

# The Performance Analysis of Self-Gravitating Gas-Dust System Using Boltzman Equations and Poisson Problems

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## ABSTRACT

A Boltzman equation is usually to present wave function, particle moment involving dust component equations, gas dynamics equations and poisson equations. The interaction between Boltzman equation and Poisson problems are obtained to present the movement of short and long wave. This paper presents the gas-dust movement system using some numerical methods. Two numerical methods are focused to discretize the Boltzman equation. The discretization of gas-dust movement modeling is based on the Variational Iteration Method (VIM) and Finite Difference Method (FDM). Comparison between VIM and FDM have investigated the performance of the mathematical modeling of gas-dust movement in terms of numerical analysis. Borland C++ Builder, MATLAB 2008a and MAPLE 13 software are used to support the computation of the numerical simulation. Computation algorithms are analyzed by the pattern and gradient of the graphs, time execution and computational complexity. The results show that the VIM and FDM are successfully in visualizing the movement of the gas-dust in reality.

**KEYWORDS:** Self-Gravitating; Gas-Dust Sytem; Boltzman Equation; Poisson Equation; VIM; FDM.

## 1 INTRODUCTION

Many differences numerical methods have been introduced in solving several of linear and nonlinear partial differential equation (PDE). The methods commonly used are Adomian Decomposition Method (ADM), Homotopy Pertubation Method (HPM), Homotopy Analysis Method (HAM) and Variational Iteration Method (VIM). On the other hand, Finite Difference Method (FDM) is also one of the most popular methods in solving linear and nonlinear PDE [1]. This is because the method is easy to apply directly to the problems.

Since the research of dust movement continues to be a big interest, the building of mathematical, analytical and numerical approaches are developing as well. In this research, we applied VIM and FDM method in solving numerical model which included with dust component, gas components and gravitational interaction [2]. Normally, the model was solved by other famous methods. Dust components usually solved by particle-in-cell (PIC) method and gas components solved by fluid-in-cell (FLIC) method [3]. The Poisson equation for gravitational interaction is solved on the grid in cylindrical coordinate system, "particle-particle" method [3] and by Fourier transforms method [2].

## 2 Boltzmann Equations and Poisson Equations

The mathematical model used in this research included a collisionless Boltzmann equation for dust component (1), gas dynamics equations (2)-(5) and Poisson equation for self-gravitation (6). All the equations will be discretizing by using finite difference method. This is the sub-heading of the paper.

$$\frac{\partial f}{\partial t} + \bar{u} \frac{\partial f}{\partial r} + \bar{a} \frac{\partial f}{\partial u} = 0 \quad (1)$$

$$\frac{\partial \rho_g}{\partial t} + \text{div}(\rho_g \bar{v}) = 0 \quad (2)$$

$$\frac{\partial}{\partial t}(\rho_g \bar{v}) + \text{div}[(\rho_g \bar{v}) \bar{v}] = -\text{gragp} - \rho_g \text{grad}\phi + k_f \rho_p \rho_g (\bar{u} - \bar{v}) \quad (3)$$

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$$\frac{\partial}{\partial t}(\rho_g E) + \text{div}[(\rho_g E)\bar{v}] = -\text{div}(p\bar{v}) - \rho g(\text{grad}\phi, \bar{v}) + k_{fr} \rho_p \rho_g (\bar{u} - \bar{v}\bar{u}) \quad (4)$$

$$\frac{\partial p}{\partial t} + \text{div}(p\bar{v}) = (\gamma - 1) \left[ -p \text{div}\bar{v} + k_{fr} \rho_p \rho_g (\bar{u} - \bar{v})^2 \right] \quad (5)$$

$$\Delta\phi_2 = 4\pi p \quad (6)$$

### 3 Variational Iteration Method and Finite Difference Method

After discretizing all the equations (1)-(6) by using central finite different method, we will use Jacobi Method in order to get the result. From equation (2) we discretize the equation becomes

$$q_{i,j,k+1} = \Delta t \left[ \begin{aligned} & -v_{i,j,k} \left( \frac{q_{i+1,j,k} - q_{i,j,k}}{\Delta x} \right) - q_{i,j,k} \left( \frac{v_{i+1,j,k} - v_{i,j,k}}{\Delta x} \right) \\ & -v_{i,j,k} \left( \frac{q_{i,j,k+1} - q_{i,j,k}}{\Delta y} \right) - q_{i,j,k} \left( \frac{v_{i,j,k+1} - v_{i,j,k}}{\Delta y} \right) \end{aligned} \right] \quad (7)$$

