallel oscillating vertical plates. The unsteady constitutive equations of differential type have been used. Uniform magnetic field is applied perpendicularly to the flow field between parallel plates. The effect of temperature is also involved in the flow field. Two different varieties of flow problems have been modelled in terms of non-linear partial differential equations with some physical conditions. Optimal Homotopy Asymptotic Method (OHAM) and Adomian Decomposition Method (ADM) have been used to obtain the analytical solution of the modelled partial differential equations. These methods are used usually due to its tremendous results for solving nonlinear differential equations arise in various applied and engineering sciences. In this work the excellent agreement of these two methods is analyzed numerically and graphically. Effect of different modeled parameters on velocity and temperature fields has been studied numerically and graphically.
KEYWORDS: Unsteady Second Grade Fluid, MHD, Temperature, Verticals plates, (OHAM) and (ADM).

INTRODUCTION

Non-Newtonian fluids have got great importance great in the field of research, especially in applied and bio mathematics, industry and engineering problems. Examples of such types of fluids are plastic trade, food processing, movement of biological fluids, wire and fibres coating, paper production, gaseous diffusion transpiration cooling, drilling mud, heat pipes etc. Several complex fluids such as polymer melts, paint, shampoo, mud, ketchup, blood, certain oils and greases, and many emulsions are involved in the class of non-Newtonian fluids. Due to industrial and technological usages non-Newtonian fluids have become its significant part. That's why researchers take a great interest in it. Islam et al. [1] studied

Couette and Poiseuille flow and there generalized form under the effect of heat analysis. For the solution of the problem they used OHAM. Hayat et al. [2] worked on the MHD steady flow of oldroyd-6 constant fluid. HAM method was used in this work for the nonlinear differential equation of three different types of flows. Attia [3] examined the MHD non Newtonian unsteady couette and poisuille flows. The effect of Hall term and physical parameters are discussed for velocity and temperature distributions. Aiyesimi et al. [4-5] calculated the solution of MHD Couette flow, Poiseuille flow problems of velocity and temperature profile by using regular perturbation method. Danish et al. [6] studied Poisuille and Coueete and poisuille flow of third grade fluid.. Rajagopal, et al.[7], examined non-Newtonian fluids between two parallel and vertical plates in the form of a Natural Convection Flow.Bhargava et al. [8], investigated Numerical solution of free convection MHD micro polar fluid flow between two parallel porous vertical plates. They have discussed the effect of various physical parameters. In recent Gul et al. [9-13] worked out on differential type fluids in variations of articles. They discussed the effect of various physical parameters on flow fields. Volume flux, skin friction, average velocity, and the temperature distribution across the film were shown in there studied. In most of their work they used two analytical techniques (OHAM and ADM) to obtained best results.Dileep and kumar [14] investigates the unsteady second grade fluid in a porous channel. They give the effect of physical parameters on the fluid motion during porous and clear region. Salah et al. [15], examined the flow of second grade fluid in a porous and rotating frame. Constant and accelerated fluid flows cases are studied in their works.Nemati et al.[16], studied the unsteady thin film flow of non-Newtonian fluid over a moving belt. The approximate solutions of velocity profile have been shown by using HAM. Iftikhar [17], examined the unsteady boundary layer flow of a second grade fluid affected by an impulsively stretching sheet. HAM method is used to get the analytical solution and the effects of the physical parameters are discussed through graphs. Chauhan and Kumar [18], examined the unsteady shear flow of a second grade fluid between two horizontal parallel plates.In their work Laplace transform method is applied to find the solution of the flow problem. Abbas et al.[19], discussed the unsteady thin liquid film of second grade fluid through stretching surface. HAM method was used for analytical solution. Kumari and Parsad [20], discussed the heat effect in Stokes second problem in unsteady cas under the effect of magnetic field. analytical result is shown for temperature field.Hameed and Ellahi [21], worked on thin film flow in a non-Newtonian fluid on a vertical moving belt.Fetecau

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[22], examined the longitudinal and torsional oscillations of second grade fluid circular cylinder. Tan et al.[23] studied Stoke first problem for a second grade fluid in a porous half space with heated boundary. They reported some good results. A variety of analytical techniques have been used by the researchers for the solution of differential equations. In the recent years the Adomian decomposition Method (ADM) and Optimal Homotopy Asymptotic Method (OHAM) are the two analytical techniques receiving more attention. The ADM was revised with some new results by Adomian [24]. Wazwaz [25] and Siqqiqui et al [26] used Adomian decomposition method in their work to get attractive results. Application of the optimal homotopy asymptotic method for solving nonlinear equations arising in heat transfer was investigated by Marinca et al. [27-29]. In another investigation Marinca et al. [30] have used optimal homotopy asymptotic method for the steady flow of a fourth-grade fluid past a porous plate.

Governing Equation

The MHD and heat equation governing the problem (momentum, mass and second order fluid equation) can be written as

$$\nabla.\mathbf{U} = 0\,, \tag{1}$$

$$\rho \frac{D\mathbf{U}}{Dt} = \boldsymbol{\nabla}.\mathbf{T} + \mathbf{J}\times\mathbf{B} + f_g\,. \tag{2}$$

$$\rho\, c_p \frac{D\Theta}{Dt} = k\,\nabla^2\Theta + tr(\boldsymbol{T.L}), \tag{3}$$

Here U represent velocity of the flow, ρ is flow density, $\frac{D}{Dt}$ is the material time derivative, and f_g is body force due to gravity. Thus,the Lorentz force perunit volume is

$$\mathbf{J}\times\mathbf{B} = \left[0, -\sigma B_0{}^2\mathrm{u}, 0\right], \tag{4}$$

Where $\mathbf{B} = (0, B_0, 0)$ is the uniform magnatic filed, B_0 is the applied magnetic field and σ is the electrical conductivity.

The current density $\mathbf{J}$ is

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{U}\times\mathbf{B}), \quad \nabla\times\mathbf{B} = \mu_0\mathbf{J}. \tag{5}$$

Here, μ_0 is the magnetic permeability, $\mathbf{E}$ is an electric field which we ignore in this work, and

The cauchy stress tensor, T is

$$\mathrm{T} = -p\mathbf{I} + \mathbf{S}, \tag{6}$$

Where $\mathbf{S}$is the extra tress tensor, $p\mathbf{I}$ is the isotropic stress.For second grade fluid

$$\boldsymbol{S} = \mu\, \boldsymbol{A_1} + \alpha_1 \boldsymbol{A_2} + \alpha_2 \boldsymbol{A}_1^2, \tag{7}$$

$$\mathbf{A}_0 = \mathbf{I}\ ,\ \mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \mathbf{L} = \mathrm{grad}\mathbf{U}\,, \tag{8}$$

$$\boldsymbol{A_1} = (\nabla \boldsymbol{u}) + (\nabla \boldsymbol{u})^T, \boldsymbol{A_1} = \frac{D\,\boldsymbol{A_{n-1}}}{Dt} + \boldsymbol{A_{n-1}}(\nabla \boldsymbol{u}) + (\nabla \boldsymbol{u})^T \boldsymbol{A_{n-1}}, n \geq 2 \tag{9}$$

$\mathbf{A}$ is the Rivlin Ericksen stress tensor and μ is the viscosity cofficient.

