



## Application of Optimal Homotopy Asymptotic Method to Convective Radiative Fin with Temperature Dependent Thermal Conductivity

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### ABSTRACT

Application of Optimal Homotopy Asymptotic Method (OHAM), a new analytic approximate technique has been applied to convective-radiative fin with temperature dependent thermal conductivity. OHAM has the beauty to control the convergence of approximate solutions when it is compared with other methods such as Homotopy Analysis Method (HAM), Adomian Decomposition Method (ADM) and Homotopy Perturbation Method (HPM).

**KEYWORDS:** OHAM, Exact, ADM, HPM, HAM, Non-linear differential equations.

### 1. INTRODUCTION

The engineering problems arising in convective-radiative fins are nonlinear. Some of analytic methods are available in literature like Adomian Decomposition Method (ADM) [1], Variational Iteration Method (VIM)[2], Differential Transform Method (DTM) [3] ,Homotopy Perturbation Method (HPM)[4,5] and Homotopy Analysis Method (HAM) [6]. These methods needs assume initial solution which is a problem. The Perturbation Methods were studied [7-9] for nonlinear boundary value problems (BVPs). These methods contain a small parameter and are difficult to found. Vasile Marinca *et al* introduce Optimal Homotopy Asymptotic Method (OHAM) [10-14] for the solution of nonlinear BVPs which is independent of the assumption of small parameter and initial guess solution.

OHAM has been proved to be a powerful technique for solution of nonlinear BVPs [15-22].The motivation of this article is to use OHAM theory for convective radiative fins BVPs.

The structure of article is as the basic idea of OHAM is discussed in section 2 and in section 3 have been implemented to BVPs.

### 2. Basic Mathematical Theory of OHAM

Consider a general form of nonlinear differential equations

$$\mathcal{L}(\chi(\varsigma)) + s(\varsigma) + \mathcal{N}(\chi(\varsigma)) = 0, \quad (2.1)$$

with

$$\Upsilon\left(\chi, \frac{d\chi}{d\varsigma}\right) = 0. \quad (2.2)$$

Where  $\mathcal{L}$  is linear,  $\chi(\varsigma)$  is unknown function,  $s(\varsigma)$  is known function,  $\mathcal{N}(\chi(\varsigma))$  is nonlinear operators.

The homotopy  $\vartheta(\varsigma, m): \Xi \times [0, 1] \rightarrow \Theta$  gives

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$$(1-m) \left[ \mathcal{L}(\vartheta(\varsigma, m)) + s(\varsigma) \right] = \\ K(m) \left[ \begin{array}{l} \mathcal{L}(\vartheta(\varsigma, m)) + s(\varsigma) \\ + \mathcal{N}(\vartheta(\varsigma, m)) \end{array} \right], \quad (2.3)$$

$$\Upsilon \left( \vartheta(\varsigma, m), \frac{\partial \vartheta(\varsigma, m)}{\partial \varsigma} \right) = 0, \quad (2.4)$$

here  $m \in [0, 1]$  is embedding parameter,  $\vartheta(\varsigma, m)$  is unknown function,  $K(m)$  is nonzero auxiliary function.

The solution  $\vartheta(\varsigma, m)$  varies from  $\chi_0(\varsigma)$  to  $\chi(\varsigma)$ , where  $u_0(t)$  is zeroth order solution for  $p = 0$ :

$$\mathcal{L}(\chi_0(\varsigma)) + s(\varsigma) = 0, \quad \Upsilon \left( \chi_0, \frac{d\chi_0}{dt} \right) = 0. \quad (2.5)$$

The auxiliary function is

$$K(m) = ml_1 + m^2 l_2 + m^3 l_3 + \dots \quad (2.6)$$

Where  $l_1, l_2, l_3, \dots$  are optimal constants.

Using Taylor's series to  $\vartheta(\varsigma, m)$  about  $m$

$$\vartheta(\varsigma, m, l_1, l_2, \dots, l_m) = \chi_0(\varsigma) + \sum_{i=1}^{\infty} \chi_i(\varsigma, l_1, l_2, \dots, l_m) m^i, \\ i = 1, 2, \dots \quad (2.7)$$

Using Eq. (2.7) into Eq. (2.1)-(2.2) and equating the coefficient of like powers of  $m$ , we obtain

$$\mathcal{L}(\chi_1(\varsigma)) = l_1 \mathcal{N}(\chi_0(\varsigma)), \\ \Upsilon \left( \chi_1, \frac{d\chi_1}{dt} \right) = 0, \quad (2.8)$$

$$\mathcal{L}(\chi_2(\varsigma)) - \mathcal{L}(\chi_1(\varsigma)) = l_1 \mathcal{N}_0(\chi_0(\varsigma)) + \\ l_1 \left[ \mathcal{L}(\chi_1(\varsigma)) + \mathcal{N}_1(\chi_0(\varsigma), \chi_1(\varsigma)) \right], \quad (2.9)$$