From equation (3) we discretize the equations becomes

$$E_{i,j,k+1} = \frac{\Delta t}{q_{i,j,k}} \left[ \begin{aligned} & -v_{i,j,k} \left( \frac{p_{i+1,j,k} - p_{i,j,k}}{\Delta x} \right) - p_{i,j,k} \left( \frac{v_{i+1,j,k} - v_{i,j,k}}{\Delta x} \right) - v_{i,j,k} \left( \frac{p_{i,j,k+1} - p_{i,j,k}}{\Delta y} \right) \\ & - p_{i,j,k} \left( \frac{v_{i,j,k+1} - v_{i,j,k}}{\Delta y} \right) - q_{i,j,k} v_{i,j,k} \left( \frac{\phi_{i+1,j,k} - \phi_{i,j,k}}{\Delta x} \right) \\ & - q_{i,j,k} v_{i,j,k} \left( \frac{\phi_{i,j,k+1} - \phi_{i,j,k}}{\Delta y} \right) + c(2u_{i,j,k}^2 - u_{i,j,k} v_{i,j,k} - u_{i,j,k} v_{i,j,k}) \\ & - E_{i,j,k} \left( \frac{q_{i,j,k+1} - q_{i,j,k}}{\Delta t} \right) - \alpha_{i,j,k} \left( \frac{v_{i+1,j,k} - v_{i,j,k}}{\Delta x} \right) - v_{i,j,k} \left( \frac{\alpha_{i,j,k+1} - \alpha_{i,j,k}}{\Delta x} \right) \\ & - \alpha_{i,j,k} \left( \frac{v_{i,j,k+1} - v_{i,j,k}}{\Delta y} \right) - v_{i,j,k} \left( \frac{\alpha_{i,j,k+1} - \alpha_{i,j,k}}{\Delta y} \right) \end{aligned} \right] + E_{i,j,k} \quad (8)$$

From equation (4) we expand the equation becomes

$$\frac{\partial}{\partial t}(qE) + \nabla(\alpha\bar{v}) = -\nabla(p\bar{v}) - q(\nabla\Phi, \bar{v}) + c(\bar{u} - \bar{v}\bar{u}) \quad (9)$$

From equation (5) we discretize the equations

$$\text{become } p_{i,j,k+1} = \left\{ (y-1) + c(2u_{i,j,k}^2 - 4u_{i,j,k} v_{i,j,k} + 2v_{i,j,k}^2) - v_{i,j,k} \left( \frac{p_{i+1,j,k} - p_{i,j,k}}{\Delta x} \right) \right. \\ \left. - p_{i,j,k} \left( \frac{v_{i+1,j,k} - v_{i,j,k}}{\Delta x} \right) - v_{i,j,k} \left( \frac{p_{i,j,k+1} - p_{i,j,k}}{\Delta y} \right) - p_{i,j,k} \left( \frac{v_{i,j,k+1} - v_{i,j,k}}{\Delta y} \right) \right\} + p_{i,j,k} \quad (10)$$

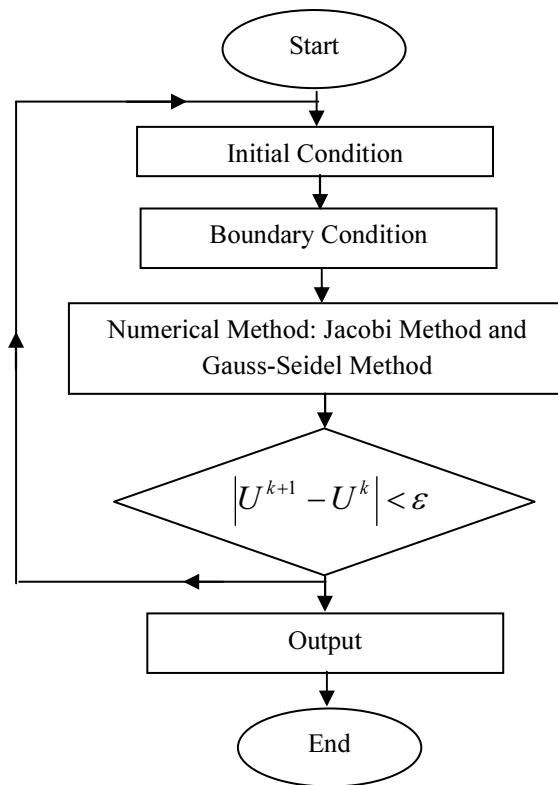
From equation (6) we discretize the equation becomes

$$\Phi_{2i+1,j} + \Phi_{2i-1,j} + \Phi_{2i,j+1} + \Phi_{2i,j-1} - \left( \frac{1}{\lambda} \right)^2 4\pi p_{i,j} \quad (11)$$