Formulation of the Couette type flow Problem

Consider two Vertical and parallel plates such that one of them is oscillating and moving with constant velocity U and the other plate kept oscillating only. The total thickness of the fluid between the plates assumed to be “2h”.Moving and oscillating plate caries with itself a liquid of width “h”.The configuration of fluid flow is along the Y-axis and perpendicular to x-axis. A transverse magmatic field applied to the belt. Gravitational force and magnetics force causes the fluid motion. We assume that the flow is un-steady, laminar and incompressible.

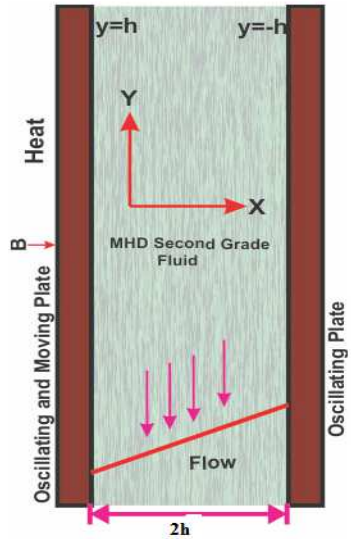


Figure: 1. Geometry of the Couette flow problem

The velocity field and boundary conditions for the problem is given as

$$\mathbf{U} = \left(0, u\left(x,t\right), 0\right), \text{ And } \Theta = \Theta(x, t) \tag{10}$$

$$\boldsymbol{U}(h,t) = U + UCos\omega t, \quad \boldsymbol{U}(-h,t) = UCos\omega t, \tag{11}$$

$$\Theta(h,t) = \Theta_0, \Theta(-h,t) = \Theta_1, \tag{12}$$

ω is for the Expressions the frequency of the oscillating plates .
Consuming (10) in (2) and (3) reduced to the form

$$\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial t} T_{xy} - \rho g - \sigma B_0^2 \text{ u}, \tag{13}$$

$$\rho c_p \left(\frac{\partial \Theta}{\partial t}\right) = k \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial}{\partial t} T_{xy}, \tag{14}$$

From (7) Cauchy stress tensor components **S** is calculated as

$$T_{xx} = -P + (2\alpha_1 + \alpha_2)\left(\frac{\partial \boldsymbol{u}}{\partial x}\right)^2, \tag{15}$$

$$T_{xy} = \mu \frac{\partial \boldsymbol{u}}{\partial x} + \alpha_1 \frac{\partial}{\partial t}\left(\frac{\partial \boldsymbol{u}}{\partial x}\right), \tag{16}$$

$$T_{yy} = -P + \alpha_2 \left(\frac{\partial \boldsymbol{u}}{\partial x}\right)^2, \tag{17}$$

Inserting of equation (14) in equation (12) and (13) give us

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + \alpha_1 \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) - \rho g - \sigma B_0^2 \text{ u}, \tag{18}$$

$$\rho c_p \left(\frac{\partial \Theta}{\partial t}\right) = k \frac{\partial^2 \Theta}{\partial x^2} + \mu \left(\frac{\partial u}{\partial x}\right)^2 + \alpha_1 \left(\frac{\partial u}{\partial x}\right) \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right), \tag{19}$$

Using non-dimensional parameters

$$\breve{u} = \frac{\boldsymbol{u}}{U}, \breve{x} = \frac{x}{h}, \breve{t} = \frac{\mu t}{\rho \delta^2}, \breve{\Theta} = \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0}, B_r = \frac{\mu U^2}{k(\Theta_1 - \Theta_0)}, M = \frac{\sigma B_0^2 \delta^2}{\mu_0},$$
$$P_r = \frac{\mu c_p}{k}, \breve{\omega} = \frac{\omega \delta^2 \rho}{\mu}, S_t = \frac{\delta^2 \rho g}{\mu U}, \alpha = \frac{\alpha_1}{\rho \delta^2}, \Omega = \frac{h^2}{\mu U}\frac{\partial p}{\partial x} \tag{20}$$

Where α is the non-dimensional variable, B_r is the Brinkman number, S_t is Stoke number and P_r is the Prandtl number, ω is the oscillating parameter, M is the magnetic parameter. Using the above dimensionless parameter in equation (18,19) and reducing bars we obtain

$$\frac{\partial u}{\partial t} = +\frac{\partial^2 u}{\partial x^2} + \alpha \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) - S_t - Mu, \tag{21}$$

$$P_r\left(\frac{\partial \Theta}{\partial t}\right) = \frac{\partial^2 \Theta}{\partial x^2} + B_r\left[\left(\frac{\partial u}{\partial x}\right)^2 + \alpha\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 u}{\partial t \partial x}\right)\right], \tag{22}$$

And the boundary conditions are

$$u(1,t) = 1 + Cos\omega t, \quad u(-1,t) = Cos\omega t, \tag{23}$$

$$\Theta(0,t) = 0, \Theta(1,t) = 1, \tag{24}$$

ADM Solution for the Couette flow problem

To obtained the analytical solution first we apply the ADM method using the above boundary conditions. The zero, first and second order problems and there is solution is given below in order

$$P^0 : \frac{\partial^2 u_0(\mathrm{x},\mathrm{t})}{\partial x^2} = S_t, \tag{25}$$

$$\frac{\partial^2 \Theta_0(\mathrm{x},\mathrm{t})}{\partial x^2} = 0, \tag{26}$$

$$P^1 : \frac{\partial^2 u_1(\mathrm{x},\mathrm{t})}{\partial x^2} = \frac{\partial u_0}{\partial x} - \alpha[A_0] + Mu_0, \tag{27}$$

$$\frac{\partial^2 \Theta_1(\mathrm{x},\mathrm{t})}{\partial x^2} = \frac{\partial^2 \Theta_0}{\partial x^2} - B_r\left([B_0] + \alpha[C_0]\right), \tag{28}$$

$$P^2 : \frac{\partial^2 u_2(\mathrm{x},\mathrm{t})}{\partial x^2} = \frac{\partial u_1}{\partial x} - \alpha[A_1] + Mu_1, \tag{29}$$

$$\frac{\partial^2 \Theta_2(\mathrm{x},\mathrm{t})}{\partial x^2} = P_r \frac{\partial^2 \Theta_1}{\partial x^2} - B_r\left([B_1] + \alpha[C_1]\right), \tag{30}$$

