Megnetohydrodynamic Flow of Casson Fluids over a Moving Boundary Surface

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Received: December 23, 2016
Accepted: February 28, 2017

ABSTRACT

The numerical study for the effects of thermal radiation and chemical reaction is considered on magneto hydrodynamic boundary layer flow of Casson fluids is considered. The fluid flows through a porous medium, near a stagnation point and over the stretching/shrinking sheet. The physical problem has been fundamental in the form of non-linear partial differential equations. The governing equations are then converted into ordinary differential form by using similarity transforms. The problem is then solved numerically to reveal the physical nature of the problem through effects of the pertinent parameter. $M$ is the magnetic parameter, $K$ is the porosity parameter, $Q$ is the heat source parameter, $\beta$ is the Casson parameter, $c$ is the shrinking parameter, $R$ is the radiation parameter, $Sc$ is the Schmidt number. The result have been represented and discussed through plots for concentration, velocity and temperature.

KEYWORDS: radiation, chemical reaction, Casson fluids, porous medium, stagnation point.

1. INTRODUCTION

Non-Newtonian fluid flow arises in many branches of chemical and material processing engineering. There are different types of non-Newton fluids like Viscoelastic fluid, couple stress fluid, micropolar fluid and power-law fluid etc. In addition with these, there is another non-Newtonian fluid model is known as the Casson fluid model. In the published literature, it is sometimes claimed that for many materials, the Casson model is better than the general visco plastic models in fitting the rheological data. So, it becomes the preferred rheological model for blood and chocolate. The influence of thermal radiation and chemical reaction on micro polar fluid flow in a rotating frame was discussed by Das [1] and concluded that an increase in the volume fraction of nano particles enhances the velocity profiles. Hayat et al., [2] discussed the cross diffusion effects on MHD Casson fluid flow. Rashidi et al., [3] analytically discussed the steady flow over a rotating disk in a porous medium by using homotopy analysis method. The effects of radiation on unsteady free convection flow of a nanofluid past an infinite plate was discussed by Sandeep et al., [4]. Further Sandeep and Sugunamma [5] studied the effect of inclined magnetic field on dusty viscous fluid between two infinite flat plates. Nandy [6] studied the heat transfer characteristics of MHD Casson fluid flow over a stretching sheet and found that an increase in the value of dimensionless thermal slip parameter reduces the velocity profiles. Sandeep and Sugunamma [7] discussed the effects of radiation and inclined magnetic field on natural convection flow over an impulsively moving vertical plate. The influence of chemical reaction on MHD flow past a stretching sheet with heat generation has been investigated by Mohan krishna et al., [8]. Nandeppanavar [9] studied the flow and heat transfer analysis with two heating conditions considering the non-Newtonian Casson fluid due to linear stretching sheet. The solution they obtained is by a power series method analytically, further Nandeppanavar [9-11] investigated the heat transfer analysis of Casson fluid due to stretching sheet with convective heating condition both Numerical and analytical results in terms of Kummer’s function and RungeKutta flourth order method with shooting technique. Attia and Ahmed[12] studied the transient Coutte flow analysis of Casson fluid between parallel plates with heat transfer analysis. Bhattacharyya et.al[13] have given an analytical solution for magnetohydrodynamic boundary layer flow of Casson fluid, they also studied the effect of wall mass transfer analysis too. Swati[14] studied the effect of thermal radiation on the flow and heat transfer analysis of Casson fluid over an unsteady stretching sheet with effect of suction and blowing. Shehzad et.al[15] investigated the mass transfer of magnetohydrodynamic flow of Casson fluid with an chemical reaction. [16] discussed the influence of non linear thermal radiation on MHD 3D Casson fluid flow with viscous dissipation. A comparative study has been done by Sandeep et al. [17] to study the heat and mass transfer characteristics in non-Newtonian nanofluid past a permeable stretching surface. Raju et al., [18] discussed the effects of thermal diffusion.

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and diffusion thermo on the flow over a stretching surface with inclined magnetic field. This paper concludes that an increase in the Soret number increases the friction factor but reduces the heat transfer rate. Very recently, the researchers [19-24] investigated the heat and mass transfer characteristics of non-Newtonian and Newtonian flows by considering various channels.

