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A Note On Killing Symmetries of Lemaitre-Tolman-Bondi Metric

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ABSTRACT

The purpose of this paper is to obtain Killing vectors on the manifold of well known Lemaitre-Tolman-Bondi metric in general relativity theory. For the purpose some algebraic and direct integration techniques are used. Different possibilities of the metric functions are considered and Killing vectors are obtained for each case. It turns out that the spacetime under consideration admits only three or four Killing vectors.

KEYWORDS: Lie derivative, direct integration technique, Killing vector fields

1. INTRODUCTION

An elegant theory of gravitation known as general relativity theory, describes gravity as a property of the geometry of the spacetime. In this theory curvature of the spacetime is sewn directly to the matter present in the spacetime through Einstein's field equations. The non linearbehaviour of these field equations restricts us to find exact solutions which clearly describe physical situation. In order to find exact solutions of Einstein's field equations and further classification of those exact solutions we require certain symmetry restrictions. The most interesting symmetry restrictions are Killing, homothetic, conformal and self similar vector fields. These symmetry restrictions provide us vital information about the physical shape of the matter content and geometrical features of the spacetime structure. Our universe allows the matter content to exhibit some conservation laws under certain conditions. These conservation laws can also be studied through different symmetries [1]. Over the past few years researchers have discussed some important symmetry of the spacetimes like Killing, homothetic, conformal and self-similar vector fields [2-8]. The purpose of this paper is to classify Lemaitre-Tolman-Bondi metric according to its Killing vector fields. It is to mention that a vector field X is called a Killing vector field if it satisfies the Killing equation given as

$$\underset{X}{L}\, g_{ab} = g_{ab,c}X^{c} + g_{cb}X^{c}_{,a} + g_{ac}X^{c}_{,b} = 0, \qquad (1)$$

where L is the Lie derivative operator along the vector field X.

2. Main Results

The line element for Lemaitre-Tolman-Bondi metric in the usual coordinate system is given by

$$ds^2 = -dt^2 + V^2(t,r)dr^2 + W^2(t,r)\left(d\theta^2 + \sin^2\theta d\phi^2\right) \qquad (2)$$

where V and W are no-where zero functions of t and r only. Now using (2) in (1) we get the Killing equations as follows:

$$X^{0}_{,0} = 0 \qquad (3)$$

$$V^2 X^{1}_{,0} - X^{0}_{,1} = 0 \qquad (4)$$

$$W^2 X^{2}_{,0} - X^{0}_{,2} = 0 \qquad (5)$$

$$W^2 \sin^2\theta\, X^{3}_{,0} - X^{0}_{,3} = 0 \qquad (6)$$

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$$V^{\bullet} X^0 + V X^1_{,1} + V' X^1 = 0 \qquad (7)$$

$$W^2 X^2_{,0} + V^2 X^1_{,2} = 0 \qquad (8)$$

$$W^2 \sin^2\theta X^3_{,1} + V^2 X^1_{,2} = 0 \qquad (9)$$

$$W' X^1 + W^{\bullet} X^0 + W X^2_{,2} = 0 \qquad (10)$$

$$\sin^2\theta X^3_{,2} + X^2_{,3} = 0 \qquad (11)$$

$$W' X^1 + W^{\bullet} X^0 + W \cot\theta X^2 + W X^3_{,3} = 0 \qquad (12)$$

where dot stands for differentiation with respect to t and a dash stands for differentiation with respect to r. On integrating equations (3) to (6), we have a system of equations as

$$X^0 = P^1(r,\theta,\phi), \qquad X^1 = P^1_r(r,\theta,\phi)\int\frac{1}{V^2}dt + P^2(r,\theta,\phi)$$

$$X^2 = P^1_\theta(r,\theta,\phi)\int\frac{1}{W^2}dt + P^3(r,\theta,\phi), \qquad X^3 = P^1_\phi(r,\theta,\phi)\operatorname{cosec}^2\theta\int\frac{1}{W^2}dt + P^4(r,\theta,\phi), \qquad (13)$$

where $P^1(r,\theta,\phi)$, $P^2(r,\theta,\phi)$, $P^3(r,\theta,\phi)$, and $P^4(r,\theta,\phi)$, are functions of integration which are to be determined. Result for each possible solution is written directly here and lengthy details are omitted. The cases when Lemaitre-Tolman-Bondi metric admits three Killing vector fields are given below as:

(Ia)	$V = V(t,r)$,	$W = W(t,r)$	(Ib)	$V = V(t)$,	$W = W(t,r)$
(Ic)	$V = \text{constant}$,	$W = W(t,r)$	(Id)	$V = V(t,r)$,	$W = \text{constant}$
(Ie)	$V = V(r)$,	$W = W(t)$	(If)	$V = V(t,r)$,	$W = W(r)$
(Ig)	$V = V(r)$,	$W = W(t,r)$	(Ih)	$V = V(t,r)$,	$W = W(t)$
(Ii)	$V = V(r)$,	$W = W(t)$	(Ij)	$V = V(t)$,	$W = W(r)$

The Killing vector fields for the above cases are obtained as:

$$X^0 = 0, \qquad X^1 = 0,$$

$$X^2 = -C_1\sin\phi + C_2\cos\phi, \qquad X^3 = -\cot\theta(C_1\cos\phi + C_2\sin\phi) + C_3, \qquad (14)$$

where $c_1, c_2, c_3 \in \Re$. The generators of the Killing algebra are $\frac{\partial}{\partial\phi}$, $-(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi})$ and $(\cos\phi\frac{\partial}{\partial\theta} - \cot\theta\sin\phi\frac{\partial}{\partial\phi})$.

The cases when Lemaitre-Tolman-Bondi metric admits four Killing vector fields are given as:

(IIa)	$V = V(t)$,	$W = \text{constant}$	(IIb)	$V = \text{constant}$,	$W = W(t)$
(IIc)	$V = \text{constant}$,	$W = W(r)$	(IId)	$V = V(r)$,	$W = \text{constant}$

The Killing vector fields for the above cases (IIa) and (IIb) are obtained as:

$$X^0 = 0, \qquad X^1 = c_4,$$
$$X^2 = -c_1 \sin\phi + c_2 \cos\phi, \qquad X^3 = -\cot\theta(c_1 \cos\phi + c_2 \sin\phi) + c_3, \tag{15}$$

where $c_1, c_2, c_3, c_4 \in \Re$. The generators of the Killing algebra are $\frac{\partial}{\partial r}, \frac{\partial}{\partial \phi}$, $-(\sin\phi \frac{\partial}{\partial \theta} + \cot\theta \cos\phi \frac{\partial}{\partial \phi})$ and $(\cos\phi \frac{\partial}{\partial \theta} - \cot\theta \sin\phi \frac{\partial}{\partial \phi})$ and the Killing vector fields for case (IIc) and (IId) are obtained as:

$$X^0 = c_4, \qquad X^1 = 0,$$
$$X^2 = -c_1 \sin\phi + c_2 \cos\phi, \qquad X^3 = -\cot\theta(c_1 \cos\phi + c_2 \sin\phi) + c_3, \tag{16}$$

where $c_1, c_2, c_3, c_4 \in \Re$. The generators of the Killing algebra are $\frac{\partial}{\partial t}, \frac{\partial}{\partial \phi}$, $-(\sin\phi \frac{\partial}{\partial \theta} + \cot\theta \cos\phi \frac{\partial}{\partial \phi})$ and $(\cos\phi \frac{\partial}{\partial \theta} - \cot\theta \sin\phi \frac{\partial}{\partial \phi})$.

3. Conclusion

In this paper we obtained Killing vector fields for Lemaitre-Tolman-Bondi spacetime in general relativity theory. Some algebraic and direct integration techniques are applied for the purpose. We studied every possibility and listed the obtained Killing vector fields and their generators for each case. Our results show that this spacetime admits only three or four Killing vector fields when it remains non flat.

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