The Adomian polynomials are defined as,

$$A_0 = \frac{\partial}{\partial t}\left(\frac{\partial^2 u_0}{\partial x^2}\right), \; A_1 = \frac{\partial}{\partial t}\left(\frac{\partial^2 u_1}{\partial x^2}\right),$$

$$B_0 = \left(\frac{\partial u_0}{\partial x}\right)^2, \; C_0 = \left(\frac{\partial u_0}{\partial x}\right)\left(\frac{\partial^2 u_0}{\partial t \partial x}\right), \; B_1 = 2\frac{\partial u_0}{\partial x}\frac{\partial u_1}{\partial x}, \tag{31}$$

$$C_1 = \frac{\partial u_1}{\partial x}\frac{\partial}{\partial t}\frac{\partial u_0}{\partial x} + \frac{\partial u_0}{\partial x}\frac{\partial}{\partial t}\frac{\partial u_1}{\partial x}, \tag{32}$$

The solution of the above components is,

$$P^0 : u_0(\mathrm{x},t) = \frac{1}{2}(1+x) + \cos[t\omega] + \frac{1}{2}\left(x^2 - 1\right). \tag{33}$$

$$\theta_0(\mathrm{x},t) = \frac{1}{2}(1+\mathrm{x}). \tag{34}$$

$$P^1 : u_1(\mathrm{x},t) = \frac{1}{24}\begin{pmatrix} M(-6-2x+6x^2+2x^3) + 12M\mathrm{Cos}[t\omega](1-x^2) - 12\omega\mathrm{Sin}[t\omega]c_1(1+x^2) \\ +12\omega\mathrm{Sin}[t\omega](1-x^2) + S_t\mathrm{M}\,(5\text{-}6x^2+x^4) \end{pmatrix} \tag{35}$$

$$\theta_1(\mathrm{x},t) = \frac{1}{24}(3\,\mathrm{B}_r(1-\mathrm{x}^2) + 4S_t\,\mathrm{x}\mathrm{B}_r(1-\mathrm{x}^2) + 2S^2{}_t\,\mathrm{B}_r(1-\mathrm{x}^4). \tag{36}$$

$$u_2(\mathrm{x},t)=\frac{1}{720}M^2\{(75+7x-90x^2-10x^3+15x^4+3x^5)+M^2\cos[t\omega](150-180x^2+30x^4)$$
$$+M^2\omega^2\cos[t\omega](150-180x^2+30x^4)+360\alpha\omega^2\cos[t\omega](x^2-1)+M\omega\sin[t\omega](300+360x^2+30x^4)$$
$$+360\alpha\omega^2\sin[t\omega](x^2-1)+S_t\mathrm{M}^2\ (61+75x^2-15x^4+x^6)\}. \tag{37}$$

$$\theta_2(\mathrm{x},t)=\frac{1}{720}\{15\,\mathrm{MB}_r(-1+4x+2\,\mathrm{x}^2-4x^3-x^4)+120\,\mathrm{B}_r\,Mx\mathrm{Cos}[t\omega](1-x^2)+60\,\mathrm{B}_r\,x\alpha\omega^2$$
$$\cos[t\omega](x^2-1)120\,\mathrm{B}_r\,x\omega\sin[t\omega](x^2-1)+60M\,\mathrm{B}_r\,x\alpha\omega\sin[t\omega](x^2-1)+M\,\mathrm{B}_r\,S_t(60-56x$$
$$+80\,\mathrm{x}^3-60x^4-24x^5)+120S_t\,\mathrm{B}_r\,M\mathrm{Cos}[t\omega](1-x^4)+60S_t\,\mathrm{B}_r\,\alpha\omega^2\cos[t\omega](x^4-1)+120S_t\,\mathrm{B}_r$$
$$\omega\sin[t\omega](x^4-1)+60M\,\mathrm{B}_r\,S_t\alpha\omega\sin[t\omega](x^4-1)+M\,\mathrm{B}_r\,S^2{}_t(-52+60x^4-8x^6). \tag{38}$$

The series solutions of velocity profile is obtained as

$$u(\mathrm{x},t)=u_0(\mathrm{x},t)+u_1(\mathrm{x},t)+u_2(\mathrm{x},t) \tag{39}$$

$$u(\mathrm{x},t)=\frac{1}{2}(1+x)+\cos[t\omega]+\frac{1}{2}(x^2-1)+\frac{1}{24}\{M(-6-2x+6x^2+2x^3)+12M\mathrm{Cos}[t\omega]$$
$$(1-x^2)-12\omega\mathrm{Sin}[t\omega]c_1(1+x^2)+12\omega\mathrm{Sin}[t\omega](1-x^2)+S_t\mathrm{M}\ (5\text{-}6x^2+x^4)\}+\frac{1}{720}M^2$$
$$\{(75+7x-90x^2-10x^3+15x^4+3x^5)+M^2\cos[t\omega](150-180x^2+30x^4)+M^2\omega^2\cos \tag{40}$$
$$[t\omega](150-180x^2+30x^4)+360\alpha\omega^2\cos[t\omega](x^2-1)+M\omega\sin[t\omega](300+360x^2+30x^4)$$
$$+360\alpha\omega^2\sin[t\omega](x^2-1)+S_t\mathrm{M}^2\ (61+75x^2-15x^4+x^6)\}.$$

The series solutions of temperature profile is obtained as

$$\theta(\mathrm{x},t)=\theta_0(\mathrm{x},t)+\theta_1(\mathrm{x},t)+\theta_2(\mathrm{x},t) \tag{41}$$

$$\theta(\mathrm{x},t)=\frac{1}{2}(1+\mathrm{x})+\frac{1}{24}\{3\,\mathrm{B}_r(1-\mathrm{x}^2)+4S_t\,\mathrm{xB}_r(1-\mathrm{x}^2)+2S^2{}_t\,\mathrm{B}_r(1-\mathrm{x}^4)\}+\frac{1}{720}\{15\,\mathrm{MB}_r(-1+4x$$
$$+2\,\mathrm{x}^2-4x^3-x^4)+120\,\mathrm{B}_r\,Mx\mathrm{Cos}[t\omega](1-x^2)+60\,\mathrm{B}_r\,x\alpha\omega^2\cos[t\omega](x^2-1)120\,\mathrm{B}_r\,x\omega\sin[t\omega]$$
$$(x^2-1)+60M\,\mathrm{B}_r\,x\alpha\omega\sin[t\omega](x^2-1)+M\,\mathrm{B}_r\,S_t(60-56x+80\,\mathrm{x}^3-60x^4-24x^5)+120S_t\,\mathrm{B}_r\,M$$
$$\mathrm{Cos}[t\omega](1-x^4)+60S_t\,\mathrm{B}_r\,\alpha\omega^2\cos[t\omega](x^4-1)+120S_t\,\mathrm{B}_r\,\omega\sin[t\omega](x^4-1)+60M\,\mathrm{B}_r\,S_t\alpha\omega\sin$$
$$[t\omega](x^4-1)+M\,\mathrm{B}_r\,S^2{}_t(-52+60x^4-8x^6).. \tag{42}$$