2. MATHEMATICAL ANALYSIS

The steady incompressible and two dimensional flow of caisson fluid is consider in the presence of uniform magnetic field of strength $B_0$. The flow of fluid due to a horizontal sheet which is porous and stretches/shrinks. The flow in plane $y > 0$, the temperature at the surface of sheet $T_w$ the fluid temperature is $T$. The temperature and velocity in the external flow are respectively $T_\infty$ and $U$. The fluid flows through a porous medium of permeability $K_1$ in the presence of a species of concentration $C$. Where concentration of species in the external flow in $C_\infty$. The fluid velocity components are $u$ and $v$ respectively in x and y directions.

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \left(1 + \frac{1}{\beta}\right) \nu \frac{\partial^2 u}{\partial y^2} + \left(1 + \frac{1}{\beta}\right) \frac{\nu}{K_1} (U - u) + \frac{\sigma e B_0^2}{\rho} (U - u)$$

(2)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{16 \alpha}{3 \beta \rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma e B_0^2}{\rho C_p} (U - u)^2$$

(3)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 u}{\partial y^2} - R (C - C_\infty)$$

(4)

Where $\rho$ is density, $\sigma$ is the electrical conductivity, $K_1$ is the permeability of the porous medium $C$ is the specific heat capacity at constant pressure, $\mu$ is dynamic viscosity, $\alpha$ is thermal diffusivity, $\mu$ is coefficient of viscosity.

The boundary conditions are:

$$u = u_w(x) = bx, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0$$

$$u = u_\infty(x) = ax, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \to \infty$$

(5)

Using similarity transformations:

The velocity components are described in terms of the stream function $\Psi (x, y)$:

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x}$$

$$\Psi(x, y) = \sqrt{\alpha x U} f(\eta), \quad \eta = y \sqrt{\frac{U}{\alpha x}}, \quad T = T_\infty + (T_w - T_\infty) \theta(\eta), \quad C = C_\infty + (C_w - C_\infty) \phi(\eta)$$

(6)

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4) we get

$$P_r \left(1 + \frac{1}{\beta}\right) f'''' + \left(1 + \frac{1}{\beta}\right) K (1 - f') + M (1 - f') + 1 = f''^2 - ff'$$

(7)
\[(1 + R_n)\theta'' + f \theta' + Q\theta + E_c P_r f'' + E_c M(1 - f')^2 = 0\]  
\[\phi'' + S_c (f \theta' - \gamma \phi) = 0\]  
\[f = 0, f' = c, \theta = 1, \phi = 1 \text{ as } \eta = 0\]  
\[f' \rightarrow 1, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty\]

where \(P_r = \frac{\nu}{\alpha}\) is the Prandtl number, \(E_c = \frac{U}{C_p(T_w - T_{\infty})}\) is Eckert number, \(K = \frac{\nu}{aK_1}\) is the permeability parameter, \(M = \frac{\sigma_c B_0^2 \nu R_e}{\rho U^2}\) is the magnetic parameter, \(R_e = \frac{Ux}{\nu}\) is the Reynolds number, \(\gamma = \frac{R}{a}\) is non-dimensional rate of solutal. \(Sc = \frac{\nu}{D}\) Schmidt number.

3. RESULTS AND DISCUSSION

The resulting set of governing equations (7) to (10) is nonlinear in higher order. It is hard to find any analytical solution of this system of equations. Thus a numerical treatment of the situation has been employed to obtain a reliable solution of the problem. The higher order derivatives have been reduced to their first order form. We take 
\[f' = u, u' = v, \theta = w, \phi = g\]  
Thus the resulting system of first order equations is as follows:

\[P_r \left(1 + \frac{1}{\beta}\right) v' + \left(1 + \frac{1}{\beta}\right) K(1 - u) + M(1 - u) + 1 = u^2 - fv\]  
\[(1 + R_n)w' + fw + Q\theta + E_c P_r u^2 + E_c M(1 - u)^2 = 0\]  
\[g' + S_c (fw - \gamma \phi) = 0\]

These equations along with the associated boundary conditions have been solved through ND solve command of computing software Mathematica version 11. We made several computations for sufficient ranges of the pertinent parameters that affect the physical nature of the problem some representative plots for, concentration function, temperature function and velocity are presented to indicate the effects of the physical parameters of interest.

The curves for velocity \(f'\) as drawn in fig.1 demonstrate the effect of magnetic field parameters on the horizontal velocity component \(f'\). It is obtained that velocity increases with magnetic field strength for stretching in the values of porosity parameter caisson increase in this velocity component nearly \(f'\) as indicated in fig.2. The effect of velocity ratio parameter \(C\) on \(f'\) is demonstrated in fig.3, here \(C < 0\) stands for shrinking case and \(C > 0\) is for stretching case.