$$\Upsilon \left( \chi_2, \frac{d\chi_2}{dt} \right) = 0, \\ \mathcal{L}(\chi_k(\varsigma)) - \mathcal{L}(\chi_{k-1}(\varsigma)) = l_k \mathcal{N}_0(\chi_0(\varsigma)) + \\ \sum_{i=1}^{k-1} l_i \left[ \mathcal{L}(\chi_{k-i}(\varsigma)) + \mathcal{N}_{k-i}(\chi_0(\varsigma), \chi_1(\varsigma), \dots, \chi_{k-1}(\varsigma)) \right], \\ \Upsilon \left( \chi_k, \frac{d\chi_k}{dt} \right) = 0, k = 2, 3, \dots, \quad (2.10)$$

where  $\mathcal{N}_{k-i}(\chi_0(\varsigma), \chi_1(\varsigma), \dots, \chi_{k-i}(\varsigma))$  is the coefficient of  $m^{k-i}$  in the expansion series

$$\mathcal{N}(\vartheta(\varsigma, m, l_i)) = \mathcal{N}_0(\varsigma_0(\varsigma)) + \sum_{k \geq 1} \mathcal{N}_k \begin{pmatrix} \varsigma_0, \varsigma_1, \varsigma_2, \\ \dots, \varsigma_k \end{pmatrix} m^k,$$

$i = 1, 2, 3, \dots$

(2.11)

It should be noted that the convergence of Eq. (2.7) depends upon  $l_1, l_2, \dots$ . If Eq. (2.7) converges at  $q = 1$ , we have

$$\tilde{\chi}(\varsigma, l_1, l_2, \dots, l_i) = \chi_0(\varsigma) + \sum_{k \geq 1} \chi_k(\varsigma, l_1, l_2, \dots, l_i). \quad (2.12)$$

Substituting Eq. (2.12) into Eq. (2.1), so the residual is

$$R(\varsigma, l_1, l_2, \dots, l_i) = \begin{bmatrix} \mathcal{L}(\tilde{\chi}(\varsigma, l_1, l_2, \dots, l_i)) + s(\varsigma) \\ + \mathcal{N}(\tilde{\chi}(\varsigma, l_1, l_2, \dots, l_i)) \end{bmatrix}. \quad (2.13)$$

Different methods like Galerkin's Method, Ritz Method, Least Squares Method and Collocation Method. are used for finding auxiliary constants,  $l_i, i = 1, 2, \dots, m$ . Here we apply the Method of Least Squares as

$$Q(l_1, l_2, \dots, l_k) = \int_c^d R^2(\varsigma, l_1, l_2, \dots, l_k) d\varsigma, \quad (2.14)$$

where  $c$  and  $d$  are two distinct values.

The auxiliary constants  $l_i, i = 1, 2, \dots, m$  may be calculated as from Eq. (2.14)

$$\frac{\partial Q}{\partial l_1} = \frac{\partial Q}{\partial l_2} = \dots = \frac{\partial Q}{\partial l_k} = 0 \quad (2.15)$$

These optimal constants can be used to find  $k$ th order approximate solution.

### 3. Application of OHAM to convective radiative fin with temperature dependent thermal conductivity

The efficiency and effectiveness of OHAM formulation are demonstrated by two models.

#### Model 3.1

Consider the non-dimensional form of the temperature dependent thermal conductivity through a fin taken from [23].

$$\frac{d^2 u}{dt^2} - n^2 u - \varepsilon u^4 = 0 \quad (3.1.1)$$

with boundary conditions

$$u(1) = 1, \frac{du(0)}{dt} = 0 \quad (3.1.2)$$

#### Zeroth Order Problem:

$$u_0''(t) - n^2 u_0(t) = 0 \quad (3.1.3) \quad u_0(1) = 1, u_0'(0) = 0. \quad (3.1.4)$$

Its solution is

$$u_0(t) = \frac{2e \cosh(t)}{1 + e^2}. \quad (3.1.5)$$

#### First Order Problem:

$$u_1''(t) = \begin{bmatrix} u_1(t) + (1+l_1)u_0''(t) \\ -l_1 u_0^4(t) - (1+l_1)u_0(t) \end{bmatrix}, \quad (3.1.6)$$

$$u_1(1) = 0, \quad u_1'(0) = 0.$$

(3.1.7)

Whose solution is

$$u_1(t) = \frac{2}{15(1+e^2)^5} \begin{bmatrix} e + 20e^3 \\ -90e^5 \\ +20e^7 + e^9 \\ Cosh(t) \\ -e^5(1+e^2) \\ -45 + \\ 20Cosh(2t) \\ +Cosh(4t) \end{bmatrix} C_1.$$

(3.1.8)

**Second Order Problem:**

$$u_2''(t) = u_2(t) + (1+C_1)u_1''(t) - (1+C_1 + 4C_1u_0^3(t))u_1(t) \\ - C_2(q_0(t) + q_0^4(t) - q_0''(t)),$$

(3.1.9)

$$u_2(1) = 1, \quad u_2'(0) = 0.$$

(3.1.10)