OHAM Solution Couette flow problem

Now here we apply OHAM method to get the required solution. Zero and first component problem for velocity and temperature profiles are

$$P^0:\frac{\partial^2 u_0(\mathrm{x},\mathrm{t})}{\partial x^2}=S_t, \tag{43}$$

$$\frac{\partial^2\Theta_0(\mathrm{x},\mathrm{t})}{\partial x^2}=0, \tag{44}$$

$$P^1:\frac{\partial^2 u_1(\mathrm{x},\mathrm{t})}{\partial x^2}=S_t(1+c_1)+c_1\frac{\partial^2 u_0}{\partial^2 x}+\frac{\partial^2 u_0}{\partial^2 x}-c_1\frac{\partial u_0}{\partial t}-\alpha c_1\frac{\partial^2}{\partial^2 t}\left(\frac{\partial u_0}{\partial t}\right)-Mc_1u_0, \tag{45}$$

$$\frac{\partial^2\Theta_1(\mathrm{x},\mathrm{t})}{\partial x^2} = -P_r c_3 \frac{\partial\Theta_0}{\partial t} + \frac{\partial^2\Theta_0}{\partial x^2} + c_3 \frac{\partial^2\Theta_0}{\partial x^2} - B_r c_3 \left(\frac{\partial u_0}{\partial x}\right)^2 + \alpha B_r c_3 \left(\frac{\partial u_0}{\partial x}\right)\left(\frac{\partial^2 u_0}{\partial t \partial x}\right), \tag{46}$$

$$P^1 : \frac{\partial^2 u_2(\mathrm{x},\mathrm{t})}{\partial x^2} = S_t c_2 + c_2 \frac{\partial^2 u_0}{\partial^2 x} + \frac{\partial^2 u_1}{\partial^2 x} + c_1 \frac{\partial^2 u_1}{\partial^2 x} - c_1 \frac{\partial u_1}{\partial t} - c_1 \frac{\partial u_1}{\partial t} + \alpha c_2 \frac{\partial^2}{\partial^2 t}\left(\frac{\partial u_0}{\partial t}\right)$$
$$+\alpha c_1 \frac{\partial^2}{\partial^2 t}\left(\frac{\partial u_1}{\partial t}\right) - Mc_2 u_0 - Mc_1 u_1, \tag{47}$$

$$\frac{\partial^2\Theta_2(\mathrm{x},\mathrm{t})}{\partial x^2} = -P_r c_4 \frac{\partial\Theta_0}{\partial t} - P_r c_3 \frac{\partial\Theta_1}{\partial t} + \frac{\partial^2\Theta_1}{\partial x^2} + c_4 \frac{\partial^2\Theta_0}{\partial x^2} + c_3 \frac{\partial^2\Theta_1}{\partial x^2} + B_r c_4 \left(\frac{\partial u_0}{\partial x}\right)^2$$
$$+\alpha B_r c_4 \frac{\partial u_0}{\partial x}\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial t}\right) + 2B_r c_3 \frac{\partial u_0}{\partial x}\left(\frac{\partial u_1}{\partial x}\right) + \alpha B_r c_3 \frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial t}\right)\frac{\partial u_1}{\partial x} + \alpha B_r c_3 \frac{\partial u_0}{\partial x}\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial t}\right). \tag{48}$$

Solutions of zero, first and second components problem using boundary conditions from equation (24,25)in equations (43-48) are given as

$$P^0 : u_0(\mathrm{x},t) = \frac{1}{2}(1+x) + \cos[t\omega] + \frac{1}{2}\left(x^2 - 1\right) S_t \tag{49}$$

$$\theta_0(\mathrm{x},t) = \frac{1}{2}(1+\mathrm{x}). \tag{50}$$

$$P^1 : u_1(\mathrm{x},t) = \frac{1}{24}\begin{pmatrix} Mc_1(-6-2x+6x^2+2x^3) + 12Mc_1 \mathrm{Cos}[t\omega](1-x^2) \\ +12c_1\omega \mathrm{Sin}[t\omega](1-x^2) + 12S_t(1-x^2) + \\ 12c_1 S_t(1-x^2) + \mathrm{M}S_t c_1\ (-5+6x^2-x^4) \end{pmatrix}, \tag{51}$$

$$\theta_1(\mathrm{x},t) = \frac{1}{24}\mathrm{B}_r \mathrm{c}_3\{3(-\mathrm{x}^2) + 4S_t\ \mathrm{x}(1-\mathrm{x}^2) + 2S^2{}_t(1-\mathrm{x}^4)\}. \tag{52}$$

u_2 is and $\Theta_2$extremely larg so we ignore to write.we can express it only graphicaly

Formulation of the Poiseuille type flow Problem

In this Problem, we consider the second grade fluid in between two vertical plates y=h and y=-h. We assumed that both plates are oscillating and the flow of fluid is due to the constant pressure gradient.

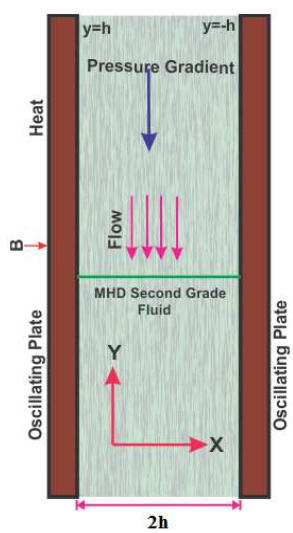


Figure: 2. Geometry of the Poiseuille type flow Problem**.**

When pressure gradient is involved then the momentum and heat equation is

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial x^2}+\alpha_1\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right)-\rho g-\sigma B_0^2\,\mathrm{u}, \tag{53}$$

$$\rho c_p\left(\frac{\partial\Theta}{\partial t}\right)=k\frac{\partial^2\Theta}{\partial x^2}+\mu\left(\frac{\partial u}{\partial x}\right)^2+\alpha_1\left(\frac{\partial u}{\partial x}\right)\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right), \tag{54}$$

Using the dimensionless parameter (19) in (20) we obtained

$$\frac{\partial u}{\partial t}=-\Omega+\frac{\partial^2 u}{\partial x^2}+\alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right)-S_t-Mu, \tag{55}$$

$$P_r\left(\frac{\partial\Theta}{\partial t}\right)=\frac{\partial^2\Theta}{\partial x^2}+B_r\left[\left(\frac{\partial u}{\partial x}\right)^2+\alpha\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 u}{\partial t\partial x}\right)\right], \tag{56}$$