### 4 Borland C++ Builder, MATLAB 2008a and MAPLE 13

In our research, two combination of software had been used to achieve a better result. Borland C++ Builder had been chosen to solve the complicated mathematical modeling that involves several parameters,

By using C++ Builder, the adjustment of the parameters will not interface with the programming's coding. On the other side, Microsoft excel work as converting the data into graphs. MATLAB 2008a and MAPLE13 are used to support the variation iteration method simulation. MAPLE package is used to generate the initial and boundary conditions. Figure 1 shows the sequential algorithm of self-gravitating gas-dust system.

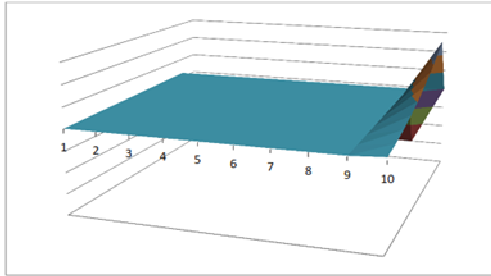


**Figure. 1.** The Flowchart of Mathematical Modeling Self-Gravitating Gas-Dust System

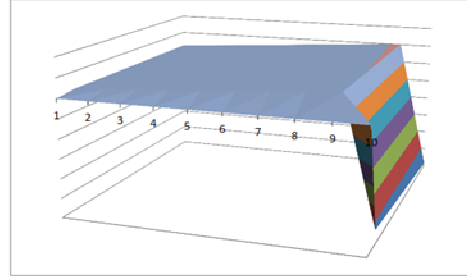
## 5 Numerical Results and Performance.

### 5.1 Result using Finite Difference Method

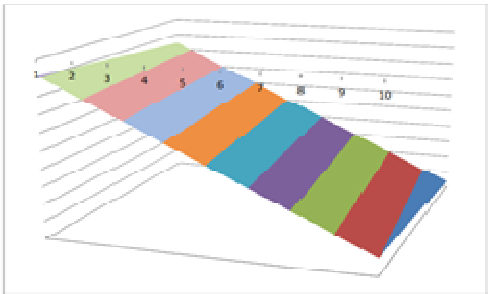
In FDM, each equation solved separately because the independent variable features. The 3D visualization of the results is shown in following graphs.



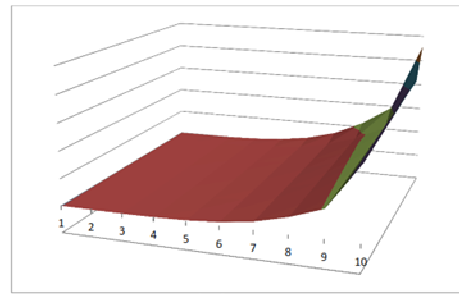
**Figure 2.** Result for dust component,  $u$  against  $t$



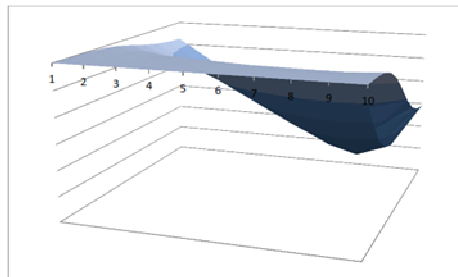
**Figure 3.** Result of gas density,  $\rho_g$  against  $t$



**Figure 4.** Result for energy,  $E$  against  $t$



**Figure 5.** Result of gas pressure,  $p$  against  $t$



**Figure 6.** Result of gravitational potential,  $\phi$  against  $t$

## 5.2 Result using Variational Iteration Method

By using VIM, all the dynamics equations are solved simultaneously. It is because the values of parameters depend on each other. The results are shown in the following graphs.