We obtain the following solution

$$u_2(t) = \frac{1}{900(1+e^2)^9} \begin{bmatrix} (1+e^{1-7t})(-60e^{2t})(1+e^2)^4(e^3+e^5-e^{3t}-e^{5t}+20e^{2+3t}+20e^{5+2t}) \\ -20e^{2+3t}+90e^{4+3t}-20e^{6+3t}-e^{8+3t}-90e^{3+4t}-90e^{5+4t}-20e^{2+5t} \\ +90e^{4+5t}-20e^{6+5t}-e^{8+5t}+20e^{3+6t}+20e^{5+6t}+e^{3+8t}+e^{5+8t}) \\ (5e^6+10e^8+5e^{10}+71e^{6t}+71e^{8t}-810e^{10+10t}-636e^{11+11t}+3860e^{12+6t} \\ +71e^{16+8t}+230e^{10+5t}+230e^{6+2t}+460e^{8+2t}-76e^{3+3t}-636e^{5+3t}+520e^{7+3t} \\ +520e^{9+3t}-636e^{11+3t}-76e^{13+3t}-810e^{6+4t}-1620e^{8+4t}-810e^{10+4t}-1520e^{3+5t} \\ -12720e^{5+5t}+10400e^{7+3t}+10400e^{9+5t}-12720e^{11+5t}-1520e^{13+5t}+1845e^{2+6t} \\ +3860e^{4+6t}+1845e^{14+6t}+71e^{16+6t}+6840e^{3+7t}+57240e^{5+7t}-46800e^{7+7t} \\ +46800e^{9+7t}+57240e^{11+7t}+6840e^{13+7t}+1845e^{2+8t}+3860e^{4+8t}+3860e^{12+8t} \\ +1845e^{14+8t}-1520e^{3+9t}-12720e^{5+9t}+10400e^{7+9t}+10400e^{9+9t}-12720e^{11+9t} \\ -1520e^{13+9t}-810e^{6+10t}-1620e^{8+10t}-76e^{3+11t}-636e^{5+11t}+520e^{7+11t}+520e^{9+11t} \\ -76e^{13+11t}+230e^{6+12t}+460e^{8+12t}+230e^{10+12t}+5e^{6+14t}+10e^{8+14t}+5e^{10+14t} \\ +10e^{6+6t}(-9179+2268t)+10e^{8+6t}(-4643+2268t)-10e^{10+8t}(4643+2268t) \\ -10e^{6+8t}(9179+2268t)-8e^{8+8t}(-14969+5670t)+8e^{8+6t}(14969+5670t) \\ 60e^{3t}(1+e^5)^5(e^3+e^5-e^{3t}-e^{5t}+20e^{3+2t}+20e^{5+2t}-20e^{2+3t}+90e^{4+3t}-20e^{6+3t} \\ -e^{8+3t}-90e^{3+4t}-90e^{5+4t}-20e^{2+5t}+90e^{4+5t}-20e^{6+5t}-e^{8+5t}+20e^{3+6t}+20e^{5+6t}+ \\ e^{3+8t}+e^{5+8t}) \end{bmatrix} C_1.$$

(3.1.11)

Adding Eqs. (3.1.5), (3.1.8) and (3.1.11) for  $\varepsilon = 1 = n$ , we have

$$\begin{aligned}
 u(t) = & \frac{1}{900 (1 + e^2)^9} \left[ \begin{array}{l} (1+e^{1-7t})(-120e^{3t}(1+e^{2t})^4(e^3+e^5-e^{3t}-e^{5t}+20e^{3+2t}+20e^{3+5t}) \\ -20e^{2+3t}+90e^{4+3t}-20e^{6+3t}-e^{8+3t}-90e^{3+4t}-90e^{5+4t}-20e^{2+5t} \\ +90e^{4+5t}-20e^{6+5t}-e^{8+5t}+20e^{3+6t}+20e^{5+6t}+e^{3+8t}+e^{5+8t} \end{array} \right] C_1 \\
 & + \left[ \begin{array}{l} (5e^6+10e^8+5e^{10}+71e^{6t}+71e^{8t}-810e^{10+10t}-636e^{11+11t}+3860e^{12+6t} \\ +71e^{16+8t}+230e^{10+5t}+230e^{6+2t}+460e^{8+2t}-76e^{3+3t}-636e^{5+3t}+520e^{7+3t} \\ +520e^{9+3t}-636e^{11+3t}-76e^{13+3t}-810e^{6+4t}-1620e^{8+4t}-810e^{10+4t}-1520e^{3+5t} \\ -12720e^{5+5t}+10400e^{7+3t}+10400e^{9+5t}-12720e^{11+5t}-1520e^{13+5t}+1845e^{2+6t} \\ +3860e^{4+6t}+1845e^{14+6t}+71e^{16+6t}+6840e^{3+7t}+57240e^{5+7t}-46800e^{7+7t} \\ -46800e^{9+7t}+57240e^{11+7t}+6840e^{13+7t}+1845e^{2+8t}+3860e^{4+8t}+3860e^{12+8t} \\ +1845e^{14+8t}-1520e^{3+9t}-12720e^{5+9t}+10400e^{7+9t}+10400e^{9+9t}-12720e^{11+9t} \\ -1520e^{13+9t}-810e^{6+10t}-1620e^{8+10t}-76e^{3+11t}-636e^{5+11t}+520e^{7+11t}+520e^{9+11t} \\ -76e^{13+11t}+230e^{6+12t}+460e^{8+12t}+230e^{10+12t}+5e^{6+14t}+10e^{8+14t}+5e^{10+14t} \\ +10e^{6+6t}(-9179+2268t)+10e^{8+6t}(-4643+2268t)-10e^{10+8t}(4643+2268t) \\ -10e^{6+8t}(9179+2268t)-8e^{8+8t}(-14969+5670t)+8e^{8+6t}(14969+5670t) \end{array} \right] C_1^2 \\
 & + \left[ \begin{array}{l} 60e^{3t}(1+e^2)^4(15e^{3t})(1+e^2)^4(1+e^{2t})+(-e^3-e^5+e^{3t}+e^{5t})-(20e^{3+2t} \\ -20e^{5+2t}+20e^{2+3t}-90e^{4+3t}+20e^{6+3t}+e^{8+3t}+90e^{3+4t}+90e^{5+4t}+20e^{2+5t} \\ -90e^{4+5t}+20e^{6+5t}+e^{8+5t}-20e^{3+6t}-20e^{5+6t}-e^{3+8t}-e^{5+8t}) \end{array} \right] C_2
 \end{aligned}$$