And the boundary conditions are

$$u(1,t)=Cos\omega t,\quad u(-1,t)=Cos\omega t, \tag{57}$$

$$\Theta(0,t)=0,\Theta(1,t)=1, \tag{58}$$

ADM Solution for the Poiseuille type flow Problem

The adomian polynomials of both problems are same

$$P^0: u_0(\mathrm{x},t)=\cos[t\omega]+\frac{1}{2}\left(x^2-1\right)(\mathrm{S}_t+\Omega). \tag{59}$$

$$\theta_0(\mathrm{x},t)=\frac{1}{2}(1+\mathrm{x}). \tag{60}$$

$$P^1: u_1(\mathrm{x},t)=\frac{1}{24}\begin{pmatrix} M(\mathrm{S}_t+\Omega)(5-6x^2+x^4)+12M\mathrm{Cos}[t\omega](1-x^2) \\ +12\omega\mathrm{Sin}[t\omega](1-x^2)+S_t\mathrm{M}\,(5\text{-}6x^2+x^4)\end{pmatrix} \tag{61}$$

$$\theta_1(\mathrm{x},t)=\frac{1}{12}(\mathrm{S}_t+\Omega)^2\,\mathrm{B}_r(1-\mathrm{x}^4). \tag{62}$$

$$P^2: u_2(\mathrm{x},t)=\frac{1}{720}\begin{pmatrix} M^2(\mathrm{S}_t+\Omega)(61+75x^2-15x^4+x^6)+M^2\cos[t\omega](150-180x^2+30x^4) \\ +\omega^2\cos[t\omega](-150-180x^2-30x^4)+360\alpha\omega^2\cos[t\omega](x^2-1)+M\omega\sin[t\omega] \\ (-300+360x^2-60x^4)+360\alpha\omega\sin[t\omega](x^2-1).\end{pmatrix} \tag{63}$$

$$\theta_2(\mathrm{x},t)=\frac{1}{180}\begin{bmatrix} 30\cos[t\omega](\mathrm{S}_t+\Omega)\mathrm{B}_r(1-\mathrm{x}^4)+15\alpha\omega^2\cos[t\omega](\mathrm{S}_t+\Omega)\mathrm{B}_r(\mathrm{x}^4-1)+ \\ 30\omega\sin[t\omega](\mathrm{S}_t+\Omega)\mathrm{B}_r(\mathrm{x}^4-1)+15M\alpha\omega\sin[t\omega](\mathrm{S}_t+\Omega)\mathrm{B}_r(\mathrm{x}^4-1)+ \\ M(\mathrm{S}_t+\Omega)^2\,\mathrm{B}_r(-13+15\,\mathrm{x}^4-1-2x^6)\end{bmatrix}. \tag{64}$$

The series solutions of velocity profile is obtained as

$$u(\mathrm{x},t)=u_0(\mathrm{x},t)+u_1(\mathrm{x},t)+u_2(\mathrm{x},t) \tag{65}$$

The series solutions of temperature profile is obtained as

$$\theta(\mathrm{x},t)=\theta_0(\mathrm{x},t)+\theta_1(\mathrm{x},t)+\theta_2(\mathrm{x},t) \tag{66}$$

$$\theta(\mathrm{x},t)=\frac{1}{2}(1+\mathrm{x})+\frac{1}{12}(\mathrm{S}_t+\Omega)^2\,\mathrm{B}_r(1-\mathrm{x}^4)+\frac{1}{180}\{30\cos[t\omega](\mathrm{S}_t+\Omega)\mathrm{B}_r(1-\mathrm{x}^4)+15\alpha\omega^2\cos[t\omega]$$
$$(\mathrm{S}_t+\Omega)B_r(\mathrm{x}^4-1)+30\omega\sin[t\omega]\mathrm{S}_t+\Omega)\mathrm{B}_r(\mathrm{x}^4-1)+15M\alpha\omega\sin[t\omega](\mathrm{S}_t+\Omega)\mathrm{B}_r(\mathrm{x}^4-1)$$
$$+M(\mathrm{S}_t+\Omega)^2\,\mathrm{B}_r(-13+15\,\mathrm{x}^4-1-2x^6).. \tag{67}$$

The OHAM Solution of Poiseuille type flow Problem.

Here we apply OHAM method to get the required solution. Solutions of zero, first and second components problem using same boundary

$$P^0 : u_0(\mathrm{x},t) = \frac{1}{2}(1+x) + \cos[t\omega] + \frac{1}{2}(x^2-1)S_t$$

(68)

$$\theta_0(\mathrm{x},t) = \frac{1}{2}(1+\mathrm{x}). \tag{69}$$

$$P^1 : u_1(\mathrm{x},t) = \frac{1}{24}\begin{pmatrix} Mc_1(-6-2x+6x^2+2x^3)+12Mc_1\mathrm{Cos}[t\omega](1-x^2)+ \\ 12c_1\omega\mathrm{Sin}[t\omega](1-x^2)+12S_t(1-x^2)+ \\ 12c_1S_t(1-x^2)+\mathrm{M}S_tc_1\ (-5+6x^2-x^4) \end{pmatrix}, \tag{70}$$

$$\theta_0(\mathrm{x},t) = \frac{1}{2}(1+\mathrm{x}). \tag{71}$$

$$\theta_1(\mathrm{x},t) = \frac{1}{24}\mathrm{B}_r\mathrm{c}_3\{3(-\mathrm{x}^2)+4S_t\,\mathrm{x}(1-\mathrm{x}^2)+2S^2{}_t(1-\mathrm{x}^4)\}. \tag{72}$$

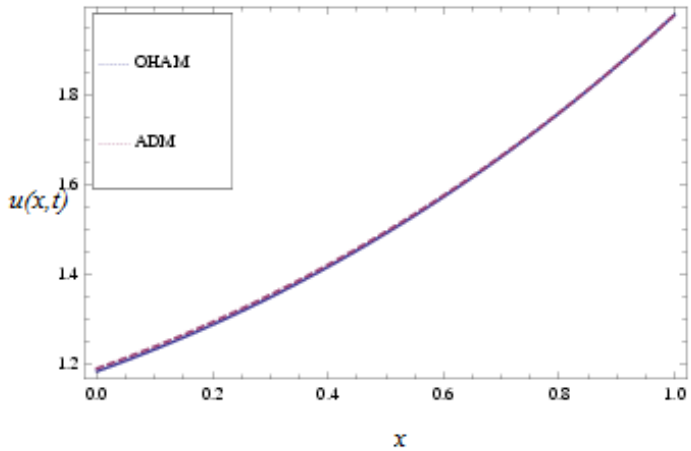
u_2 is and $\Theta_2$extremely larg so we ignore to write.we can express it only graphicaly.The series solutions of velocity profile is obtained as

"Table 1" Comparison of OHAM and ADM for the velocity profile in case of Couette flow, when $\omega = 0.2, \alpha = 0.02, M = 0.5, t = 1, S_t = 0.5, B_r = 0.4, c_1 = -1.0016691, c_2 = -0.000189541.$

x	OHAM	ADM	Absolute Error
0.0	1.18356	1.18997	-0.0064089
0.1	1.23259	1.23886	-0.00627247
0.2	1.28762	1.29356	-0.00594448
0.3	1.34891	1.35435	-0.00543829
0.4	1.41674	1.42151	-0.00543829
0.5	1.49142	1.49541	-0.00398994
0.6	1.57328	1.5764	-0.00312076
0.7	1.66268	1.6649	-0.00222002
0.8	1.76	1.76134	-0.00134948
0.9	1.86565	1.86623	-0.000581616
1.0	1.98007	1.98007	1.97373×10^{-17}