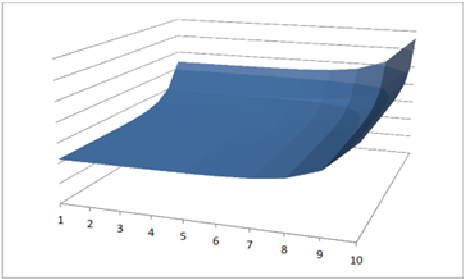


Figure 7. Result for dust component,  $u$  against  $t$

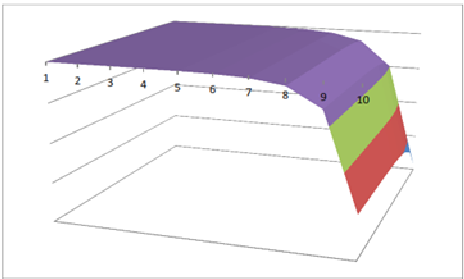


Figure 8. Result of gas density,  $\rho_g$  against  $t$

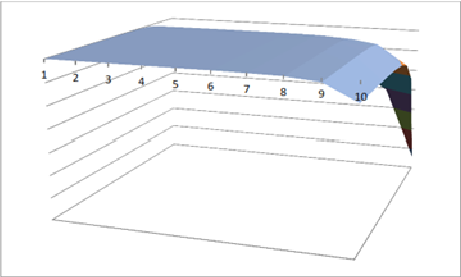


Figure 9. Result for energy,  $E$  against  $t$

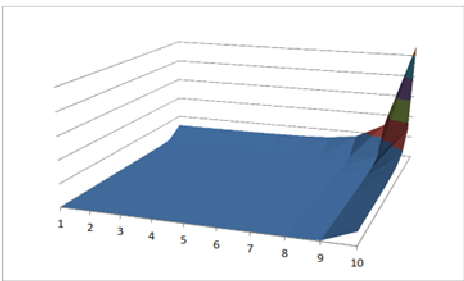


Figure 10. Result of gas pressure,  $p$  against  $t$

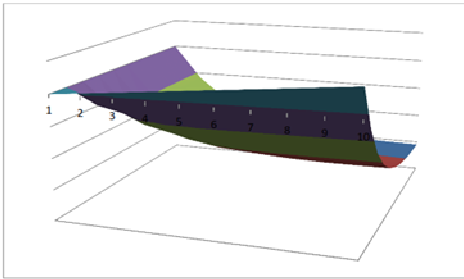


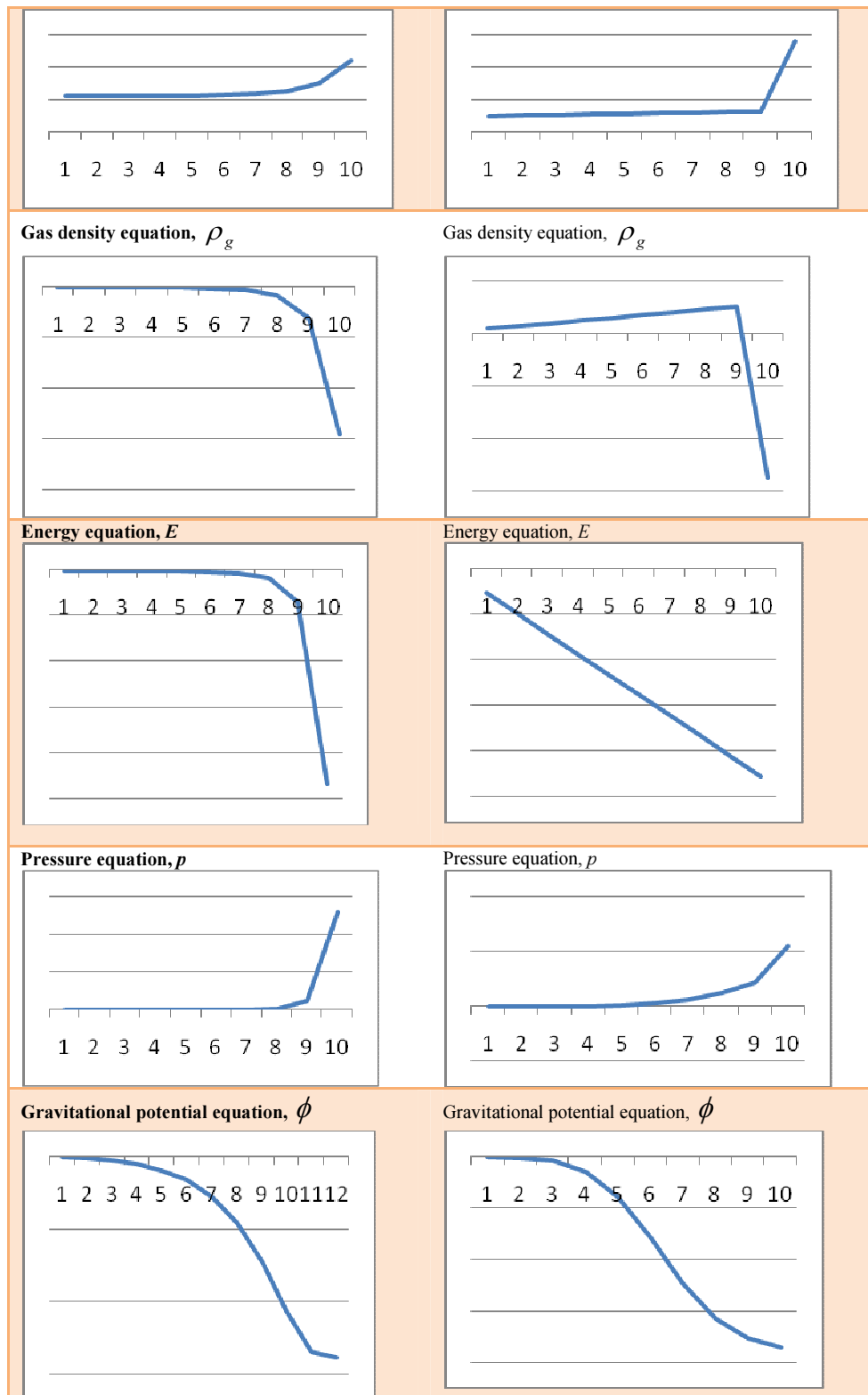
Figure 11. Result of gravitational potential,  $\phi$  against  $t$