(3.1.12)

Using Eq. (3.1.12) in (3.1.1) and applying procedure in Eqs. (2.13)-(2.15), we have

$$C_1 = -0.767024893148472 \quad \text{and} \quad C_2 = -0.03274310523383026.$$

Substituting these in Eq. (3.1.12), we obtain

$$\begin{aligned}
 u(t) = & -0.0515508+0.0229114 \cosh(2t)-0.000397156 \cosh(3t) \\
 & +0.00114557 \cosh(4t)+0.000112773 \cosh(5t)+2.45158 \times 10^{-6} \cosh(7t) \\
 & +0.610094 \cosh(t)-0.0111204 t \sinh(t)-1.0842 \times 10^{-19} \sinh(4t).
 \end{aligned}$$

(3.1.13)

**Solution by Double Optimal Linearization Method (DOLM) [23]**

$$u(t)_{-DOLM} = \frac{\cosh V_1 t}{\cosh V_1}, \quad (3.1.14)$$

$$\text{where } V_1 = \left[ n^2 + \frac{\varepsilon}{60} \frac{(150 \sinh(n) + 25 \sinh(3n) + 3 \sinh(5n))}{(2n + \sinh(2n)) \cosh^3(n)} \right]^{\frac{1}{2}}.$$

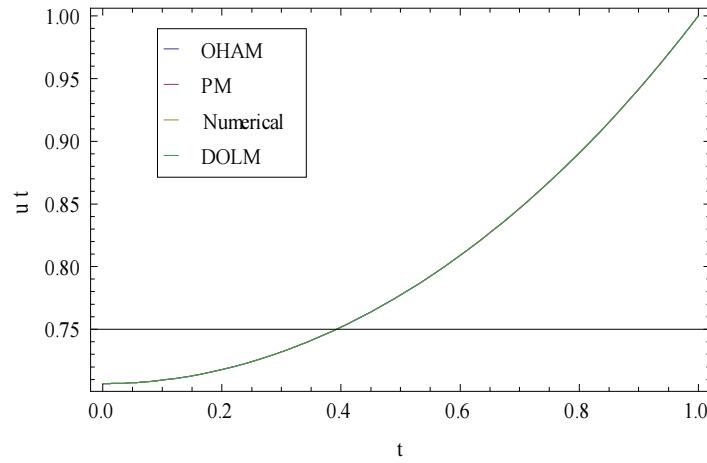


Fig. 1. Comparison of OHAM and exact result for  $\varepsilon = 0.2$ ,  $n = 1$ .

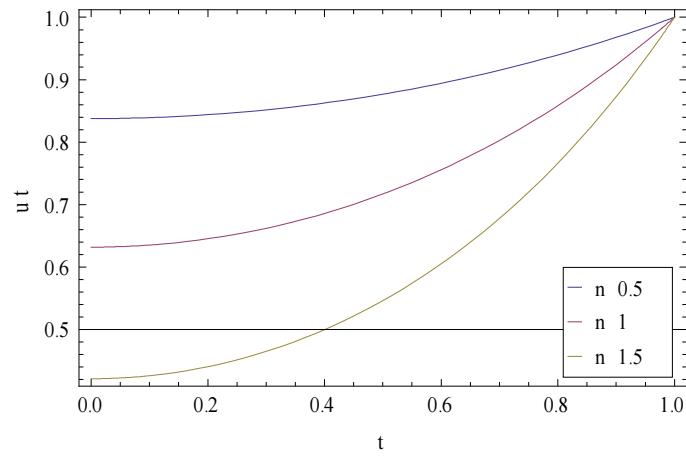


Fig. 2. OHAM result  $u(t)$  for different values of  $n$  and  $\varepsilon = 0.2$ .