"Table 2" Comparison of OHAM and ADM for the velocity profile, when $\omega = 0.2, \alpha = 0.02, M = 0.5, t = 0.5,, B_r = 0.4, c_1 = -1.001669146322, c_2 = -0.0001895412688948.$

x	OHAM	ADM	Absolute Error
0.0	0.6337	0.628305	0.00539562
0.1	0.637212	0.631831	0.00538123
0.2	0.647771	0.642425	0.00534641
0.3	0.665414	0.660139	0.00527499
0.4	0.690196	0.685057	0.00513964
0.5	0.7222	0.717299	0.00490173
0.6	0.76153	0.757019	0.00451114
0.7	0.862698	0.804406	0.00390596
0.8	0.76153	0.859686	0.00301219
0.9	0.924862	0.923119	0.00174333
1.0	0.995004	0.995004	2.31296×10^{-17}



"Figure 3" Graphical Comparison of OHAM and ADM for the velocity profile in Couette flow problem, when $\omega = 0.2, \alpha = 0.02, M = 0.5, t = 1, S_t = 0.5, B_r = 0.4, c_1 = -1.0016691, c_2 = -0.000189541,$

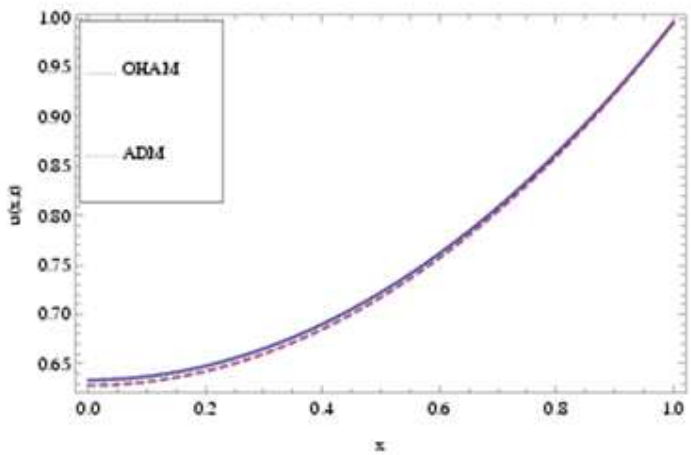


Figure 4: Comparison graph of OHAM and ADM for the velocity profile of Poiseuille type flow Problem, when

$\omega = 0.2, \alpha = 0.02, M = 0.5, t = 0.5,, B_r = 0.4, c_1 = -1.001669146322, c_2 = -0.0001895412688948.$

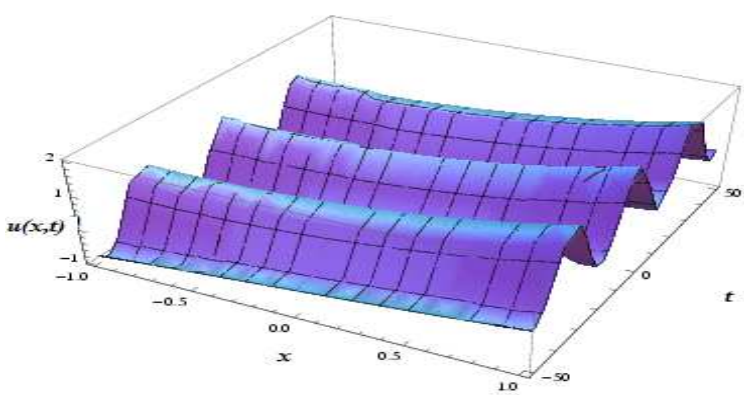


Figure 5: 3D graphs for the fluid flow during different time level in Couette flow problem. When $\omega = 0.2, \alpha = 0.02, M = 0.5, S, \Omega = 0.3,$

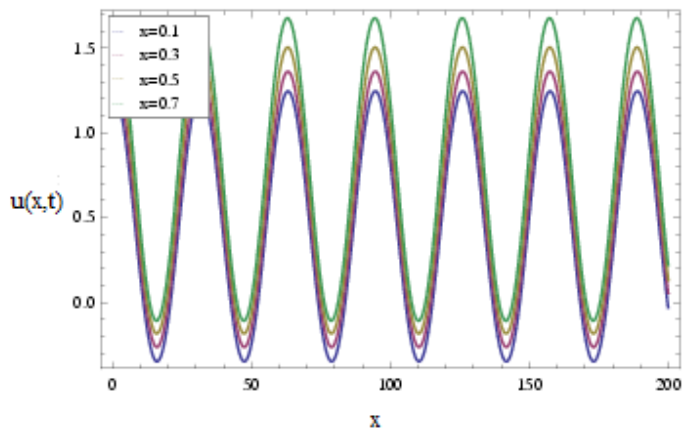


Figure 6: Velocity distribution graphs durind oscillation at different time level in Couette flow problem. When $\omega = 0.2, \alpha = 0.02, M = 0.5, \mathrm{S}, \Omega = 0.3,$

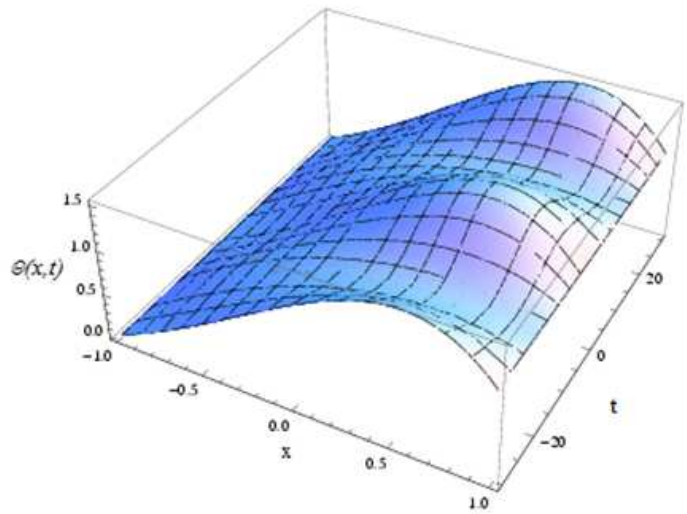


Figure 7: 3D graph for temperature distribution in Couette flow problem. When

$$\omega = 0.2, \alpha = 0.02, M = 0.5, \mathrm{S}_t = 0.3, \mathrm{B}_r = 4, P_r = 0.6;$$