Fig 4. Show the effect of Prandtl number \(Pr\) on velocity \(f'\) in case of stretching sheet. It is observed that increase in Prandtl number causes reduction in the magnitude of the velocity and increase in momentum boundary layer thickness. But opposite effect of the casson parameter \(\beta\) is noticed through Fig 5., Here the velocity magnitude increase with in \(\beta\) and the boundary layer thickness decrease.

The effect of velocity ratio parameter \(C\) on temperature distribution is persevered in fig 6. It is notices that temperature function \(\theta(\eta)\) increase with increase in shrinking of the sheet \((C < 0)\) and decrease with increase in stretching of the sheet \((C > 0)\). Moreover the temperature distribution is greater in magnitude for shrinking case than the stretching case. The impact of heat sink source parameter \(\theta(\eta)\) on the thermal distribution is presented in fig 7.

The increase in heat sink \(Q < 0\) parameter causes decrease in the magnetic of \(\theta(\eta)\) but the increase in the value of heat source parameter \(Q < 0\) causes increase in the magnetic of \(\theta(\eta)\).Fig 8 demonstrate the effect of Prandtl number \(Pr\) on temperature function \(\theta(\eta)\). It is obtained that temperature distribution increases with increase in the
value of Pr. similarity, the fig.9 indicate the effect of radiation parameter Rn on $\theta(\eta)$. The increase in the magnitude of $\theta(\eta)$ as shown in fig.10.

The impact of solutal parameter $\gamma$ on concentration function $\theta(\eta)$ is shown in fig 11. The increasing value of $\gamma$, increase the function $\theta(\eta)$ significantly. But opposite effect is observed for the Schmidt number on $\theta(\eta)$ as plotted in fig 12.

Table 1- Numerical values of $f'(0)$ for different values of $c, P_r, K, M$

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<th>$P_r$</th>
<th>$K$</th>
<th>$M$</th>
<th>Santosh result</th>
<th>Presents result</th>
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Table 2- Numerical values of $\theta'(0)$ for different values of $c, P_r, K, M, E_c$

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<th>$K$</th>
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Fig.1: The plot for curves of $f'$ under the effect of magnetic parameter $M$ when $c=-0.1, P_r=0.5, \text{ and } K=0.1$
Fig. 2: The plot for curves of $f''$ under the effect of porosity $K$ when $c=-0.1$, $Pr=0.5$.

Fig. 3: The plot for curves of $f''$ under the effect of parameter $c$ when $K=0.1$, $M=0.1$, $Pr=0.5$.

Fig. 4: The plot for curves of $f''$ under the effect of parameter $Pr$ when $c=-0.1$, $K=0.1$, $M=0.1$. 
Fig. 5: The plot for curves of $f'$ under the effect of Casson parameter $\beta$ when $c=0.1$, $K=0.1$, $M=0.1$, $Pr=0.5$.

Fig. 6: The plot for curves of $f'$ under the effect of parameter $c$ when $c=0.1$, $K=0.1$, $M=0.1$, $Pr=0.5$.

Fig. 7: The plot for curves of $f'$ under the effect of heat source parameter $Q$ when $c=0.1$, $K=0.1$, $M=0.1$, $Pr=0.5$. 
Fig. 8: The plot for curves of $\theta$ under the effect of Prandtl number $Pr$ when $c=-0.1, M=0.1, E_c=0.1$ and $R_n=0.1$.

Fig. 9: The plot for curves of $\theta$ under the effect of Radiation parameter $R_n$ when $c=-0.1, M=0.1, E_c=0.1$ and $Pr=0.5$.

Fig. 10: The plot for curves of $\theta$ under the effect of Eckert number $E_c$ when $c=-0.1, M=0.1, Pr=0.5$ and $R_n=0.1$. 
Fig. 11: The plot for curves of $\phi$ under the effect of Solutal number $\gamma$ when $c=-0.1, M=0.1, P_r=0.1$ and $R_n=0.1$.

Fig. 12: The plot for curves of $\phi$ under the effect of Schmidt number $S_c$ when $c=-0.1, M=0.1, P_r=0.5$ and $R_n=0.1$.

REFERENCES