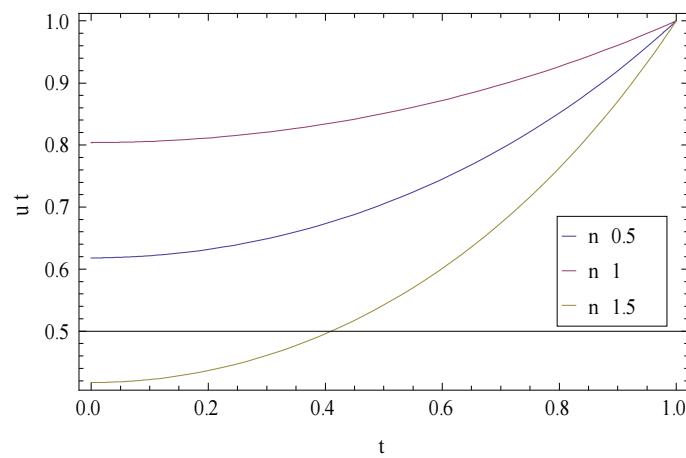


Fig. 3. OHAM result  $u(t)$  for different values of  $n$  and  $\varepsilon = 0.4$ .

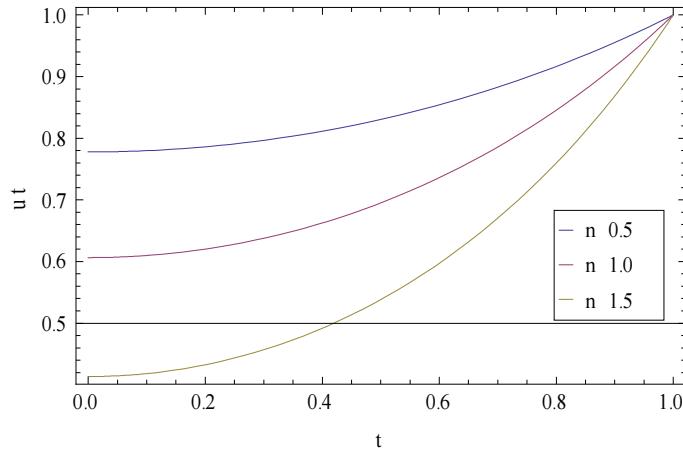


Fig. 4.  $u(t)$  for different values of  $n$  and  $\varepsilon = 0.6$ .

### Model 3.2 [23]

Assuming the thermal conductivity of the form

$$k = k_0(1 + \beta T), \quad (3.2.1)$$

where the constant  $\beta$  is a measure of the thermal conductivity. Using

$$\frac{k}{k_0} = 1 + \varepsilon u, \quad (3.2.2)$$

where  $\varepsilon = \beta T_b$ . The differential equations now become

$$\frac{d}{dt} \left[ \left( 1 + \varepsilon u(t) \frac{du(t)}{dt} \right) \right] - n^2 u(t) - \varepsilon u^4(t) = 0, \quad (3.2.3)$$

with boundary conditions

$$u(1) = 1, \quad \frac{du}{dt}(0) = 0 \quad (3.2.4)$$

According to Eq. (2.1), we define the operators

$$\mathcal{L}(u(t)) = u''(t) - n^2 u(t), \quad g(t) = 0, \quad \mathcal{N}(u(t)) = \varepsilon(u(t)u''(t) + u'^2(t) - u^4(t)), \quad (3.2.5)$$

where  $u'(t)$  and  $u''(t)$  represent the first and second derivatives of  $u(t)$  with respect to  $t$ .

$$\text{Zeroth Order Problem: } u_0''(t) - n^2 u_0(t) = 0, \quad (3.2.6)$$

$$u_0(1) = 1, \quad u_0'(0) = 0. \quad (3.2.7)$$

$$\text{Its solution is } u_0(t) = \frac{2e\cosh(t)}{1+e^2}. \quad (3.2.8)$$

### First Order Problem:

$$u_1''(t) = u_0''(t) + u_1(t) + C_1 \left( u_0''(t) + (u_0')^2(t) \right) - u_0(t) \left[ 1 + C_1 (1 - u_0''(t) + u_0^3(t)) \right], \quad (3.2.9)$$

$$u_1(1) = 0, \quad u_1'(0) = 0. \quad (3.2.10)$$

$$\text{Whose solution is } u_1(t) = \frac{1}{15(1+e^2)^5} \begin{bmatrix} -2e(9 + 110e^4 + 9e^8)\cosh(t) + \\ 10e(1+e^2)(1+e^4) \cosh(2t) \\ -e^2[-45+\cosh(4t)]C_1 \end{bmatrix}. \quad (3.2.1)$$

**Second Order Problem:**

$$\begin{aligned} u_2''(t) &= u_2(t) + 2C_1 u_0'(t) u_1'(t) + \left[ 1 + C_1(1+u_0(t)) u_1''(t) \right] \\ &- u_1(t) \left[ 1 + C_1(1+4u_0^3(t)) - u_0''(t) \right] - C_2 \left[ u_0(t) + u_0^4(t) - (u_0')^2(t) - (1+u_0(t)) u_0''(t) \right], \end{aligned} \quad (3.2.12)$$