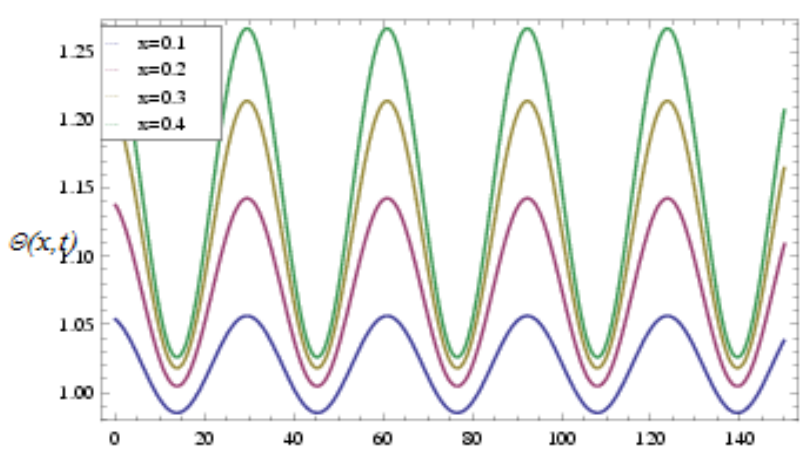


Figure 8: temperature distribution at different time level in Couette flow problem. When

$$\omega = 0.2, \alpha = 0.02, M = 0.5, \mathrm{S}_t = 0.3, \mathrm{B}_r = 4, P_r = 0.6;$$

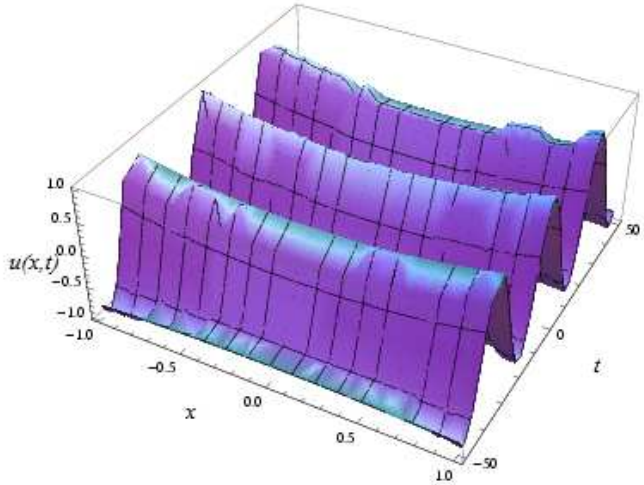


Figure 9: The fluid flow during different time level of Poiseuille type flow Problem. When $\omega = 0.2, \alpha = 0.02, M = 0.5, \mathrm{S}, \Omega = 0.3,$

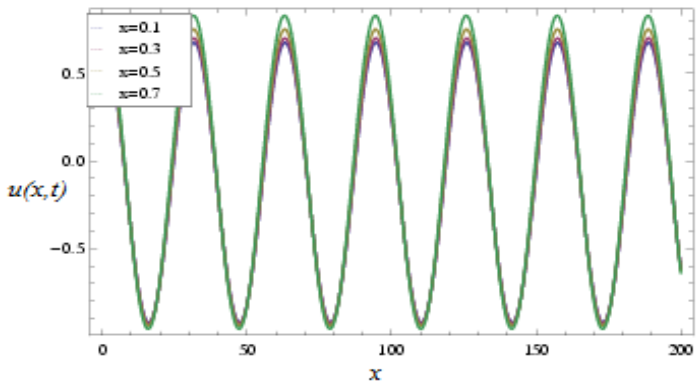


Figure 10: Effect of velocity profile of Poiseuille type flow Problem. When $\omega = 0.2, \alpha = 0.02, M = 0.5, \mathrm{S}, \Omega = 0.3,$

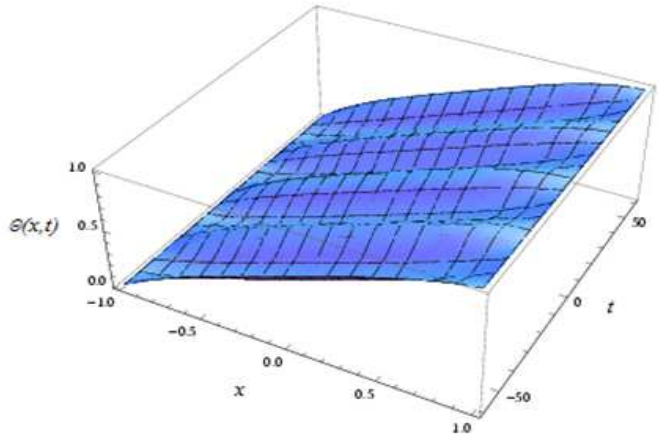


Figure 11: 3D Effect of temperature in Poiseuille type flow Problem. When $\omega = 0.2, \alpha = 0.02, M = 0.5, \mathrm{S}_t = 0.3, \mathrm{B}_r = 4, P_r = 0.6;$

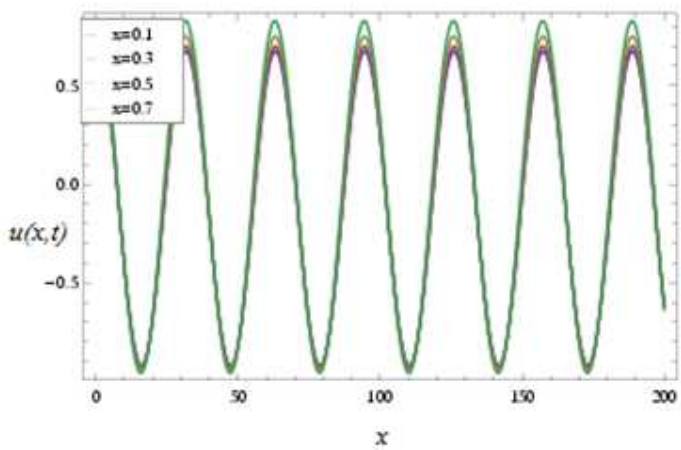


Figure 12: Effect of temperature distribution at different time level inPoiseuille type flow Problem. When

$$\omega = 0.2, \alpha = 0.02, M = 0.5, S_t = 0.3, B_r = 4, P_r = 0.6;$$

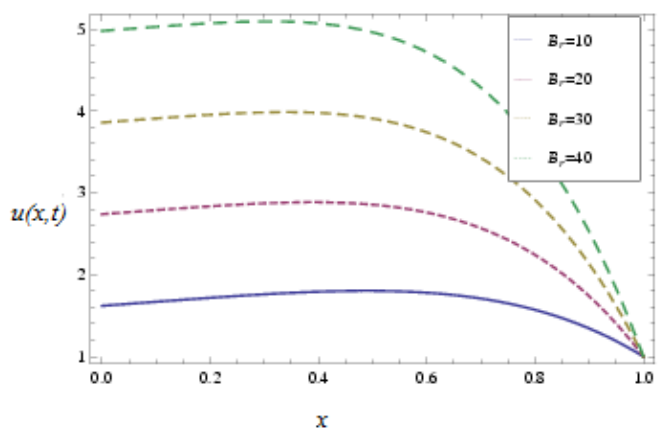


Figure 13: Physical illustration of Magnetic parameter "M" for Poiseuille flow when