5.3 Comparison between FDM and VIM

The result obtained by FDM and VIM are compared with term of pattern of the graph (1D), the gradient of the graph and time execution. The comparisons are shown in Table 1, 2 and 3 .

Table 1. Comparison VIM and FDM based on the pattern of the graph

VIM	FDM
Dust equation, $u$	Dust equation, $u$



In Table 2, we clearly see that the visualization for both results are almost similar curve. For the dust equations, both methods showed the upward movement even though the time step is quite difference. This is because the initial values taken for FDM and VIM are not exactly similar. Gas density equation and Energy equation also showed the same downward movements. FDM gives a linear movement compared to VIM. Both methods presented same pattern for pressure equation which moving upward. Next, graphs for gravitational potential equation are smoothly going downward. In general, we can say that the graph of VIM is more smooth compared to FDM's graph which only have straight line (linear).

**Table 2.** Comparison VIM and FDM based on gradient of the graph

Method/equation	VIM	FDM
Dust equation	Positive (+)	Positive (+)
Gas density equation	Negative (-)	Negative (-)
Energy equation	Negative (-)	Negative (-)
Pressure equation	Positive (+)	Positive (+)
Gravitational potential equation	Negative (-)	Negative (-)

The second comparison is based on the gradient of the graph. Table 2 shows that the gradient for both methods are same. This implies that the movement of the graph is similar. Positive gradient indicated the upward movement and negative gradient present downward movements. Another comparison is based on time execution. Table 3 shows the time taken for each equation to compute the result. From the table, we can see the time for VIM is faster than FDM.

**Table 3.** Comparison VIM and FDM based on time execution

Method/time execution(second, s)	VIM (second, s)	FDM (second, s)
Dust equation	0.01	0.01
Gas density equation	0.05	0.12
Energy equation	0.76	1.00
Pressure equation	1.40	1.60
Gravitational potential equation	0.09	0.10

## 6 Conclusion

In this research, the equations are solved by using VIM and FDM with selected initial conditions. The results are computed by using Borland C++ Builder, MATLAB version 2008a, MAPLE 13 and Microsoft excel. We have discussed the result based on visualization of the graph. The graphs are analyzed based on their pattern. At the same time, the gradient of the graph and time execution also compared. Based on the result, we clearly can say that both methods are suitable in solving the Boltzmann equation of the gas-dust model. Both methods were successfully in visualizing the pattern of the movement dust.

On the other hand, VIM seems to be more powerful method in solving the equations compared to FDM. VIM is better since it does not involve discretization of the variables and hence is free from rounding off errors and does not require large computer memory and time. VIM was used in the direct way without using linearization, perturbation or restrictive assumptions. Differ to FDM, it was used discretization and restricted to the simple grids and does not conserve momentum, energy and mass on coarse grids. As we know, numerical methods which used discretization gives rise to rounding off errors causing loss of accuracy. Besides, VIM provides solutions in the form of rapidly convergent series whereas FDM gives solutions on the grid points. VIM converges rapidly to its exact solution and obtains high accuracy if the number of iterations increased.

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