$$u_2(1) = 0 \quad u_2'(0) = 0. \quad (3.2.13)$$

We obtain the following solution

$$\begin{aligned} u_2(t) &= -\frac{e^{1-7t}}{1800(1+e^2)^9} \begin{bmatrix} (120e^{3t})(1+e^2)^4(e^3+e^5+9e^{3t}+9e^{5t}-10e^{(1+2t)}-10e^{(3+2t)}) \\ -10e^{(5+2t)}-10e^{(7+2t)}+110e^{(4+3t)}+9e^{(8+3t)}-90e^{(3+4t)} \\ -90e^{(5+4t)}+110e^{(4+5t)}+9e^{(8+5t)}-10e^{(1+6t)}-10e^{(3+6t)} \\ -10e^{(5+6t)}-10e^{(7+6t)}+e^{(3+8t)}+e^{(5+8t)} \end{bmatrix} C_1 \\ &+ \begin{bmatrix} -(10e^6+20e^8+10e^{10}-963e^{6t}-963e^{8t}+90e^{10+10t}-312e^{11+11t}-325e^{12+12t}) \\ -963e^{16+8t}+3465e^{12+3t}-240e^{15+3t}-840e^{10+2t}-325e^{12+2t}-325e^{8+2t}-840e^{6+2t} \\ -1030e^{8+2t}+168e^{3+3t}-312e^{5+3t}+2320e^{7+3t}+2320e^{9+3t}-312e^{11+3t} \\ +168e^{13+3t}+1350e^{2+4t}+3465e^{4+4t}+90e^{6+4t}-4050e^{8+4t}+90e^{10+4t} \\ +1350e^{14+4t}-240e^{1+5t}+7440e^{3+5t}-2960e^{5+5t}+34160e^{7+5t}+34160e^{9+5t} \\ -2960e^{11+5t}+7440e^{13+5t}-963e^{16+6t}-15120e^{3+7t}+28080e^{5+7t}-208800e^{7+7t} \\ -208800e^{9+7t}+28080e^{11+7t}-15120e^{13+7t}-240e^{1+9t}+7440e^{3+9t} \\ -2960e^{5+9t}+34160e^{7+9t}+34160e^{9+9t}-2960e^{11+9t}+7440e^{13+9t}-240e^{15+9t} \\ -1350e^{2+10t}+3465e^{4+10t}+90e^{6+10t}-4050e^{8+10t}+3465e^{12+10t}+1350e^{14+10t} \\ +168e^{3+11t}-312e^{5+11t}+2320e^{7+11t}+2320e^{9+11t}+168e^{11+11t}-325e^{4+12t} \\ -840e^{6+12t}-1030e^{8+12t}-840e^{10+12t}+10e^{6+14t}+20e^{8+14t}+10e^{10+14t} \\ +e^{8+8t}(268714-111120t)-60e^{12+8t}(-479+30t)-60e^{4+8t}(-379+30t) \\ +60e^{4+6t}(379+30t)+60e^{12+6t}(479+30t)+10e^{14+8t}(-941+60t) \\ +10e^{2+8t}(-821+60t)-10e^{2+6t}(821+60t)-10e^{14+6t}(941+60t) \\ +360e^{10+6t}(-114+161t)-360e^{10+8t}(114+161t)+120e^{6+6t}(-1228+483t) \\ -120e^{6+8t}(1228+483t)+2e^{8+6t}(134357+55560t) \end{bmatrix} C_2 \end{aligned}$$

$$+ \left[ \begin{array}{l} (120e^{3t})(1+e^2)^4(e^3+e^5+9e^{3t}+9e^{5t}-10e^{(1+2t)}-10e^{(3+2t)}-10e^{(5+2t)} - \\ 10e^{(7+2t)} + 110e^{(4+3t)} + 9e^{(8+3t)} - 90e^{(3+4t)} - 90e^{(5+4t)} + 110e^{(4+5t)} + \\ 9e^{(8+5t)} - 10e^{(1+6t)} - 10e^{(3+6t)} - 10e^{(5+6t)} - 10e^{(7+6t)} + e^{(3+8t)} + e^{(5+8t)} \end{array} \right] C_2. \quad (3.2.14)$$

From Eqs. (3.3.8), (3.3.11) and (3.3.14) by adding, we obtain:

$$\begin{aligned} u(t) = & \frac{1}{15(1+e^2)^5} \left[ \begin{array}{l} (30e(1+e^2)^4 \text{Cosh}(t) - 2e(9 + 110e^4 + 9e^4) \text{Cosh}(t)) \\ + 10(e+e^3)(1+e^4) \text{Cosh}(2t) - e^3(45-\text{Cosh}(2t)) \end{array} \right] C_1 \\ & - \frac{e^{1-7t}}{1800(1+e^2)^9} \left[ \begin{array}{l} (120e^{3t})(1+e^2)^4(e^3+e^5+9e^{3t}+9e^{5t}-10e^{(1+2t)}-10e^{(3+2t)} - \\ - 10e^{(5+2t)} - 10e^{(7+2t)} + 110e^{(4+3t)} + 9e^{(8+3t)} - 90e^{(3+4t)} \\ - 90e^{(5+4t)} + 110e^{(4+5t)} + 9e^{(8+5t)} - 10e^{(1+6t)} - 10e^{(3+6t)} \\ - 10e^{(5+6t)} - 10e^{(7+6t)} + e^{(3+8t)} + e^{(5+8t)}) \end{array} \right] C_1 \\ & \left[ \begin{array}{l} -(10e^6 + 20e^8 + 10e^{10} - 963e^{6t} - 963e^{8t} + 90e^{10+10t} - 312e^{11+11t} - 325e^{12+12t} \\ - 963e^{16+8t} + 3465e^{12+3t} - 240e^{15+3t} - 840e^{10+2t} - 325e^{12+2t} - 325e^{8+2t} - 840e^{6+2t} \\ - 1030e^{8+2t} + 168e^{3+3t} - 312e^{5+3t} + 2320e^{7+3t} + 2320e^{9+3t} - 312e^{11+3t} \\ + 168e^{13+3t} + 1350e^{2+4t} + 3465e^{4+4t} + 90e^{6+4t} - 4050e^{8+4t} + 90e^{10+4t} \\ + 1350e^{14+4t} - 240e^{1+5t} + 7440e^{3+5t} - 2960e^{5+5t} + 34160e^{7+5t} + 34160e^{9+5t} \\ - 2960e^{11+5t} + 7440e^{13+5t} - 963e^{16+6t} - 5120e^{3+7t} + 28080e^{5+7t} - 08800e^{7+7t} \\ - 208800e^{9+7t} + 28080e^{11+7t} - 15120e^{13+7t} - 240e^{1+9t} + 7440e^{3+9t} \\ - 2960e^{5+9t} + 34160e^{7+9t} + 34160e^{9+9t} - 2960e^{11+9t} + 7440e^{13+9t} - 240e^{15+9t} \\ + 1350e^{2+10t} + 3465e^{4+10t} + 90e^{6+10t} - 4050e^{8+10t} + 3465e^{12+10t} + 1350e^{14+10t} \\ + 168e^{3+11t} - 312e^{5+11t} + 2320e^{7+11t} + 2320e^{9+11t} + 168e^{11+11t} - 325e^{4+12t} \\ - 840e^{6+12t} - 1030e^{8+12t} - 840e^{10+12t} + 10e^{6+14t} + 20e^{8+14t} + 10e^{10+14t} \\ + e^{8+8t}(268714 - 111120t) - 60e^{12+8t}(-479 + 30t) - 60e^{4+8t}(-379 + 30t) \\ + 60e^{4+6t}(379 + 30t) + 60e^{12+6t}(479 + 30t) + 10e^{14+8t}(-941 + 60t) \\ + 10e^{2+8t}(-821 + 60t) - 10e^{2+6t}(821 + 60t) - 10e^{14+6t}(941 + 60t) \\ + 360e^{10+6t}(-114 + 161t) - 360e^{10+8t}(114 + 161t) + 120e^{6+6t}(-1228 + 483t) \\ - 120e^{6+8t}(1228 + 483t) + 2e^{8+6t}(134357 + 55560t) \end{array} \right] C_1 \\ & + \left[ \begin{array}{l} (120e^{3t})(1+e^2)^4(e^3+e^5+9e^{3t}+9e^{5t}-10e^{(1+2t)}-10e^{(3+2t)}-10e^{(5+2t)} - \\ 10e^{(7+2t)} + 110e^{(4+3t)} + 9e^{(8+3t)} - 90e^{(3+4t)} - 90e^{(5+4t)} + 110e^{(4+5t)} + \\ 9e^{(8+5t)} - 10e^{(1+6t)} - 10e^{(3+6t)} - 10e^{(5+6t)} - 10e^{(7+6t)} + e^{(3+8t)} + e^{(5+8t)}) \end{array} \right] C_2. \quad (3.2.15) \end{aligned}$$

Using the technique as discussed in Eqs. (2.13)- (2.15), we obtain

$$C_1 = -0.48731589994908137, C_2 = 0.009192225410854144.$$

Eq. (3.2.15) becomes

$$\begin{aligned} u(t) = & -0.0759244 - 0.0975583 \cosh(2t) + 0.00758987 \cosh(3t) \\ & + 0.00168721 \cosh(4t) - 0.000260795 \cosh(5t) + 9.89574 \times 10^{-7} \cosh(7t) \\ & + \cosh(t)(0.87112 - 4.33681 \times 10^{-19} t) + (0.00419216 t) \sinh(t). \end{aligned} \quad (3.2.16)$$

**Solution by Double Optimal Linearization Method (DOLM) [101]**

$$u(t)_{\text{-DOLM}} = \frac{\cosh V_2 t}{\cosh V_2}, \quad (3.2.17)$$

$$V_1 = \left[ n^2 + \frac{\varepsilon}{60} \frac{(150 \sinh(n) + 25 \sinh(3n) + 3 \sinh(5n))}{(2n + \sinh(2n)) \cosh^3(n)} \right]^{\frac{1}{2}},$$

where

$$V_2 = \left[ 1 + \frac{2\varepsilon}{3} \frac{(3 \sinh V_1 + 25 \sinh 3V_1)}{(2V_1 + \sinh 2V_1) \cosh V_1} \right]^{-\frac{1}{2}}.$$