$$\omega = 0.2, \alpha = 0.02, M = 0.5, S_t = 0.3, P_r = 0.6;$$

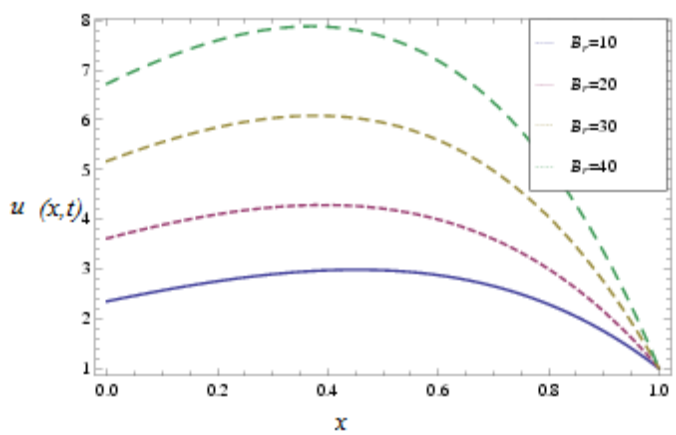


Figure 14: Physical illustration of Magnetic parameter "M" for Coueete flow when

$$\omega = 0.2, \alpha = 0.02, M = 0.5, S_t = 0.3, P_r = 0.6;$$

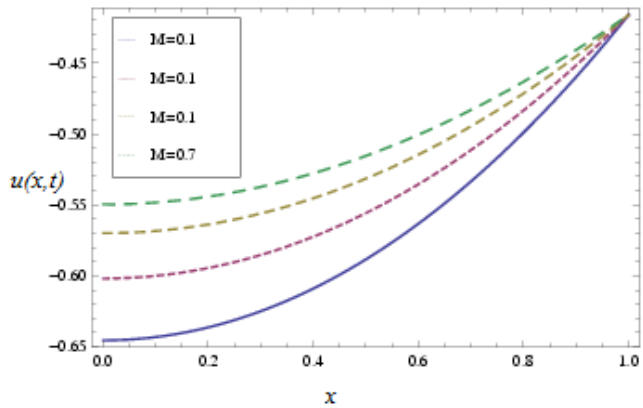


Figure 15: Physical illustration of Magnetic parameter "M" for poiseuille flow when

$$\omega = 0.2, \alpha = 0.02, S_t = 0.3, t = 10, \Omega = 0.4$$

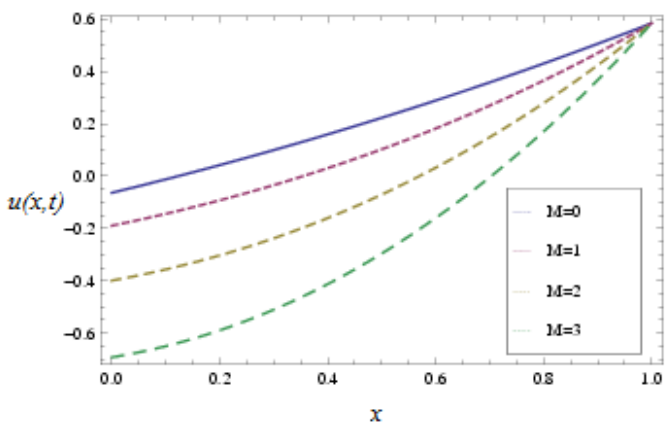


Figure 16: Physical illustration of Magnetic parameter "M" for **Couette** flow when

$$\omega = 0.2, \alpha = 0.02, S_t = 0.3, t = = 5.$$

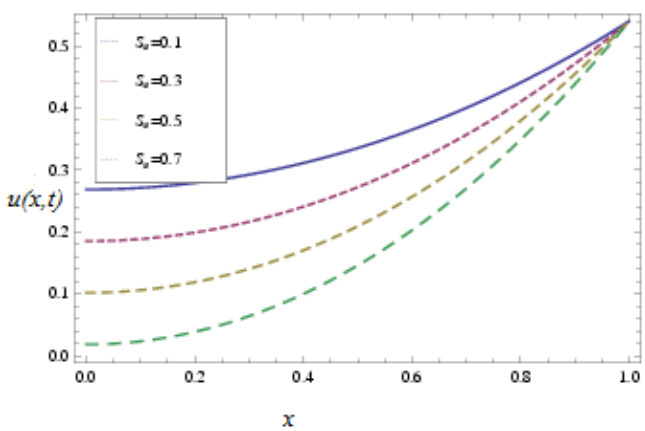


Figure 17: Physical illustration of Stoke number "S_t" for poiseuille flow problems flow

Where $\omega = 0.2, \alpha = 0.02, M = 0.5, \Omega = 0.4; t = 5$

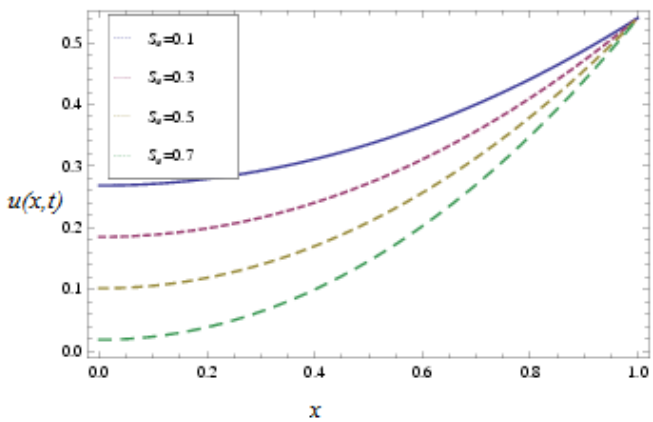


Figure 18: Physical illustration of Stoke number "S_t" for Couette flow when $\omega = 0.2, \alpha = 0.02, M = 0.5, \Omega = 0.4; t = 5$

RESULTS AND DISCUSSION

Figures 1 and 2 show the geometry of Couette and poiseuille flow problems. Tables 1, 2 and Figures 3,4 are ploted for the comparson of ADM and OHAM methods. Figures 5-12 show the effect of velocity and temperature fields at different time level for both Couette and poiseuille flow problems respectually. The influence of different dimensionless physical parameters (Stock number S_t ,Brinkman number B_r ,and some other parematers are described in Figures.13–18. The effect of Brinkman number B_r for both problems have been shown in Figures 13,14. Increasing Brinkman number increases fluid motion. The reason is that the cohesive forces reduces wich increase the fluid motion. The effect of stock number and magnetic field for both broblems have been shown in Figures 15-18. Increasing these parameters reduces fluid motion. Because the resistance force increases wich decrease the fluid motion.

Conclusion

In this article, the modelled partial differential equations have been solved analytically by using ADM and OHAM methods. The comparison of these methods analysed numerically and graphically. We have concluded excellent agreement of these two methods.

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