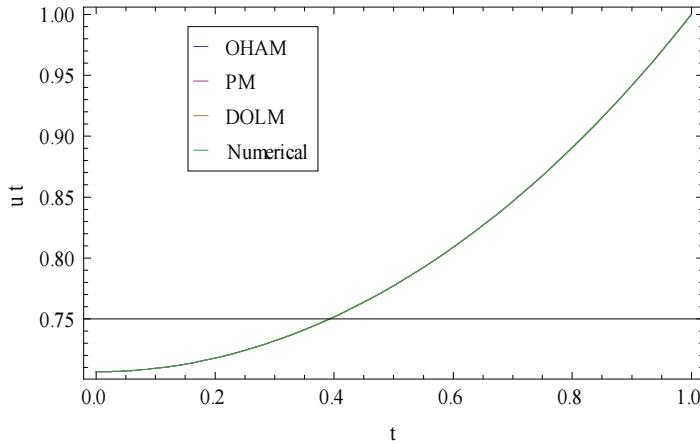


Fig. 5 Comparison of OHAM to Exact for  $n = 1, \varepsilon = 0.2$

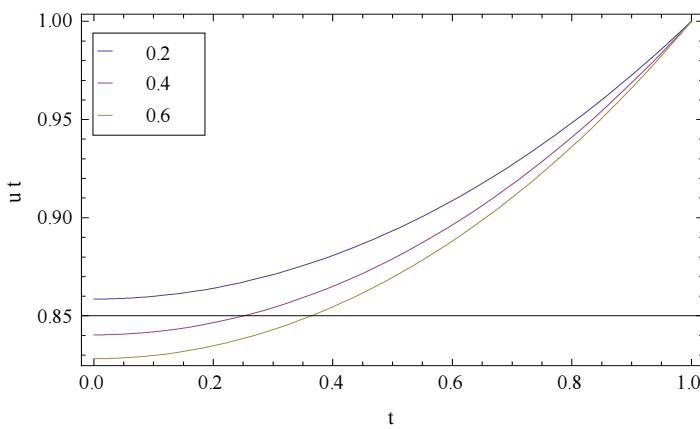


Fig. 6 Plot of  $u(t)$  with respect to  $t$  for  $n = 0.5$ .

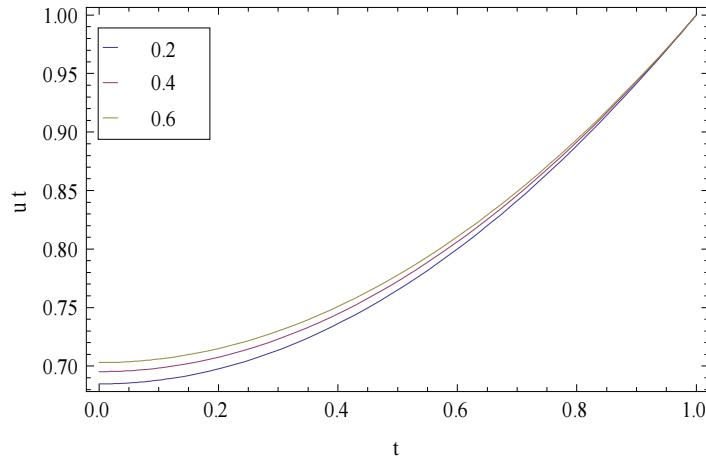


Fig. 7 Plot of  $u(t)$  with respect to  $t$  for  $n=1$ .

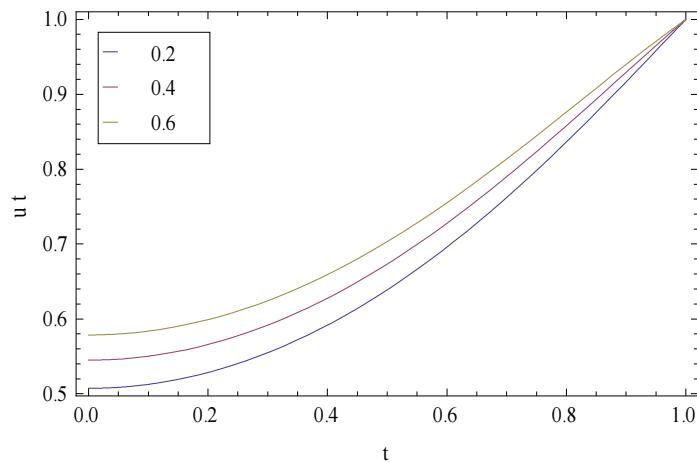


Fig. 8 Plot of  $u(t)$  with respect to  $t$  for  $n=1.5$ .

#### 4. RESULTS AND DISCUSSIONS

The OHAM theory gives well correct solution of the problems given in 3. For computational purpose we used Mathematics 7. Figs. 1 and 5 give the comparisons of OHAM and Exact solutions for Model 3.1 and 3.2 respectively. While Figs. 2-4 gives the variation of  $u(t)$  against  $t$  for different values  $n$  at  $\varepsilon = 0.2, 0.4, 0.6$  respectively for model 3.1. The behavior of solution of the problem in model 3.2 can be seen in Figs. 5-8 for different values of  $\varepsilon$  at  $n = 0.5, 1.0, 1.5$  respectively. To verify the accuracy of this method, OHAM results are compared with exact solution in Figs. 1 and 5 proving its effectiveness and efficiency.

#### 5. Conclusion

In this article, OHAM has been proved a powerful tool to solve the boundary value problem arising from convective-radiative fin with temperature dependent thermal conductivity. We have operated the mathematical theory of OHAM to model 3.1 and 3.2, found it simpler, easy to convergence and contain less computational work. Hence OHAM shows its potential for solving strongly nonlinear BVPs.

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